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AN ANALYTICAL METHOD TO ELIMINATE THE REDUNDANT PARAMETERS IN ROBOT CALIBRATION

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Abstract

Model based error compensation of a robotic manipulator, also known as robot calibration, requires the identification of its generalized errors. These errors are found from measured data and used to predict, and compensate for, the end-point errors as a function of configuration. However, the generalized error formulation introduces redundant parameters, often non-intuitive, that may compromise the robustness of the calibration. The existing numerical methods to eliminate such errors are formulated on a case-by-case basis. In this paper, the general analytical expressions and physical interpretation of the redundant parameters are developed for any serial link manipulator, expressed through its Denavit-Hartenberg parameters. These expressions are used to eliminate the redundant parameters from the error model of any manipulator prior to the identification process, allowing for systematic robot calibration with improved accuracy. Simulations are conducted to verify the theory presented in the paper.

1 Introduction

Robot calibration is a process to enhance robot position accuracy using compensation software. It involves identifying a more accurate functional relationship between the joint transducer readings and the workspace position of the end-effector [9]. The process requires the identification of the manipulator generalized errors from calibration measurements. Generalized errors characterize the relative position and orientation of frames defined at the manipulator links. These errors are found from measured data and used to predict, and compensate for, the end-point errors as a function of configuration.

Since some generalized errors result in end-effector errors in same direction, it is not possible to distinguish the amount of the error contributed by each generalized error. As a result, only linear combinations of the generalized errors can be identified, independently of the identification method used. All linear combinations must be eliminated from the error model prior to the identification process, otherwise the robustness of the calibration may be compromised [4, 6].

To eliminate redundant error parameters, a number of coordinate system representations have been considered. The four-parameter representations (such as the Denavit-Hartenberg representation) are attractive since they are the minimal parameter set required to locate the reference frames of the joints [9]. Such representation reduces the number of error combinations to be found, however the redundant parameters are not necessarily eliminated. In addition, the Denavit-Hartenberg (D.H.) error representation does not model some of the generalized errors in the presence of parallel joints (see section 3.5). The entire calibration can be compromised if such errors are significant. Also, the D.H. representation is ill-conditioned when neighboring joint axes are nearly parallel. Incorporating Hayati's proposed modification to the D.H. parameterization [2] eliminates the ill-conditioning problem, however it has a singularity when axes are nearly perpendicular [3]. Some authors have proposed a five-parameter representation [5], however this parameterization has a sensitivity problem when neighboring coordinate origins are close together [12].

Many papers have abandoned the D.H. representation of the errors, treating the general case of two coordinate systems related by six parameters [3]. The six-parameter representation of the errors, called generalized error model, does not present the sensitivity problems of the D.H. representation. However, it has the disadvantage of increased redundancy [3]. Numerical methods have been proposed to eliminate redundant errors [1, 10, 11], however up to now they must be formulated in a case-by-case basis [4]. An analytical algorithm has been proposed to eliminate the redundant errors in the D.H. error representation [7], however it cannot be applied to the generalized error formulation.

In this paper the analytical expressions and physical interpretation of the linear combinations of the generalized errors are developed for any serial link manipulator. The six-parameter representation is used to define the errors, and the linear combination coefficients are expressed through the robot's D.H. parameters. The error combinations using the D.H. four-parameter error representation are also derived from the general expressions. A non-singular form of the Identification Jacobian matrix is then obtained using these expressions, allowing for systematic calibration with improved accuracy of any serial link manipulator.

2 Analytical Background

2.1 Model Based Error Compensation

Physical errors, such as machining tolerances, assembly errors and elastic deformation, cause the geometric properties of a manipulator to be different from their ideal values. As a result, the frames defined at the manipulator joints are slightly displaced from their expected, ideal locations, creating significant end-effector errors. As seen in Figure 1, the position and orientation of a frame F_i^{real} with respect to its ideal location F_i^{ideal} is represented by a 4x4 homogeneous matrix $\mathbf{E}_i$. The translational part of matrix $\mathbf{E}_i$ is composed of 3 coordinates ε_{i_1}, ε_{i_2} and ε_{i_3} (along the X, Y and Z axes respectively, defined using the Denavit-Hartenberg representation). The rotation part of matrix $\mathbf{E}_i$ is the result of the product of three consecutive rotations ε_{i_4}, ε_{i_5} and ε_{i_6} around the Y, Z and X axes respectively (see Figure 2). These are the Euler angles of F_i^{real} with respect to F_i^{ideal}, representing spin (yaw), roll, and pitch, respectively. The 6 parameters ε_{i_1}, ε_{i_2}, ε_{i_3}, ε_{i_4}, ε_{i_5} and ε_{i_6} are called generalized error parameters, which can be a function of the system geometry and joint variables. For an n^{th} degree of freedom manipulator, there are 6(n+1) generalized errors which can be written in vector form as $\varepsilon = [..., \varepsilon_{i_1}, \varepsilon_{i_2}, \varepsilon_{i_3}, \varepsilon_{i_4}, \varepsilon_{i_5}, \varepsilon_{i_6}, ...]$, with i ranging from 0 to n. Here it is assumed that both the manipulator and the location of its base are being calibrated. If only the manipulator is being calibrated, then the error matrix $\mathbf{E_0}$ (which models the generalized errors of the robot base location) is not considered, reducing the number of generalized errors to 6n.

