Dynamic Optimization of Geneva Mechanisms

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ABSTRACT

The Geneva wheel is the simplest and most widely used mechanism to provide intermittent motion from a continuously rotating input. However, the dynamic properties of Geneva mechanisms are not ideal, and typically lead to step changes in acceleration. A four-bar linkage with the drive pin located at a coupler point proves to be an appealing solution to reduce acceleration and jerk. This paper proposes a highly efficient method to generate a four-bar linkage Geneva wheel drive with optimized dynamic characteristics. Results are presented for six Geneva wheels, demonstrating the high effectiveness of the approach.

1. INTRODUCTION

Among several mechanisms producing intermittent rotary motion, the Geneva mechanism is the simplest and most widely used for its accuracy and self-locking function (3). Geneva mechanisms are easy to manufacture, in contrast to cams, which require machining of complex shapes with very small tolerances. Geneva wheels are usually driven by a crank, rotating with constant velocity, and a crankpin, enabling very long dwells between rapid indexes, making it attractive to both low- and high-speed machinery and manufacturing systems. A wide variety of applications are derived from the Geneva mechanism, such as indexing in automatic machinery, peristaltic pump drives in integrated circuit manufacturing, intermittent advance of films in motion-picture projectors, and discrete motion drives with high load capacity in robotic manipulators (1, 6, 10, 11).

The main disadvantage of Geneva mechanisms is the discontinuity in the acceleration at the start and the end of the intermittent motion. At these points, the normal acceleration of the rotating crankpin is transmitted to the wheel with an impact, leading to large jerks and undesirable vibrations in the mechanism. Several methods have been proposed to decrease the wheel acceleration in order to reduce the inertia forces and the consequent wear. Among these is the idea of using a curved slot, which reduces the acceleration, but it increases the deceleration and consequently the wear on the slot (4). Other approaches involve substantial changes in the slot design, such as using different radii of curvature of the entry and exit curves, using grooved cams to drive/guide the crankpin in a specific path, and using spring elements between the slot and the driving pin (3, 4). Such approaches reduce the acceleration force in both the entry and exit stages, at the cost of implementing complex drive mechanisms. Many authors have considered minimizing the inertial forces on the Geneva wheel using a four-bar linkage with the drive pin located at a coupler (7, 8, 12, 13). Zero jerk can then be achieved for an appropriate coupler point and path, however the existing ad-hoc methods to generate the corresponding four-bar linkages must be calculated on a case-by-case basis (2, 9, 11).

This paper proposes a highly efficient method to generate a four-bar linkage Geneva wheel drive with optimized dynamic characteristics. The acceleration period is considered equal to the deceleration period, giving a symmetric coupler curve. The problem is reduced to determining the dimensions of the four-bar linkage in such a way that the tangent to the symmetric coupler curve makes a given angle with the axis of symmetry. The dynamics of the external and internal crank driven Geneva wheel are presented, as well as the design for a coupler driven Geneva wheel. The kinematic and constraint equations, used in generating the four-bar linkage for the desired trajectory, are developed. Numerical optimization of the search space determines the ideal link lengths in order to minimize the jerk of the system. Results are presented for the cases of 30°, 45°, 60°, 72°, 90° and 120° Geneva slot angles.

2. CRANK DRIVEN GENEVA WHEEL

2.1 External Geneva Wheel

In the external Geneva wheel mechanism of any number of slots, the dwell period exceeds the motion period. The opposite is true about the internal Geneva wheel. The lowest possible number of slots is three, whereas the upper limit, in theory, is unlimited. In practice, the three-slot Geneva wheel is seldom used because of the very high acceleration values encountered. Geneva wheels with more than 18 slots are also infrequent because they require large wheel diameters. For the external Geneva wheel, the crank center lies at a distance from the wheel center greater than the wheel radius. For proper operation, the drive pin (crankpin) must enter and leave the slot tangentially. In other words, the centerline of the slot and the line connecting the crankpin to the crank rotation center must form a right angle when the crankpin enters or leaves the slot (see Figure 1).

