COMPARING OVERLOAD-INDUCED RETARDATION MODELS ON FATIGUE CRACK PROPAGATION

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Summary

The modeling of load interaction effects on mode I fatigue crack propagation is discussed in this paper. Overload-induced retardation effects on the crack growth rate are considered, based on the crack closure idea, and a taxonomy of the load interaction models is presented. Modifications to the traditional retardation models are proposed to better model such effects as crack arrest, crack acceleration due to compressive underloads, and the effect of small cracks. These models are implemented on the **ViDa** software, developed to automate the fatigue dimensioning process by all the traditional methods used in mechanical design. Using this software, the presented load interaction models and the proposed modifications are compared with experimental results from various load spectra. In particular, the proposed modifications to the Wheeler model showed a good agreement with the experimental data. In a companion paper, these ideas are expanded to 2D crack propagation.

Keywords: fatigue, crack propagation, sequence effects.

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1. Introduction

It is a well known fact that load cycle interactions can have a very significant effect in the prediction of fatigue crack growth. There is a vast literature proving that tensile overloads, when applied over a constant amplitude loading, can cause retardation or arrest in the crack growth, and that even compressive overloads can sometimes affect the rate of subsequent crack propagation [1, 2, 3].

Neglecting these effects in fatigue life calculations can completely invalidate the predictions. In fact, only after considering overload induced retardation effects can the life reached by real structural components be justified when modeling many practical problems. However, the generation of a universal algorithm to quantify these effects is particularly difficult, due to the number and to the complexity of the mechanisms involved in fatigue crack retardation, such as plasticity-induced crack closure, blunting and/or bifurcation of the crack tip, residual stresses and/or strains, strain-hardening, crack face roughness, and oxidation of the crack faces. Besides, depending on the case, several of these mechanisms may act concomitantly or competitively, as a function of factors such as crack size, material microstructure, dominant stress state, and environment. Moreover, the relative importance of the several mechanisms can vary from case to case, and there is so far no universally accepted single equation capable of describing the whole problem.

On the other hand, the principal characteristic of fatigue cracks is to propagate cutting a material that has already been deformed by the plastic zone that always accompanies their tips. The fatigue crack faces are embedded in an envelope of (plastic) residual strains and, consequently they compress their faces when completely discharged, and open alleviating in a progressive way the (compressive) load transmitted through their faces.

In this work, a review of plasticity induced crack closure is presented, along with the associated physical models. A taxonomy of load interaction models based on the crack closure idea is introduced, and improvements to the traditional retardation models are proposed to model crack arrest, crack acceleration after compressive underloads, and even the effect of small cracks. These models are extended to 2D cracks in [1]. A review of the crack closure models is presented next.

2. Crack Closure

In the early seventies, Elber [4] discovered that fatigue cracks can remain closed for loads substantially higher than the minimum applied load. This was attributed to the deformation of the crack plane remaining behind the crack tip, as the crack propagated through its plastic zone, a phenomenon termed *plasticity-induced fatigue crack closure*. According to him, only after the load completely opened the crack at a stress intensity factor $\mathbf{K_{op}} > \mathbf{0}$, would the crack tip be stressed: the bigger the $\mathbf{K_{op}}$, the less would be the effective stress intensity range $\mathbf{\Delta K_{eff}} = \mathbf{K_{max}} - \mathbf{K_{op}}$, and this $\mathbf{\Delta K_{eff}}$ instead of $\mathbf{\Delta K}$ would be the crack propagation rate controlling parameter. Elber was able to show by experiment on 2024T3 aluminum that the effective stress intensity $\mathbf{\Delta K_{eff}}$, and consequently $\mathbf{\Delta K_{th}}$, is mainly dependent on the stress ratio $\mathbf{R}$. To calculate crack propagation under constant amplitude loads, taking into account the crack closure concept, Elber proposed a modification to the Paris law by using this effective stress-intensity range, $\mathbf{da/dN} = \mathbf{A}\cdot(\mathbf{\Delta K_{eff}})^{\mathbf{m}}$, where $\mathbf{A}$ and $\mathbf{m}$ are the growth rate constants. From Elber's original expression, it follows that:

$$\frac{da}{dN} = A \cdot (\Delta K_{eff})^m = A \cdot \left(\frac{(K_{max} - K_{op})(1-R)}{1-R} \right)^m = A \cdot \left(\frac{\Delta K - \Delta K_{th}}{1-R} \right)^m \qquad (1)$$

where ΔK_{th} is the threshold stress intensity factor range and ΔK is the stress intensity factor range. The threshold stress intensity factor range used in the above model can be determined for any positive stress ratio $R > 0$ using [5]

$$\Delta K_{th} = (4/\pi) \cdot \Delta K_0 \ \arctan(1-R) \qquad (2)$$

where ΔK_0 is the threshold value of the stress-intensity factor range for $R = 0$ tests.