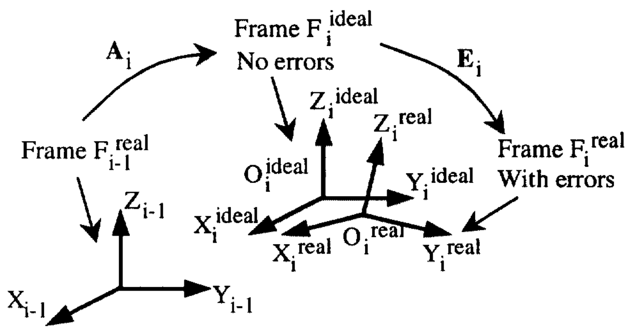

Figure 1 – Frame Translation and Rotation Due to Errors for i^{th} Link

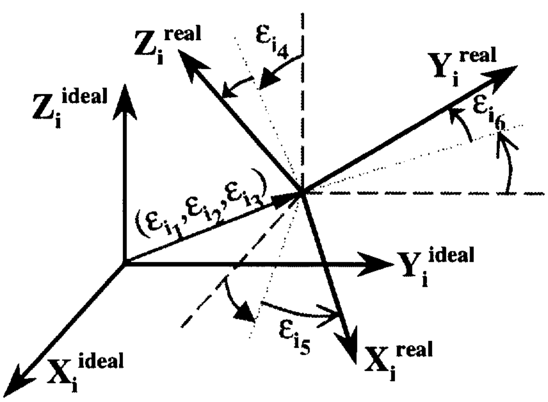

Figure 2 – Definition of the Translational and Rotational Generalized Errors for i^{th} Link

The end-effector position and orientation error $\Delta\mathbf{X}$ is defined as the 6x1 vector that represents the difference between the real position and orientation of the end-effector and the ideal or desired one:

$$\Delta\mathbf{X} = \mathbf{X}^{real} - \mathbf{X}^{ideal} \tag{1}$$

where $\mathbf{X}^{real}$ and $\mathbf{X}^{ideal}$ are the 6x1 vectors composed of the three positions and three orientations of the end-effector reference frame in the inertial reference system for the real and ideal cases, respectively. Since the generalized errors are small, $\Delta\mathbf{X}$ can be calculated by the following linear equation in ε:

$$\Delta\mathbf{X} = \mathbf{J}_e\, \varepsilon \tag{2}$$

where $\mathbf{J}_e$ is the 6x6(n+1) Jacobian matrix of the end-effector error $\Delta\mathbf{X}$ with respect to the elements of the generalized error vector ε, also known as Identification Jacobian matrix. Also, a first order approximation can then be applied to the trigonometric functions and products of the generalized errors. After the first order approximation, matrix $\mathbf{E}_i$ has the form:

$$\mathbf{E}_i = \begin{bmatrix} 1 & -\varepsilon_{i_5} & \varepsilon_{i_4} & \varepsilon_{i_1} \\ \varepsilon_{i_5} & 1 & -\varepsilon_{i_6} & \varepsilon_{i_2} \\ -\varepsilon_{i_4} & \varepsilon_{i_6} & 1 & \varepsilon_{i_3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

If the generalized errors, ε, are known then the end-effector position and orientation error can be calculated using Equation (2). The method to obtain ε is explained in the next section.

2.2 Identification of the Generalized Errors

The identification method to calculate ε is based on the assumption that some components of vector $\Delta\mathbf{X}$ can be measured at a finite number of different manipulator configurations. However, since position coordinates are much easier to measure in practice than orientations, in many cases only the three position coordinates of $\Delta\mathbf{X}$ are measured. In this case, twice the number of measurements is required to obtain the same calibration accuracy.

Assuming that all 6 components of $\Delta\mathbf{X}$ can be measured, for an n degree of freedom manipulator, its 6(n+1) generalized errors ε can be calculated by measuring $\Delta\mathbf{X}$ at m different configurations, defined as $\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_m$, and then writing Equation (2) m times:

$$\Delta\mathbf{X}_t = \begin{bmatrix} \Delta\mathbf{X}_1 \\ \Delta\mathbf{X}_2 \\ \ldots \\ \Delta\mathbf{X}_m \end{bmatrix} = \begin{bmatrix} \mathbf{J}_e(\mathbf{q}_1) \\ \mathbf{J}_e(\mathbf{q}_2) \\ \ldots \\ \mathbf{J}_e(\mathbf{q}_m) \end{bmatrix} \cdot \varepsilon = \mathbf{J}_t \cdot \varepsilon \tag{4}$$

where $\Delta\mathbf{X}_t$ is the m x 1 vector formed by all measured vectors $\Delta\mathbf{X}$ at the m different configurations and $\mathbf{J}_t$ is the 6m x 6(n+1) Total Identification Jacobian matrix formed by the m Identification Jacobian matrices $\mathbf{J}_e$ at the m configurations. To compensate for the effects of measurement noise, the number of measurements m is in general much larger than n.

If the generalized errors ε are constant, then a unique least-squares estimate $\hat{\varepsilon}$ can be calculated by:

$$\hat{\varepsilon} = \left(\mathbf{J}_t^T \mathbf{J}_t\right)^{-1} \mathbf{J}_t^T \cdot \Delta \mathbf{X}_t \quad (5)$$

If the Identification Jacobian matrix $\mathbf{J}_e$ contains linear dependent columns, then Equation (5) can give estimates with poor accuracy due to singularities. This occurs if there is redundancy in the error model, which is always true for the six-parameter representation. In this case, it is not possible to distinguish the amount of the error contributed by each generalized error ε_{i_j} by measuring the end-effector position and orientation, and only linear combinations of ε_{i_j} can be obtained. If only the position coordinates of vector $\Delta\mathbf{X}$ are measured, then additional linear combinations may be present.