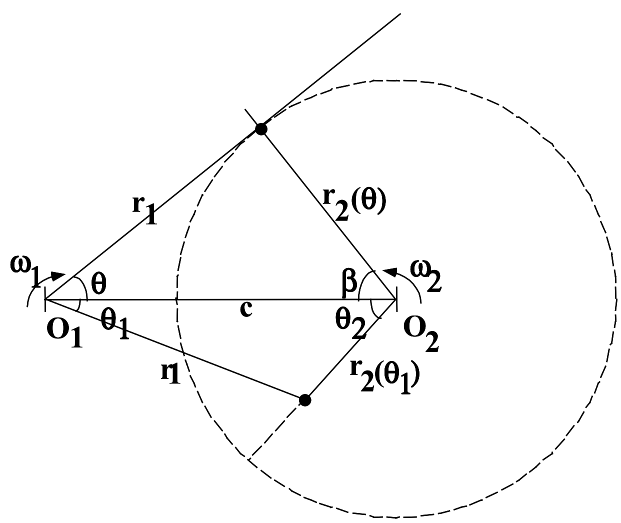

Figure 1: Schematic layout of external Geneva crank (r_1) and wheel (r_2)

The design of the Geneva mechanism is then initiated by specifying the crank radius, the crankpin diameter, and the number of slots. The angle β is defined as half the angle subtended by adjacent slots, n is the number of slots, r_1 is the crank radius, and c is the center distance (given by $r_1/\sin\beta$). The actual Geneva wheel radius is greater than an ideal one with a zero radius crankpin. This is due to the difference between the sine and the tangent of the angle subtended by the crankpin, measured from the wheel center. For the work presented here it is assumed that the crankpin radius is negligible. After the crankpin enters the slot, the drive angle formed is given by θ_1. The corresponding wheel angle is given by θ_2, related by:

$$\tan\theta_2 = \frac{r_1 \sin\theta_1}{c - r_1 \cos\theta_1} = \frac{\sin\theta_1}{(c/r_1) - \cos\theta_1} \tag{1}$$

Differentiating Equation (1) with respect to time gives the angular velocity of the wheel:

$$\frac{d(\tan\theta_2)}{dt} = \dot{\theta}_2 = \dot{\theta}_1 \frac{(c/r_1)\cos\theta_1 - 1}{1 + (c^2/r_1^2) - 2(c/r_1)\cos\theta_1} \tag{2}$$

The maximum angular velocity occurs when the crank angle is zero (with respect to the centerline c). The angular acceleration of the wheel is then obtained by differentiating the expression for the wheel angular velocity with respect to time, giving:

$$\alpha_2 = \ddot{\theta}_2 = \varpi_1^2 \frac{(c/r_1)\sin\theta_1 \left(1 - c^2/r_1^2\right)}{\left(1 + (c/r_1)^2 - 2(c/r_1)\cos\theta_1\right)^2} \tag{3}$$

The maximum angular acceleration occurs when the crank angle satisfies:

$$\cos\theta_1 = \sqrt{\left(\frac{1 + (c/r_1)^2}{4c/r_1}\right)^2 + 2} - \left(\frac{1 + (c/r_1)^2}{4c/r_1}\right) \tag{4}$$

The dwell angle is easily found from Figure 1 to be $\pi + 2\beta$.

Figure 2 shows a plot of the angular acceleration of the wheel with respect to the crank angle for 3, 4, 5, 6, 8, 10, and 12 slotted Geneva wheels with unit crank link length (r_1=1). Note that Figure 2 is symmetric about the origin, but for clarity it has been limited to a smaller window. It can be seen that there exists a non-zero angular acceleration component as the crankpin makes contact with the Geneva slot. This leads to a singularity in the third derivative of the position and hence an infinite jerk upon contact.

