On the Finite Element Representation of Hydrodynamic Bearings

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ABSTRACT

This paper deals with a numerical formulation for the representation of hydrodynamic bearings which is adequate to Finite Element modeling of shaft-rotor-bearing type rotating system. This system dynamic response is significantly affected by the reaction forces on the bearings which are dependent of the shaft transverse displacements, transverse linear velocities and angular velocity. Hydrodynamic bearing relations are, thus, represented through the equation of Reynolds which, under the Ocvirk simplified conditions for short length bearings, provides a closed form solution for the oil pressure distribution. This pressure distribution is integrated over the bearing surface allowing for the calculation of nonlinear stiffness and damping matrices associated to the shaft finite element degrees-of-freedom, at the bearing nodal point.

Plain Hydrodynamic Bearings

Hydrodynamic bearing reaction forces on rotating shafts result from pressure developed by the relative motion of parallel surfaces retaining a thin oil film, see Fig.1. Considering the Reynolds equation for an incompressible Newtonian fluid under laminar flow conditions, with no body forces present but constant viscosity μ, the pressure field $P(\theta,z)$ in the bearing results from

$$\frac{\partial}{\partial\theta}\left(h^3\frac{\partial P}{\partial\theta}\right)+R^2\frac{\partial}{\partial Z}\left(h^3\frac{\partial P}{\partial Z}\right)=12\mu\left(\frac{R}{C_r}\right)^2\left(\frac{\omega}{2}\frac{\partial h}{\partial\theta}+\frac{\partial h}{\partial t}\right) \qquad (1)$$

where ω is the rotor angular velocity, ε is the normalized rotor eccentricity, C_r is the radial clearance, $h(\theta,t)$ is the instant clearance along the bearing circumference and R is the shaft radius.

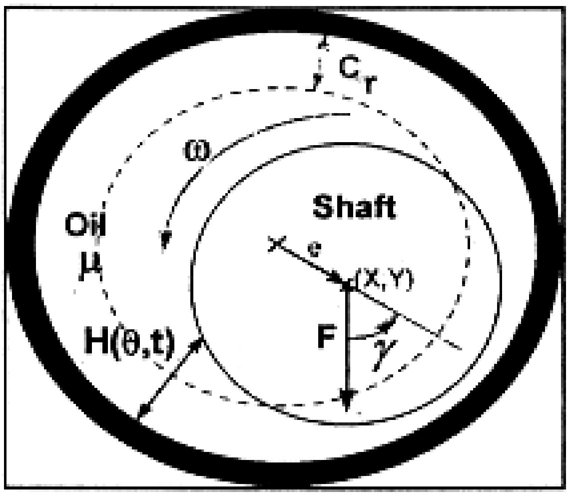


Figure 1 - Plain Hydrodynamic Bearing Geometric Parameters

Bearing reaction forces are obtained by integrating the pressure field, from equation (1), along the shaft surface. A closed form solution may be obtained, only under two simplified conditions (Cardinali [1]) :

a) Long Bearing Condition (Sommerfeld) with pressure gradient in the axial direction being much less than its gradient in the circumferential direction. In this case, the second term on the left of equation (1) is negligible. This condition won't be considered here.

b) Short Bearing Condition (Ocvirk) with pressure gradient in the circumferential direction being much less than its gradient in the axial direction. Thus, the first term on the left of equation (1) is neglected and, under this condition, the Reynolds equation reduces to

$$C_r^2\frac{\partial}{\partial Z}\left(h^3\frac{\partial P}{\partial Z}\right)=12\mu\left(\frac{\omega}{2}\frac{\partial h}{\partial\theta}+\frac{\partial h}{\partial t}\right) \qquad (2)$$

The pressure field is then obtained

$$P(z,\theta)=\frac{3}{4}\mu\omega\frac{L^2}{C_r^2}\frac{(x-2\dot{y})\sin\theta-(y+2\dot{x})\cos\theta}{h^3}(4z^2-1) \qquad (3)$$

with x, y, z, $\dot{x}$ and $\dot{y}$ being defined nondimensional parameters (Meggiolaro, [2]).