If the columns of $\mathbf{J}_e$ are reduced to a linear independent set, then the non-singular form of the Identification Jacobian matrix, called $\mathbf{G}_e$, is obtained. In this case, the generalized errors must be grouped into a smaller independent set, in accordance with the columns of the submatrix $\mathbf{G}_e$. If $\mathbf{J}_e$ is replaced by its submatrix $\mathbf{G}_e$ in Equation (4), then Equation (5) can be applied resulting in estimates with improved accuracy. The next section shows how to obtain the non-singular Identification Jacobian matrix $\mathbf{G}_e$.

3 Eliminating the Redundant Errors

To perform robot calibration with improved accuracy, redundant errors must be eliminated from the error model prior to the identification process. Section 3.1 presents the linear combinations of the columns of the Identification Jacobian matrix and the method to obtain the non-singular Identification Jacobian matrix. Section 3.2 discusses the physical interpretation of each linear combination. Section 3.3 presents the additional linear combinations introduced when only the end-effector position is measured. Section 3.4 shows the number of independent error parameters for any serial link manipulator. Section 3.5 extends the results obtained using the six-parameter representation to the Denavit-Hartenberg error parameterization. It also shows that the D.H. representation of the errors does not model some of the generalized errors in the presence of parallel joints, which can affect the identification process.

3.1 Linear Combinations of the Columns of the Identification Jacobian matrix

In this section, the linear combinations of the columns of the Identification Jacobian matrix $\mathbf{J}_e$ are presented. Defining $\mathbf{J}_{i_j}$ as the column of $\mathbf{J}_e$ associated to the generalized error ε_{i_j}, then Equation (2) is rewritten as

$$\Delta\mathbf{X} = [\mathbf{J}_{1_1}, \ldots, \mathbf{J}_{i_1}, \mathbf{J}_{i_2}, \mathbf{J}_{i_3}, \mathbf{J}_{i_4}, \mathbf{J}_{i_5}, \mathbf{J}_{i_6}, \ldots, \mathbf{J}_{n_6}] \cdot [\varepsilon_{1_1}, \ldots, \varepsilon_{i_1}, \varepsilon_{i_2}, \varepsilon_{i_3}, \varepsilon_{i_4}, \varepsilon_{i_5}, \varepsilon_{i_6}, \ldots, \varepsilon_{n_6}]^T \quad (6)$$

For each link i ($1 \le i \le n$), the following linear combinations are always valid (see Appendix A for proof):

$$\mathbf{J}_{(i-1)_3} \equiv \sin\alpha_i \mathbf{J}_{i_2} + \cos\alpha_i \mathbf{J}_{i_3} \quad (7)$$

$$\mathbf{J}_{(i-1)_5} \equiv a_i \cos\alpha_i \mathbf{J}_{i_2} - a_i \sin\alpha_i \mathbf{J}_{i_3} + \sin\alpha_i \mathbf{J}_{i_4} + \cos\alpha_i \mathbf{J}_{i_5} \quad (8)$$

where the manipulator parameters are defined using the D.H. representation: link lengths a_i, joint offsets d_i, joint angles θ_i, and skew angles α_i. If joint i is prismatic, then additional combinations of the columns of $\mathbf{J}_e$ are found:

$$\mathbf{J}_{(i-1)_1} \equiv \mathbf{J}_{i_1} \quad (9)$$

$$\mathbf{J}_{(i-1)_2} \equiv \cos\alpha_i \mathbf{J}_{i_2} - \sin\alpha_i \mathbf{J}_{i_3} \quad (10)$$

The linear combinations shown above are always present, independently of the values of a_i and α_i, even for degenerate cases (such as a_i=0). As shown in Appendix A, if the full pose of the end-effector (both position and orientation) is measured, then Equations (7-10) are the only linear combinations for link i.

To obtain the non-singular Identification Jacobian matrix $\mathbf{G}_e$, columns $\mathbf{J}_{(i-1)_3}$ and $\mathbf{J}_{(i-1)_5}$ must be eliminated from the matrix $\mathbf{J}_e$ for all values of i ($1 \le i \le n$). Columns $\mathbf{J}_{(i-1)_1}$ and $\mathbf{J}_{(i-1)_2}$ must also be eliminated if and only if joint i is prismatic. For an n-d.o.f. manipulator with r rotary joints and p (=n-r) prismatic joints, 2r+4p columns are eliminated from the Identification Jacobian $\mathbf{J}_e$ to form its submatrix $\mathbf{G}_e$. This means that 2r+4p generalized errors are not obtainable by measuring the end-effector pose.

If the vector ε^* containing the independent errors is constant, then the matrix $\mathbf{G}_e$ can be used to replace $\mathbf{J}_e$ in Equation (4), and Equation (5) is applied to calculate the estimate of the independent generalized errors ε^*, completing the identification process. Note that the independent errors ε^* are a subset of the generalized errors ε. However, if non-geometric factors are considered (e.g. link compliance, gear eccentricity), then it is necessary to further model the parameters of ε^* as a function of the system configuration prior to the identification process [8].

By definition, non-obtainable parameters do not affect the end-effector error, resulting in the identity

$$\Delta\mathbf{X} = \mathbf{J}_e\, \varepsilon \equiv \mathbf{G}_e\, \varepsilon^* \quad (11)$$

Using the above identity and the linear combinations of the columns of $\mathbf{J}_e$ from Equations (7-10), it is possible to obtain all relationships between the generalized error set ε and its independent subset ε^*. These expressions are presented in Appendix A.