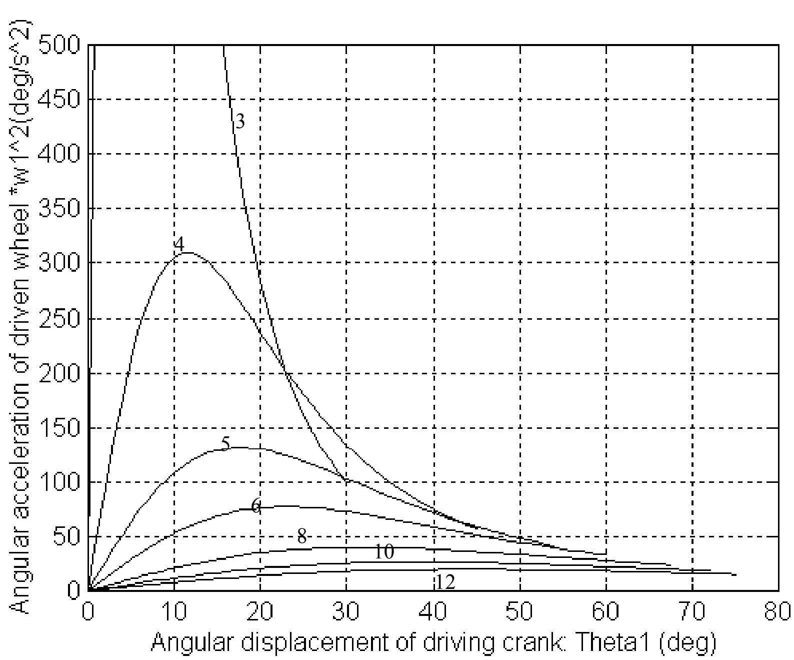

Figure 2: Angular acceleration of driven member

2.2 Internal Geneva Wheel

When the dwell period must be less than 180°, other intermittent drive mechanisms must be used. The internal Geneva wheel is one way of obtaining this form of motion. The main advantage of the internal Geneva wheel, other than its smooth operation, is its sharply defined dwell period. A disadvantage is the relatively large size of the driven member, which increases the inertial forces resisting acceleration/deceleration. For proper operation, the drive pin (crankpin) must enter and leave the slot tangentially.

Structurally, the internal Geneva wheel differs from the external Geneva wheel in that the distance of the crank center from the wheel center is less than the wheel radius. However, this leads to significant differences in the mechanics of the system. The dwell period of all internal Geneva wheels is less than 180°, leaving more time for the star wheel to reach maximum velocity, lowering the acceleration. The highest value of acceleration occurs when the crankpin enters or leaves the slot, however the acceleration curve does not reach a peak within the range of motion of the driven wheel. The geometrical maximum would occur in the continuation of the curve, but this continuation has no significance since the driven member will have entered the dwell phase associated with the high angular displacement of the driving member. This geometrical maximum falls into the region representing the motion of the external Geneva wheel.

The design of the internal Geneva mechanism is very similar to that of the external mechanism. The maximum angular velocity occurs when the crank angle is zero with respect to the centerline c. The maximum angular acceleration occurs when the crank enters the slot. Figure 3 shows a plot of the angular acceleration of the wheel with respect to the crank angle for 3, 4, 5, 6, 8, 10, and 12 slotted Geneva wheels with unit crank link length (r_1=1). It can be seen that there exits a non-zero angular acceleration component as the crankpin makes contact with the Geneva slot. In fact this is the maximum angular acceleration of the system during the non-dwell phase. Once again this leads to a singularity, hence an infinite jerk upon contact.