The equation (3) provides negative pressure values for $\alpha-\pi < \theta < \alpha$, where

$$\alpha = tg^{-1}\frac{(y+2\dot{x})}{(x-2\dot{y})} - \frac{\pi}{2}\text{sign}\frac{(y+2\dot{x})}{(x-2\dot{y})} - \frac{\pi}{2}\text{sign}(y+2\dot{x}) \tag{4}$$

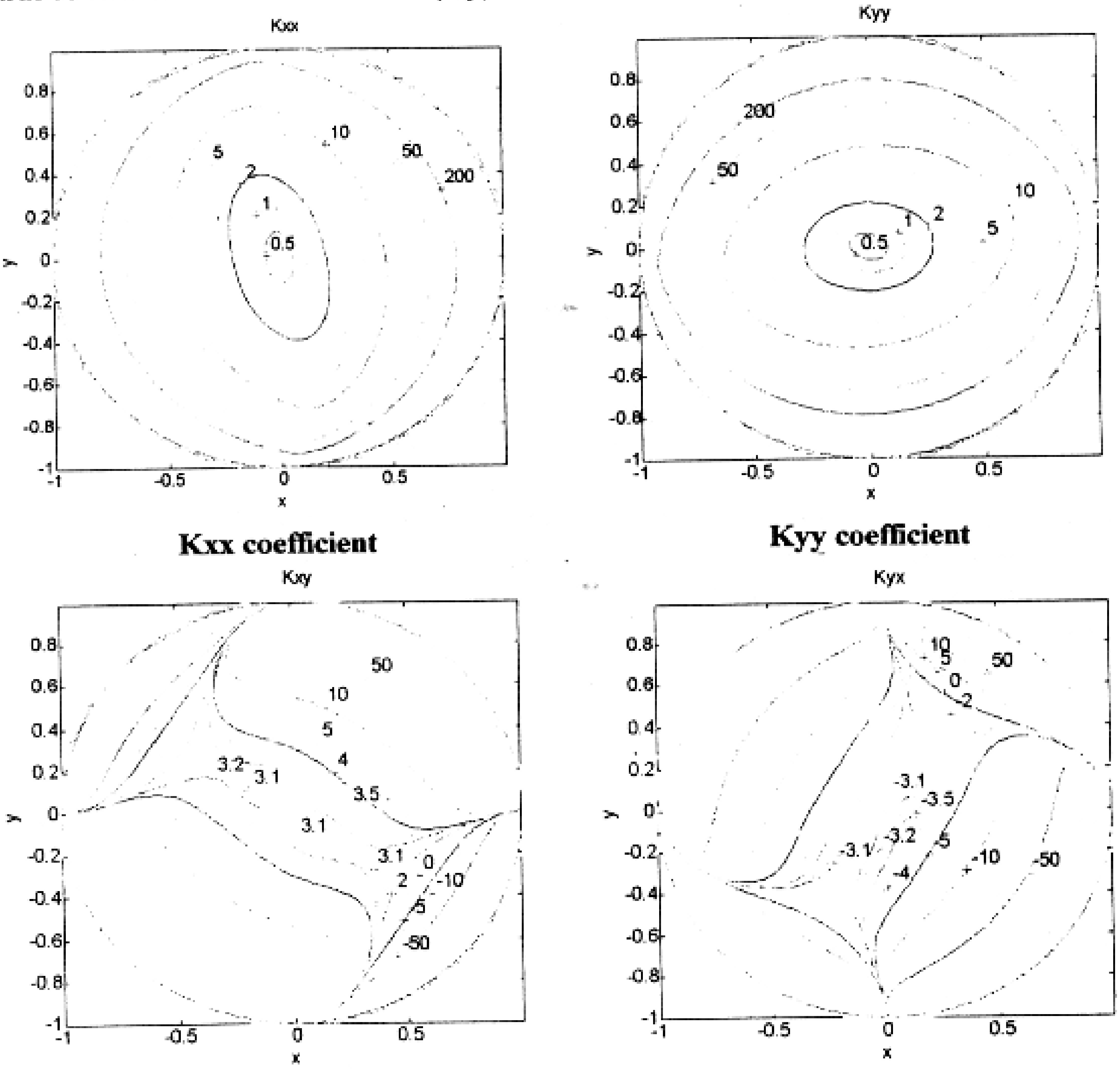
Due to cavitation in the oil film, pressure distribution at this θ-value range is assumed null (Gümbel condition), and reaction forces from the bearings are obtained by integrating the pressure distribution from equation (3) within the following domain : $-0.5<z<0.5$ and $\alpha<\theta<\alpha+\pi$ (Capone, [2])