3.2 Physical Interpretation of the Combinations

In this section the physical interpretation of Equations (7-10) is presented. Each equation associates a generalized error from link i-1 with a combination of errors from link i that result in end-effector errors in same magnitude and direction. Since it is not possible to distinguish the amount of the error contributed by each generalized error, these errors associated with link i-1 are indistinguishable.

Equation (7) reflects the fact that the translational error along the Z-axis of frame i-1 has the same effect as a combination of the translational errors along the Y and Z axes of frame i (see Figure 3). This relation is easily explained by the fact that the skew angle α_i between the axes of joints i-1 and i is constant.

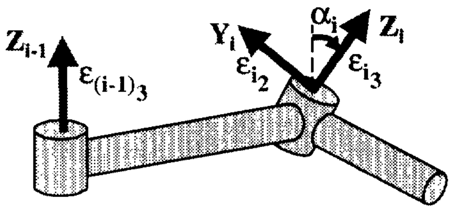

Figure 3 – Linear Combination of Translational Generalized Errors

Equation (8) states that the rotational error along the Z-axis of frame i-1 has the same effect as a combination of the rotational and translational errors along the Y and Z axes of frame i. For simplicity, a planar manipulator is used to explain this combination (see Figure 4). The top figure shows the end-effector translational and rotational errors ΔX_t and ΔX_r caused by the rotational generalized error $\varepsilon_{(i-1)_5}$ of frame i-1. The bottom figure shows that the same end-effector errors can be reproduced by a specific combination of the translational error ε_{i_2} and the rotational error ε_{i_5} of frame i. To obtain the same end-effector errors, it is required that $\varepsilon_{i_2} = \varepsilon_{(i-1)_5} \cdot a_i$ and $\varepsilon_{i_5} = \varepsilon_{(i-1)_5}$ in this case (see the relationship between ε^* and ε in Appendix A).

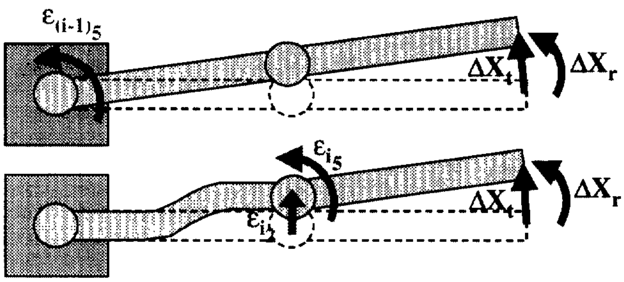

Figure 4 – Error Combinations Resulting in Same End-Effector Errors

If joint i is prismatic, then Equations (9-10) are also valid. These combinations simply state that the effects of the generalized errors along the X and Y axes of frame i-1 can always be reproduced by a combination of the three translational generalized errors of frame i (see Figure 5). This is always true for prismatic joints, since such joints only move along the Z-axis of frame i-1 (using the D.H. frame definition).

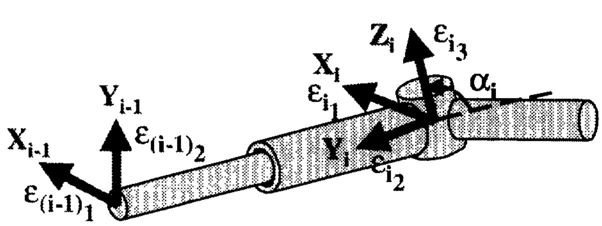

Figure 5 – Linear Combinations of Generalized Errors in Prismatic Joints

3.3 Partial Measurement of End-Effector Pose

The linear combinations of the columns of the Identification Jacobian matrix $\mathbf{J}_e$ shown in Equations (7-10) are obtained when both position and orientation of the end-effector are considered. In the case where only the end-effector position is measured, its orientation can take any value, resulting in additional linear combinations. In this case, the three last columns of $\mathbf{J}_e$ are zero vectors (see Appendix A):

$$\mathbf{J}_{n_4} \equiv \mathbf{J}_{n_5} \equiv \mathbf{J}_{n_6} \equiv \mathbf{0} \quad (12)$$

Equation (12) means, as expected, that the three rotational errors of the end-effector frame ε_{n_4}, ε_{n_5} and ε_{n_6} do not influence the end-effector position (they only affect the orientation, which is not being measured). As a result, these generalized errors are not obtainable.

If the last joint is prismatic, then no further linear combinations are found. However, if the last joint is revolute and its link length a_n is zero, then three more linear combinations are present (see Appendix A):

$$\mathbf{J}_{n-1_4} \equiv d_n \mathbf{J}_{n-1_1},\ \mathbf{J}_{n-1_6} \equiv -d_n \mathbf{J}_{n-1_2},\ \mathbf{J}_{n-1_5} \equiv 0 \quad (13)$$

meaning that the effects of ε_{n-1_4} and ε_{n-1_6} cannot be distinguished from the ones caused by ε_{n-1_1} and ε_{n-1_2}, and also the generalized error ε_{n-1_5} is not obtainable. If both link length a_n and joint offset d_n are zero, then the origin of frames n-1 and n coincide at the end-effector position. In this case, Equations (12-13) can be recursively applied to frames n-1, n-2, and so on, as long as the origin of these frames all lie at the end-effector position. See Appendix A for more details.

3.4 Number of Independent Generalized Errors

As a corollary of Equations (7-13), the number of independent generalized errors for a generic serial link manipulator can be calculated. Upper bounds of this number have been presented in the literature [1, 11], but not its exact value. Table 1 shows the number of generalized errors N, the number of linear dependencies D, and the number of independent generalized errors I (=N–D) for both robot calibration (without modeling its base frame errors) and robot+base location calibration.