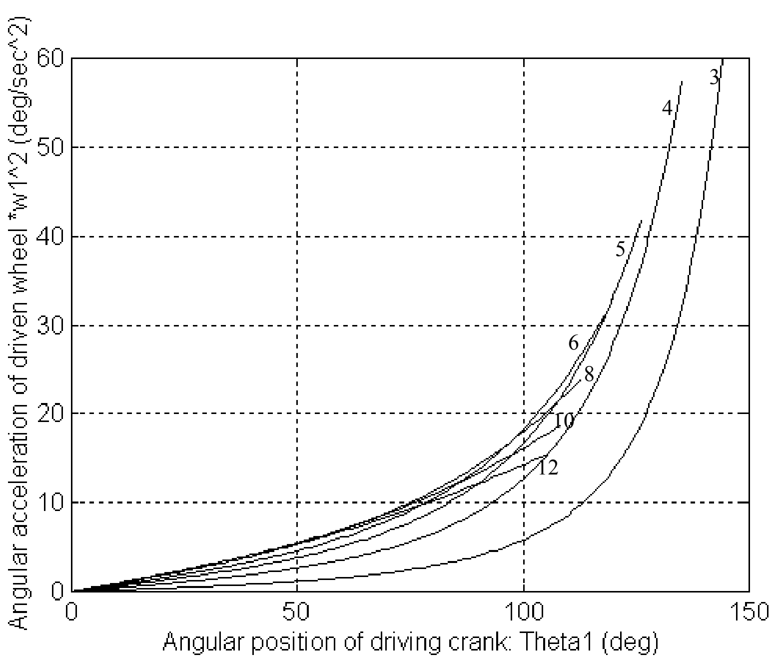

Figure 3: Angular Acceleration of Driven Member

3. COUPLER DRIVEN GENEVA WHEELS

3.1. Four bar linkage design

Kinematic linkage synthesis gives well behaved solutions to the angular acceleration and jerk, with significant improvement from crank driven Geneva wheels. In generating the coupler curve, one is only concerned with the position of the coupler point (P, in Figure 4) and its time derivatives. In designing a 4 bar linkage mechanism, one must first consider Grashof's law that states *for a planar four-bar linkage, the sum of the shortest and longest link lengths cannot be greater than the sum of the remaining two link lengths if there is to be continuous relative rotation between two members.* This eliminates the double rocker design option. Additionally, due to practical limitations, the drag-link design option can also be ignored. This leaves only the crank-rocker design and the following two design restrictions (see Figure 4): $s + l \leq p + q$ and $s + l + p \geq q$.

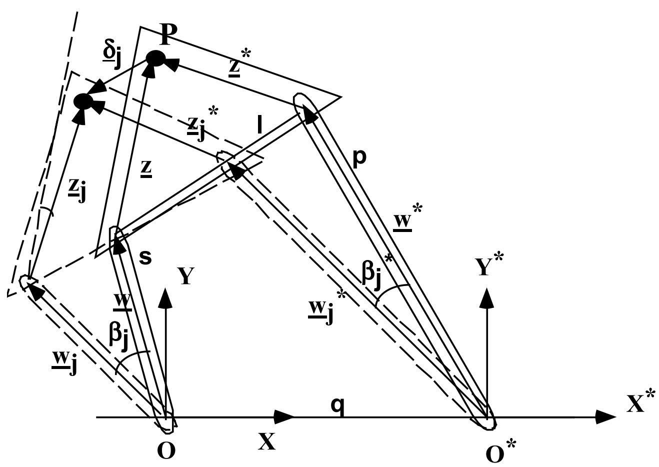

Figure 4: Layout and notation of a 4-bar linkage where s is the crank, l the coupler, p the rocker, q the ground link and P the coupler point under consideration

In order to find a solution for the four link lengths, the method of complex variables is chosen. Here, each vector is described in complex coordinates in the form of x+iy. For the original position of the coupler point, P, a left hand dyad and right hand dyad (comprising of $\underline{w} + \underline{z}$ and $\underline{w}^* + \underline{z}^*$ respectively, see Figure 4) are constructed. For each unique new position of P, the left hand and right hand dyads are rotated and vector loop closure applied to give:

$$\begin{aligned} \underline{w}(e^{i\beta_j} - 1) + \underline{z}(e^{i\alpha_j} - 1) &= \delta_j \\ \underline{w}^*(e^{i\beta_j^*} - 1) + \underline{z}^*(e^{i\alpha_j} - 1) &= \delta_j \end{aligned} \tag{5}$$

In these equations δ_j is considered to be known. Based on the number of scalar equations and the number of unknowns, the number of free choices or additional constraints that can be applied can then be determined.