$$\vec{F} = \begin{Bmatrix} F_X \\ F_Y \end{Bmatrix} = -\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{\alpha}^{\alpha+\pi} P(z,\theta) \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} (R\partial\theta)(L\partial z) \right] = -k\frac{n}{m}\bar{W} \tag{5}$$

The stiffness and damping matrices coefficients are readily obtained by deriving the reaction force components, in Eq.(5), with respect to x , y , $\dot{x}$, $\dot{y}$. Therefore,

$$\begin{pmatrix} K_{XX} \\ K_{YX} \end{pmatrix} = -\frac{\partial\vec{F}}{\partial X}, \begin{pmatrix} K_{XY} \\ K_{YY} \end{pmatrix} = -\frac{\partial\vec{F}}{\partial Y}, \begin{pmatrix} C_{XX} \\ C_{YX} \end{pmatrix} = -\frac{\partial\vec{F}}{\partial\dot{X}}, \begin{pmatrix} C_{XY} \\ C_{YY} \end{pmatrix} = -\frac{\partial\vec{F}}{\partial\dot{Y}} \tag{6}$$

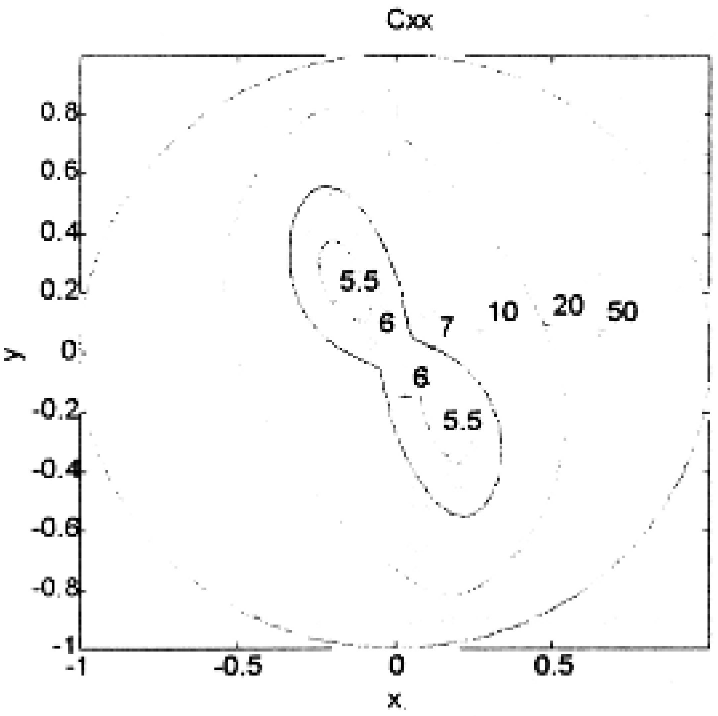
are the stiffness and damping matrices coefficients as function of the bearing dimensions, oil viscosity, rotor angular velocity ω, rotor position (X,Y) and precession velocity ($\dot{X}, \dot{Y}$). Figures 2 and 3 present contour plots for these matrix coefficients as function of the (x,y) coordinates of the rotor, for null precession velocities. In this