Table 1 – Number of Independent Generalized Errors

	Robot+Base Calibration	Robot Calibration
N	6(n+1)	6n
D	2r + 4p + k	2r' + 4p' + k
I	6(n+1) – (2r + 4p + k)	6n – (2r' + 4p' + k)

where

n : # of joints in the manipulator
r ; r' : # of revolute joints including/excluding joint 1
p ; p': # of prismatic joints including/excluding joint 1

$$k:\begin{cases}0 \text{ if measuring end - effector position and orientation} \\ 3 \text{ if only measuring end - effector position and either} \\ \quad \text{the last joint is prismatic or } a_n \neq 0 \\ 3+2q \text{ if only measuring end - effector position,} \\ \quad \text{the last q joints are revolute, and} \\ \quad a_{n-q+1} \equiv a_{n-q+2} \equiv a_{n-q+3} \equiv \ldots \equiv a_n \equiv 0 \\ \quad \text{and } d_{n-q+2} \equiv d_{n-q+3} \equiv \ldots \equiv d_n \equiv 0\end{cases}$$

3.5 Extension to Four-Parameter Error Representations

The six-parameter representation of the errors is used above to obtain all linear combinations of the generalized errors. If a four-parameter representation is chosen for the identification process, the previous results can still be applied to eliminate the redundant parameters, through an adaptation of Equations (7-10). The extension of the results to four-parameter error representations is easily accomplished because such parameterizations are a subset of the generalized error representation.

The four Denavit-Hartenberg error parameters of link i are exactly the rotational and translational errors along the Z-axis of frame i-1 and the X-axis of frame i of the six-parameter representation. Namely, the errors along the link lengths a_i, skew angles α_i, joint offsets d_i, and joint angle offsets θ_i of link i are respectively mapped to the generalized errors ε_{i_1}, ε_{i_6}, $\varepsilon_{(i-1)_3}$, and $\varepsilon_{(i-1)_5}$. This implies that the translational and rotational errors along the Y-axis of every frame, ε_{i_2} and ε_{i_4}, are not modeled when the D.H. representation is used. If the manipulator does not have parallel joints, then ε_{i_2} and ε_{i_4} can be replaced by a combination of the errors along d_i and θ_i, see Equation (30). However, if joints i-1 and i are parallel, then the rotation error ε_{i_4} cannot be obtained from the D.H. representation. In addition, if the link length a_i=0, then the translation error ε_{i_2} is also non-obtainable. This means that the entire calibration can be compromised if the manipulator has significant errors in those directions. Hayati's modification of the D.H. representation [2] only partially solves this problem, because it introduces an angular alignment parameter that models the Y-axis rotation error ε_{i_4}, but not the translation error ε_{i_2}.

When using the D.H. error representation, the column vectors $\mathbf{J}_{i_2}$ and $\mathbf{J}_{i_4}$ are not present in the Identification Jacobian, because the generalized errors ε_{i_2} and ε_{i_4} are not modeled. So, Equations (7-10) can only be applied to the D.H. representation if both $\mathbf{J}_{i_2}$ and $\mathbf{J}_{i_4}$ are not present in the linear combination. This is never true for Equations (8) and (10), but Equation (7) can be applied if $\sin(\alpha_i)$ is null, due to parallel joints. In this case, the following linear combinations of the D.H. error parameters are valid:

$$\begin{cases}\varepsilon^*_{i_3} = \varepsilon_{i_3} + \varepsilon_{i-1_3} \\ \varepsilon^*_{i_5} = \varepsilon_{i_5} + \varepsilon_{i-1_5}\end{cases} \Rightarrow \begin{cases}\delta d^*_{i+1} = \delta d_{i+1} + \delta d_i \\ \delta\theta^*_{i+1} = \delta\theta_{i+1} + \delta\theta_i\end{cases} \quad (14)$$

Finally, if joint i is prismatic, then Equation (9) results in one additional linear combination

$$\varepsilon^*_{i_1} = \varepsilon_{i_1} + \varepsilon_{i-1_1} \Rightarrow \delta a^*_i = \delta a_i + \delta a_{i-1} \quad (15)$$

As seen in Equations (14-15), even though the D.H. representation results in fewer linear combinations, redundant parameters may still be present.

In summary, the D.H. error representation does not model some of the physical errors if parallel joints are present and still presents linear combinations that need to be eliminated. Since the redundancy of the six-parameter error representation can be eliminated with the method described in this paper, the use of such parameterization in robot calibration is recommended.

4 Simulation Results

Simulations were performed on a PUMA 560 and on an Adept SCARA manipulator. The six-parameter error representation was used and its redundant parameters were eliminated using Equations (7-10). Simulated measurements were obtained and the introduced error parameters were identified using Equation (5).

For the calibration of a PUMA 560 robot and its base by measuring the end-effector position only, 27 error parameters were identified. This result agrees with [6] and also with Table 1 (using n=6, r=6, p=0 and k=3).

For an Adept SCARA robot, 20 error parameters were identified by measuring both end-effector position and orientation, which agrees with Table 1 (using n=4, r=3, p=1 and k=0). Although the D.H. error representation also models 20 parameters in this case, only 15 of these parameters are independent and identifiable. The error parameters ε_{1_4}, ε_{2_4}, ε_{3_4}, ε_{4_2} and ε_{4_4} cannot be identified using the D.H. representation due to parallel joints in the system. Even if Hayati's modification [2] is introduced, the translation error ε_{4_2} still remains unmodeled, showing that only the six-parameter representation can identify all 20 parameters in this case.