3.2. Design selection with 3 coupler point positions

To form a coupler curve, the number of unique coupler point positions must be selected and appropriate choices/constraints must be applied to give a unique solution. It is desirable to form a symmetric coupler curve, resulting in similar kinematic characteristics for the acceleration and deceleration stages. This is accomplished by setting $|\underline{w}^*| = |\underline{z}^*| = |\underline{z}-\underline{z}^*| = b$. For simplicity, the axis of symmetry is set perpendicular to the ground link, passing through the base point of the right hand dyad. These three required constraints are defined as the *symmetry constraints*.

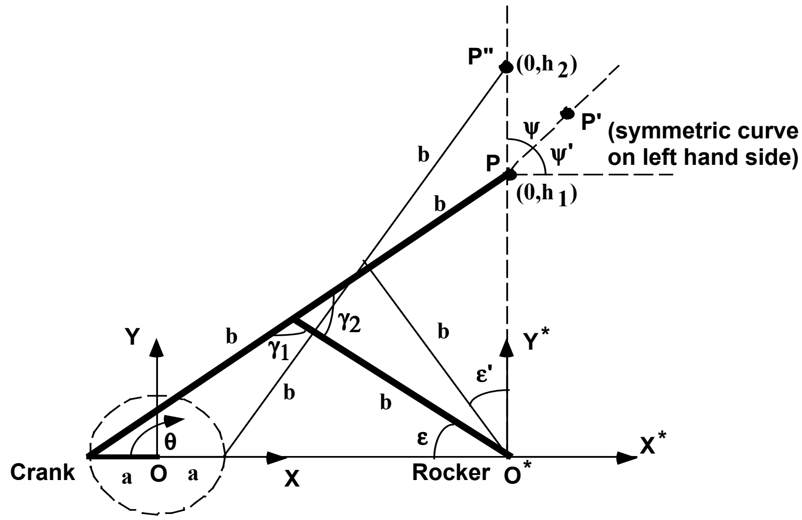

Figure 5: Layout with extremum positions of 4-bar linkage design

The first approach is selecting 3 known positions forming a triangle with included angle, $\psi^{'}$, equal to half the required inter slot angle (see Figure 5). This gives 8 scalar equations (4 for each dyad, as discussed above), where $\beta_2=-\pi/2$, $\beta_3=-\pi$ and $\underline{w}=-a+0i$, resulting in constraints 4, 5, 6 and 7. Note that β_3 is selected based on coupler curve symmetry. Constraint 8 requires that all points on the coupler curve lie within the prescribed triangle, resulting in $(y-h_1)/x \geq \tan\psi$. Finally, the goal of minimizing the jerk ($d^3\psi/dt^3$) forms the last constraint. Further, x and y are represented as $x=r\cos(\psi)$ and $y=r\sin(\psi)$, introducing a new unknown r. From Figure 5 it can also be seen that:

$$h_1 = 2b\sin\varepsilon = 2b\frac{\sqrt{b^2-\left(\frac{1+a}{2}\right)^2}}{b} = 2\sqrt{b^2-\left(\frac{1+a}{2}\right)^2}$$

$$h_2 = 2b\sin\varepsilon^{'} = 2b\frac{\sqrt{b^2-\left(\frac{1-a}{2}\right)^2}}{b} = 2\sqrt{b^2-\left(\frac{1-a}{2}\right)^2} \tag{6}$$

$$\gamma = \gamma_1+\gamma_2 = (\pi-2\varepsilon)+\left(\pi-2\left(\frac{\pi}{2}-\varepsilon\right)\right) = \pi$$

where the ground link has unity length. This gives 9 constraints with only 6+1 (due to r) free choices available, resulting in an over-constrained mechanism. Without applying constraints 8 and 9, solving the system of equations gives:

$$\underline{w} = -a+0i \qquad \underline{w}^* = -\left(\frac{a+1}{2}\right)+\frac{h_1}{2}i$$

$$\underline{z} = (1+a)+h_1 i \qquad \underline{z}^* = \left(\frac{a+1}{2}\right)+\frac{h_1}{2}i \tag{7}$$