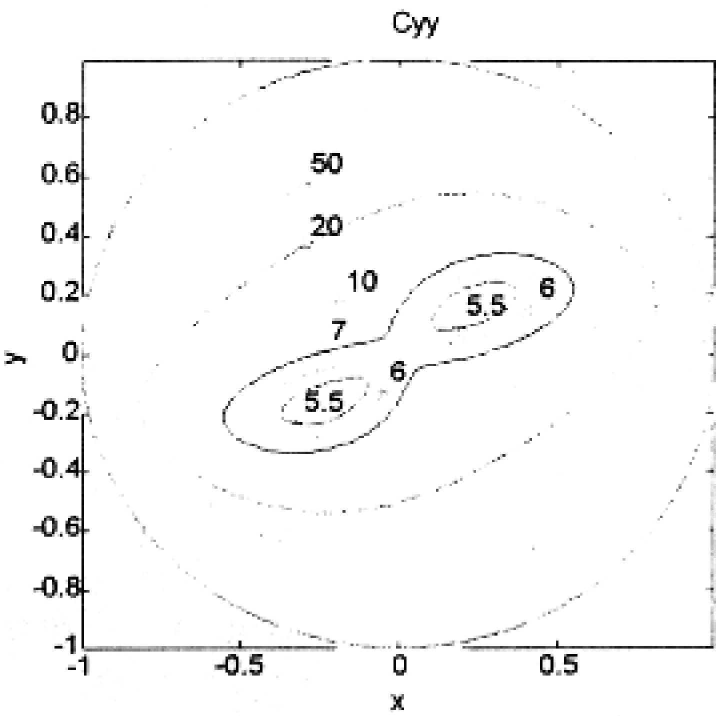
Kxx coefficient **Kyy coefficient**

Kxy coefficient **Kyx coefficient**

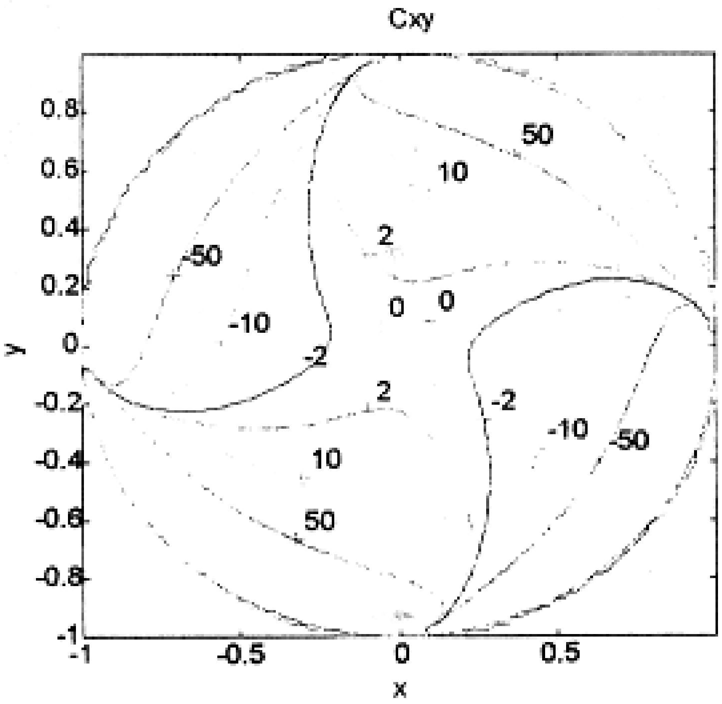
Figure 2 - Contour Plots for Stiffness Matrix Coefficients ($\dot{x} = \dot{y} = 0$)



Cxx coefficient



Cyy coefficient



Cxy coefficient (=Cyx)

Figure 3 - Contour Plots for Damping Matrix Coefficients ($\dot{x} = \dot{y} = 0$)

case, Kxx and Kyy stiffness matrix coefficients imply that stiffness increases for increasing eccentricity. Also, a nonsimmetric stiffness matrix is obtained. On the other hand damping matrix is symmetric with strong coupling coefficients. Some numerical test results are presented to evaluate the above methodology in the dynamic analysis of mass-rotor-bearing systems.

References

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[3] - Capone, G., "Descrizione analitica del campo di forze fluidodinamico nei cuscinetti cilindrici lubrificati", L'Energia Elettrica, n.3, 1991.