5 Conclusions

This paper presents a general analytical method to eliminate the redundant error parameters in robot calibration. These errors, often non-intuitive, must be eliminated from the error model prior to the identification process, otherwise the robustness of the calibration can be compromised. The analytical expressions and physical interpretation of the linear combinations present in the generalized error parameterization are developed. The non-redundant form of the Identification Jacobian matrix is then obtained using these expressions, allowing for the systematic calibration with improved accuracy of any serial link manipulator.

Acknowledgments

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Appendix A – Linear Dependency Calculations

This appendix contains the proof of the linear combination expressions of the columns of the Identification Jacobian matrix $\mathbf{J}_e$. These combinations are obtained from the symbolic form of $\mathbf{J}_e$, expressed through the manipulator's Denavit-Hartenberg parameters. It is shown that the general expressions can be broken down into combinations of the columns associated with each pair of consecutive links. This in turn allows for great simplification of the proof.

Define the position and orientation of a reference frame F_i with respect to the previous reference frame F_{i-1} as a 4x4 matrix $\mathbf{A}_i$ using the D.H. parameters:

$$\mathbf{A}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

When the generalized errors defined in section 2.1 are considered in the model, the manipulator loop closure equation takes the form:

$$\mathbf{A}_{LC} = \mathbf{E}_0\mathbf{A}_1\mathbf{E}_1\mathbf{A}_2\mathbf{E}_2\ldots\ldots\mathbf{A}_n\mathbf{E}_n \quad (17)$$

where $\mathbf{A}_{LC}$ is a 4x4 homogeneous matrix that describes the position and orientation of the end-effector frame F_n with respect to the inertial reference frame F_0.

The Identification Jacobian matrix $\mathbf{J}_e$ is determined by taking the derivative of the loop closure matrix $\mathbf{A}_{LC}$ with respect to each generalized error ε_{i_j}, resulting in 4x4 sensitivity matrices $\mathbf{L}_{i_j}$

$$\mathbf{L}_{i_j} \equiv \left.\frac{\partial\left(\mathbf{E}_0\mathbf{A}_1\mathbf{E}_1\mathbf{A}_2\mathbf{E}_2\cdots\mathbf{A}_n\mathbf{E}_n\right)}{\partial\varepsilon_{i_j}}\right|_{\varepsilon_{k_m}=0,\, k\neq i \text{ or } m\neq j} \quad (18)$$

Clearly, the linear combinations of the columns $\mathbf{J}_{i_j}$ of the Identification Jacobian matrix (defined in section 3.1) are the same as the ones of the $\mathbf{L}_{i_j}$ matrices. Since $\mathbf{E}_i$ is the only matrix that depends on the generalized error ε_{i_j}, then $\mathbf{L}_{i_j}$ can be simplified to

$$\mathbf{L}_{i_j} = \mathbf{A}_1\cdots\mathbf{A}_{i-2}\cdot\mathbf{M}_{i_j}\cdot\mathbf{A}_{i+1}\cdots\mathbf{A}_n,\quad \mathbf{M}_{i_j} \equiv \mathbf{A}_{i-1}\mathbf{A}_i\frac{\partial\mathbf{E}_i}{\partial\varepsilon_{i_j}} \quad (19)$$

Applying the same result for joint i-1, then

$$\mathbf{L}_{(i-1)_j} = \mathbf{A}_1\cdots\mathbf{A}_{i-2}\cdot\mathbf{M}_{(i-1)_j}\cdot\mathbf{A}_{i+1}\cdots\mathbf{A}_n,\quad \mathbf{M}_{(i-1)_j} \equiv \mathbf{A}_{i-1}\frac{\partial\mathbf{E}_{i-1}}{\partial\varepsilon_{(i-1)_j}}\mathbf{A}_i \quad (20)$$

Since both products $\mathbf{A}_1\ldots\mathbf{A}_{i-2}$ and $\mathbf{A}_{i+1}\ldots\mathbf{A}_n$ do not depend on the coordinates of frames i-1 and i, Equations (19-20) show that the linear combinations of the $\mathbf{L}_{i_j}$ matrices are also the same as the ones of the $\mathbf{M}_{i_j}$ matrices. Hence, it is not necessary to develop the lengthy Equations (18-20), since the desired linear combinations can be obtained from the much simpler $\mathbf{M}_{i_j}$ matrices. In addition, all linear combination expressions can then be broken down into expressions involving the generalized errors of each two consecutive links, since the considered system is a serial link manipulator.

Consider the following general expression for the linear combinations associated with two consecutive links, represented by the unknown coefficients $c_1,\ldots,c_{12}$:

$$c_1\mathbf{J}_{i-1_1} + c_2\mathbf{J}_{i-1_2} + c_3\mathbf{J}_{i-1_3} + c_4\mathbf{J}_{i-1_4} + c_5\mathbf{J}_{i-1_5} + c_6\mathbf{J}_{i-1_6} \equiv c_7\mathbf{J}_{i_1} + c_8\mathbf{J}_{i_2} + c_9\mathbf{J}_{i_3} + c_{10}\mathbf{J}_{i_4} + c_{11}\mathbf{J}_{i_5} + c_{12}\mathbf{J}_{i_6} \quad (21)$$