Also, α_2 and α_3 can be found by plugging in the expressions for w, z, δ_2 and δ_3 into the original vector closure equations and solving:

$$
\begin{aligned}
&a+(1+a)(\cos\alpha_2-1)-h_1\sin\alpha_2=r\cos\psi \\
&2a+(1+a)(\cos\alpha_3-1)-h_1\sin\alpha_3=0
\end{aligned}
\tag{8}
$$

3.3. Design modification with 2 coupler point positions

In order to keep the system fully determined, the number of constraints needs to be reduced (and/or the number of free choices increased). This can be accomplished by selecting 2 (rather than 3) coupler point positions, $(0,h_1)$ and $(0,h_2)$. This gives 4 scalar equations and 12 unknowns (w, z, w^*, z^*, β_2, β^*_2, α_2, h_1), resulting in 8 free choices. The constraints are:

- 3 from symmetry (as before)
- $\beta_2 = -\pi$
- w=-a+0i
- min $d^3\psi/dt^3$
- $(y-y')/x \geq \tan\psi$ where y' is the point of intersection of the tangent to the coupler curve to the y axis and forming an angle ψ' with the vertical

The jerk is only computed after the coupler point passes the tangent point (the point where the coupler curve meets the tangent line forming an angle ψ with the vertical). This gives the 8 constraints and the system is fully determined.

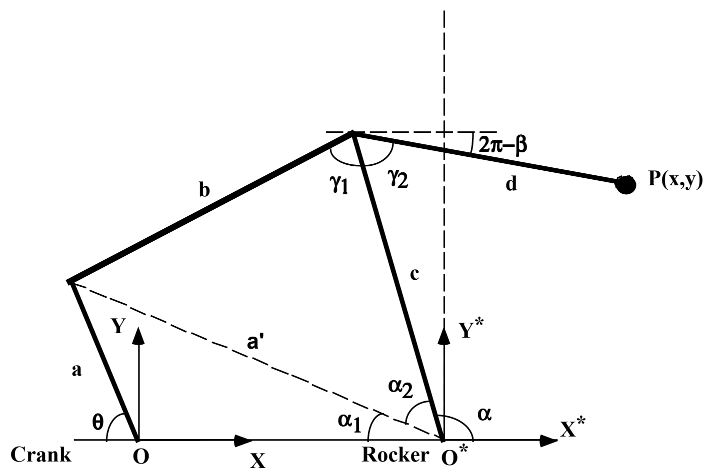

Figure 6: Layout of arbitrary 4-bar linkage with notation for curve synthesis

Given an arbitrary 4 bar linkage and coupler point P with coordinates (x,y) in the right hand dyad base coordinate frame (see figure 6), the motion of P can be easily determined. For an input crank at angle θ:

$$
\begin{aligned}
&a'^2 = 1 + a^2 + 2a\cos\theta & & \\
&\sin\alpha_1 = \frac{a}{a'}\sin\theta & &\gamma_2 = \gamma-\gamma_1 \\
& & &\alpha = \pi-(\alpha_1+\alpha_2) \\
&\cos\alpha_2 = \frac{a'^2+c^2-b^2}{2a'c} & &\beta = \pi+\alpha+\gamma_2 \\
& & &x = c\cos\alpha + d\cos\beta \\
&\cos\gamma_1 = \frac{b^2+c^2-a'^2}{2bc} & &y = c\sin\alpha + d\sin\beta
\end{aligned}
\tag{9}
$$

Equations (9) can be simplified by setting b=c=d and $\gamma=\pi$. Given (x,y), then the angle ψ' (the angle made by the Geneva slot with respect to the horizontal in Figure 5) is $\psi' = \mathrm{atan}((y-y')/x)$. Clearly, by differentiating ψ with respect to time, the velocity, acceleration and the jerk of the Geneva wheel can be obtained. In order to determine the unknowns (a,b), a simulation program is used to apply the final constraints. From the values obtained for (a,b) the code also generates the value for y'.