To determine the unknown coefficients, the column vectors $\mathbf{J}_{i_j}$ can be replaced by the matrices $\mathbf{M}_{i_j}$ in the above expression, since it was shown that both $\mathbf{J}_{i_j}$ and $\mathbf{M}_{i_j}$ sets have the same linear combinations. The $\mathbf{M}_{i_j}$ matrices are obtained by substituting Equations (3) and (16) into Equations (19-20). Equation (21) then results in

$$\begin{cases} c_4(\mathbf{w}_i\cos\theta_i) + c_5(\mathbf{v}_i\cos\theta_i - \mathbf{u}_i\sin\theta_i) + c_6(\mathbf{w}_i\sin\theta_i) \\ \quad + c_{10}(\mathbf{u}_i\sin\theta_i\sin\alpha_i - \mathbf{v}_i\cos\theta_i\sin\alpha_i + \mathbf{w}_i\cos\alpha_i) \\ \quad - c_{11}(-\mathbf{u}_i\sin\theta_i\cos\alpha_i + \mathbf{v}_i\cos\theta_i\cos\alpha_i + \mathbf{w}_i\sin\alpha_i) \equiv 0 \\ c_4(-\mathbf{w}_i\sin\theta_i\cos\alpha_i + \mathbf{u}_i\sin\alpha_i) - c_5\cos\alpha_i(\mathbf{v}_i\sin\theta_i + \mathbf{u}_i\cos\theta_i) \\ \quad + c_6(\mathbf{w}_i\cos\theta_i\cos\alpha_i - \mathbf{v}_i\sin\alpha_i) + c_{11}(\mathbf{u}_i\cos\theta_i + \mathbf{v}_i\sin\theta_i) \\ \quad + c_{12}(\mathbf{u}_i\sin\theta_i\sin\alpha_i - \mathbf{v}_i\cos\theta_i\sin\alpha_i + \mathbf{w}_i\cos\alpha_i) \equiv 0 \\ c_4(\mathbf{w}_i\sin\theta_i\sin\alpha_i + \mathbf{u}_i\cos\alpha_i) + c_5\sin\alpha_i(\mathbf{v}_i\sin\theta_i + \mathbf{u}_i\cos\theta_i) \\ \quad - c_6(\mathbf{w}_i\cos\theta_i\sin\alpha_i - \mathbf{v}_i\cos\alpha_i) - c_{10}(\mathbf{u}_i\cos\theta_i + \mathbf{v}_i\sin\theta_i) \\ \quad + c_{12}(-\mathbf{u}_i\sin\theta_i\cos\alpha_i + \mathbf{v}_i\cos\theta_i\cos\alpha_i + \mathbf{w}_i\sin\alpha_i) \equiv 0 \\ c_1\mathbf{u}_i + c_2\mathbf{v}_i + c_3\mathbf{w}_i + c_4(\mathbf{w}_i a_i\cos\theta_i + \mathbf{u}_i d_i) \\ \quad + c_5(\mathbf{v}_i a_i\cos\theta_i - \mathbf{u}_i a_i\sin\theta_i) \\ \quad + c_6(\mathbf{w}_i a_i\sin\theta_i - \mathbf{v}_i d_i) - c_7(\mathbf{u}_i\cos\theta_i + \mathbf{v}_i\sin\theta_i) \\ \quad - c_8(-\mathbf{u}_i\sin\theta_i\cos\alpha_i + \mathbf{v}_i\cos\theta_i\cos\alpha_i + \mathbf{w}_i\sin\alpha_i) \\ \quad - c_9(\mathbf{u}_i\sin\theta_i\sin\alpha_i - \mathbf{v}_i\cos\theta_i\sin\alpha_i + \mathbf{w}_i\cos\alpha_i) \equiv 0 \end{cases} \quad (22)$$

where

$$\mathbf{u}_i \equiv \begin{bmatrix} \cos\theta_{i-1} \\ \sin\theta_{i-1} \\ 0 \\ 0 \end{bmatrix},\quad \mathbf{v}_i \equiv \begin{bmatrix} -\sin\theta_{i-1}\cos\alpha_{i-1} \\ \cos\theta_{i-1}\cos\alpha_{i-1} \\ \sin\alpha_{i-1} \\ 0 \end{bmatrix},\quad \mathbf{w}_i \equiv \begin{bmatrix} \sin\theta_{i-1}\sin\alpha_{i-1} \\ -\cos\theta_{i-1}\sin\alpha_{i-1} \\ \cos\alpha_{i-1} \\ 0 \end{bmatrix} \quad (23)$$

Solving Equation (22) results in

$$c_4 = c_6 = c_{12} = 0,\ c_1 = c_7,\ c_{10} = c_5\sin\alpha_i,\ c_{11} = c_5\cos\alpha_i,$$
$$c_8 = c_3\sin\alpha_i + (c_2 + c_5 a_i)\cdot\cos\alpha_i,$$
$$c_9 = c_3\cos\alpha_i - (c_2 + c_5 a_i)\cdot\sin\alpha_i \quad (24)$$

Then, substituting Equation (24) in (22) results in Equations (7-8). If joint i is revolute, then $c_1 = c_2 = c_7 = 0$, and no other combinations are present. However, if joint i is prismatic, then Equation (24) also results in Equations (9-10).

In the particular case where only the end-effector position is measured, only the last equation in (22) must hold, resulting in

$$c_1 = c_4 = c_6 = c_7, \quad c_8 = c_3 \sin\alpha_n + (c_2 + c_5 \cdot a_n) \cdot \cos\alpha_n,$$
$$c_9 = c_3 \cos\alpha_n - (c_2 + c_5 \cdot a_n) \cdot \sin\alpha_n \tag{25}$$

Substituting Equation (25) in (21) results in

$$c_3 \mathbf{J}_{n-1_3} + c_5 \mathbf{J}_{n-1_5} \equiv (c_3 \sin\alpha_n + c_5 \cdot a_n \cos\alpha_n)\mathbf{J}_{n_2} +$$
$$(c_3 \cos\alpha_n - c_5 \cdot a_n \sin\alpha_n)\mathbf{J}_{n_3} + c_{10}\mathbf{J}_{n_4} + c_{11}\mathbf{J}_{n_5} + c_{12}\mathbf{J}_{n_6} \tag{26}$$

Equation (26) must hold for all values of c_3, c_5, c_{10}, c_{11}, c_{12}, resulting in Equation (12).