4. RESULTS

The simulation is run for Geneva wheels with 3, 4, 5, 6, 8 and 12 slots with input crank velocity of 1°/s. The final selection of link lengths is summarized in the table below:

Table 1 - Optimized link lengths for various numbers of slots

Geneva Angle (# slots)	a/d	b/d	Y'/d	Max Jerk (°/s^3)
30° (12)	0.18	0.6	0.2059	0.0016
45° (8)	0.2	0.64	0.3849	0.0014
60° (6)	0.26	0.68	0.4647	0.0009
72° (5)	0.32	0.72	0.5235	0.0015
90° (4)	0.36	0.78	0.6862	0.0018
120° (3)	0.38	0.88	1.0144	0.0014

The base link length, d, is set to unity. Figure 7 shows a plot of the angular acceleration of the wheel with respect to the crank angle for 3, 4, 5, 6, 8, and 12 slotted Geneva wheels. Note that this plot is symmetric about θ=180°. From the table above and the plot it can be seen that the jerk experienced is very low compared to those seen in section 2, showing the effectiveness of the proposed design methodology.

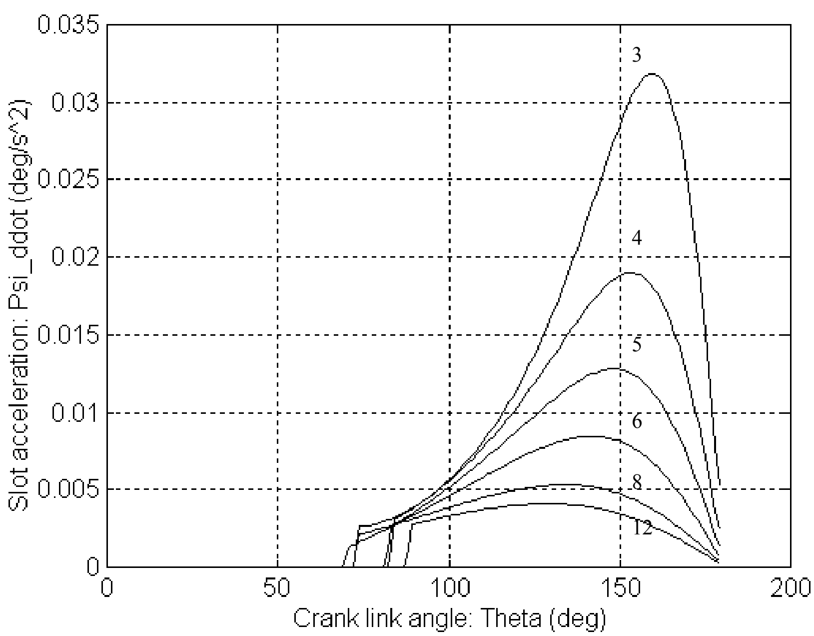

Figure 7: Angular acceleration of driven member

5. CONCLUSIONS

This paper presents the design of a Geneva wheel drive mechanism that performs significantly better than conventional crank drive approach (both external or internal). The design is based on a 4 bar linkage where the coupler point drives the slotted wheel in a prescribed intermittent fashion, based on uniform angular motion of an input crank. The design goal is to minimize the maximum angular jerk experienced by the wheel. The synthesis of the 4 bar linkage mechanism uses a complex variable approach for mechanism synthesis. By applying appropriate constraints in the form of free choices in the complex variable equations, the synthesis is reduced to the selection of only two link length parameters. Computer simulation is then used to minimize the maximum jerk felt by the wheel, and results are presented for 6 cases of inter slot angle ranging from 30 degrees to 120 degrees. On average there is an improvement in maximum jerk from ~15 deg/s^3 to ~0.002 deg/s^3 in the case of the 12 slotted wheel, and from ~100 deg/s^3 to ~0.002 deg/s^3 in the case of the 3 slotted wheel (with input crank velocities of 1 deg/s). In both the crank driven and linkage driven mechanisms the maximum jerk is found at the tangent point of the drive pin path with the slot. For possible future endeavors in improving the system, it would prove interesting to explore the added constraint of a predefined dwell angle. Additionally, the requirement of symmetry axis orthogonality and location with respect to the ground link may be relaxed and synthesis carried out.

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