If joint n is prismatic, then c_4 and c_6 are always zero (even if $a_n \equiv 0$) and no other combinations are present. However, if joint n is revolute and the link length a_n is zero, then c_4 and c_6 are different than zero, and two other linear combinations are present, namely

$$\begin{cases} c_1 + c_4 \cdot d_n = 0 \\ c_2 - c_6 \cdot d_n = 0 \end{cases} \Rightarrow \begin{cases} (-c_4 \cdot d_n)\mathbf{J}_{n-1_1} + c_4 \mathbf{J}_{n-1_4} = 0 \\ (c_6 \cdot d_n)\mathbf{J}_{n-1_2} + c_6 \mathbf{J}_{n-1_6} = 0 \end{cases} \Rightarrow \begin{cases} \mathbf{J}_{n-1_4} = d_n \mathbf{J}_{n-1_1} \\ \mathbf{J}_{n-1_6} = -d_n \mathbf{J}_{n-1_2} \end{cases} \tag{27}$$

Equations (8) and (12) also imply that $\mathbf{J}_{(n-1)_5} \equiv \mathbf{0}$ for this case. If the joint offset d_n is also zero, then reference frames n and n-1 have common origins at the manipulator end-effector, and Equations (8) and (13) imply that

$$\mathbf{J}_{n-1_4} \equiv \mathbf{J}_{n-1_5} \equiv \mathbf{J}_{n-1_6} \equiv \mathbf{0} \tag{28}$$

Equations (12-13) are then recursively applied for link n-1.

Once the linear combinations of the columns of the Jacobian matrix $\mathbf{J}_e$ are calculated, the independent generalized error set is obtained. From Equation (7),

$$\mathbf{J}_{i-1_3}\varepsilon_{i-1_3} + \mathbf{J}_{i_2}\varepsilon_{i_2} + \mathbf{J}_{i_3}\varepsilon_{i_3} \equiv \mathbf{J}_{i_2}(\varepsilon_{i_2} + \varepsilon_{i-1_3}\sin\alpha_i) +$$
$$\mathbf{J}_{i_3}(\varepsilon_{i_3} + \varepsilon_{i-1_3}\cos\alpha_i) \equiv \mathbf{J}_{i_2}\varepsilon^*_{i_2} + \mathbf{J}_{i_3}\varepsilon^*_{i_3} \tag{29}$$

showing that the generalized error ε_{i-1_3} can be incorporated into ε_{i_2} and ε_{i_3} if joint i is revolute, resulting in the combined generalized errors $\varepsilon^*_{i_2}$ and $\varepsilon^*_{i_3}$. Using Equations (7-8) and the approach described above, the combined generalized errors are obtained:

$$\begin{cases} \varepsilon^*_{i_2} = \varepsilon_{i_2} + \varepsilon_{i-1_3}\sin\alpha_i + \varepsilon_{i-1_5} \cdot a_i \cos\alpha_i \\ \varepsilon^*_{i_3} = \varepsilon_{i_3} + \varepsilon_{i-1_3}\cos\alpha_i - \varepsilon_{i-1_5} \cdot a_i \sin\alpha_i \\ \varepsilon^*_{i_4} = \varepsilon_{i_4} + \varepsilon_{i-1_5}\sin\alpha_i \\ \varepsilon^*_{i_5} = \varepsilon_{i_5} + \varepsilon_{i-1_5}\cos\alpha_i \end{cases} \tag{30}$$

which holds for both revolute and prismatic joints.

If joint i is prismatic, then Equations (9-10) are combined with Equation (30), resulting in

$$\begin{cases} \varepsilon^*_{i_1} = \varepsilon_{i_1} + \varepsilon_{i-1_1} \\ \varepsilon^*_{i_2} = \varepsilon_{i_2} + \varepsilon_{i-1_2}\cos\alpha_i + \varepsilon_{i-1_3}\sin\alpha_i + \varepsilon_{i-1_5} \cdot a_i \cos\alpha_i \\ \varepsilon^*_{i_3} = \varepsilon_{i_3} - \varepsilon_{i-1_2}\sin\alpha_i + \varepsilon_{i-1_3}\cos\alpha_i - \varepsilon_{i-1_5} \cdot a_i \sin\alpha_i \\ \varepsilon^*_{i_4} = \varepsilon_{i_4} + \varepsilon_{i-1_5}\sin\alpha_i \\ \varepsilon^*_{i_5} = \varepsilon_{i_5} + \varepsilon_{i-1_5}\cos\alpha_i \end{cases} \tag{31}$$

In the particular case where the end-effector orientation is not considered, Equation (12) implies that the generalized errors ε_{n_4}, ε_{n_5} and ε_{n_6} do not affect the end-effector measurements. Furthermore, if the last joint is revolute and $a_n \equiv 0$ then Equation (27) results in the combined generalized errors

$$\begin{cases} \varepsilon^*_{n-1_1} = \varepsilon_{n-1_1} + \varepsilon_{n-1_4} \cdot d_n \\ \varepsilon^*_{n-1_2} = \varepsilon_{n-1_2} - \varepsilon_{n-1_6} \cdot d_n \end{cases} \tag{32}$$

and also implies that ε_{n-1_5} does not influence the end-effector position.

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