Most load interaction models are, directly or indirectly, based on Elber's original crack closure idea. This implicates in the supposition that the main retardation mechanism is caused by plasticity induced crack closure: in these cases, the opening load should **increase** due to the plastic zone ahead of the crack tip, **reducing** the ΔK_{eff} and delaying the crack growth.

However, it is very important to emphasize that crack closure is by no means the only mechanism that can induce crack retardation. For example, Castro & Parks [6] showed that, under dominant plane strain conditions, overload induced fatigue crack retardation or stop can occur while ΔK_{eff} **increases**. It was found that just after the overload the opening load **decreased**, a behavior incompatible with Elber-type crack closure. The main retardation mechanism in those cases was bifurcation of the crack tip. The next section presents analytical models to account for load interaction.

3. Load Interaction Models

Several mathematical models have been developed to account for load interaction in crack propagation based on Elber’s crack closure idea. In these methods, the retardation mechanism is only considered within the plastic zone situated in front of the crack tip. According to these procedures, a larger plastic zone is created by means of an overload. When the overload is removed, an increased compressive stress state is set up in the volume of its plastic zone, reducing crack propagation under a smaller succeeding load cycle.

Perhaps the best-known models in the literature are those developed by Wheeler [7] and Willenborg [8]. They used the same model to decide whether crack propagation will be retarded or not. In both methods, retardation takes place as long as the crack **a** with its accompanying plastic zone remains within the plastic zone created by the peak load. That is, while the current plastic zone Z_i is embedded in the overload zone Z_{ol}, the crack growth retardation depends on the distance from the border of Z_{ol} to the crack tip, see Figure 1.

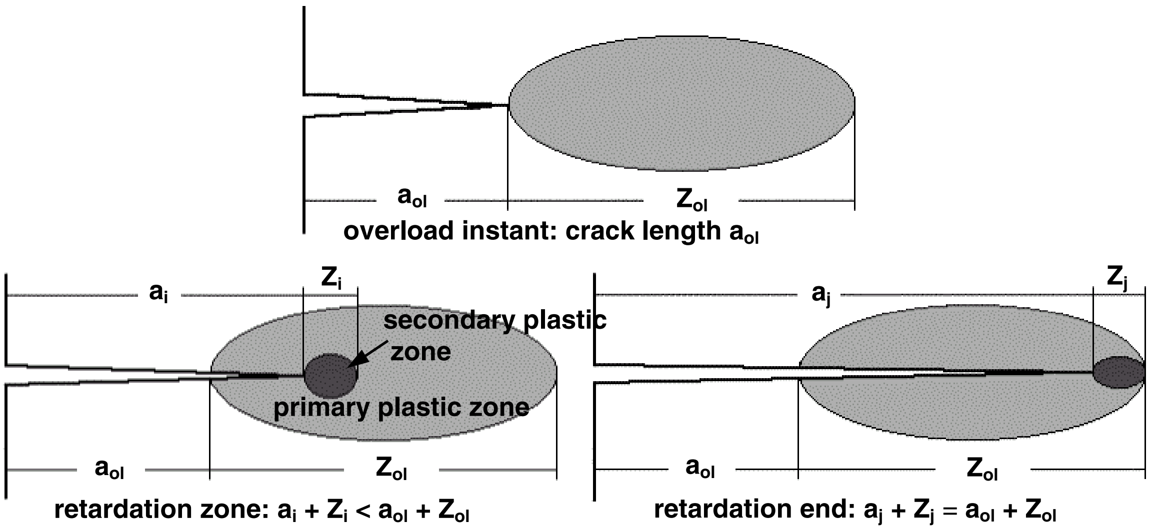


Figure 1 – Yield zone retardation model used by Wheeler and Willenborg

The main difference between the Wheeler and Willenborg procedures is that Willenborg takes account of the retardation effect by reducing the maximum and minimum stress intensity factors acting on the crack tip, while Wheeler takes account of the retardation effect by direct reduction of the crack propagation rate **da/dN** using a retardation function. Based on this and other differences, the load interaction models presented in this paper are divided in 4 categories: (i) **da/dN** models, such as the Wheeler model, which use retardation functions to directly reduce the calculated crack propagation rate **da/dN**; (ii) ΔK models, which use retardation functions to reduce the value of the stress intensity factor range ΔK; (iii) R_{eff} models, such as the Willenborg model, which introduce an effective stress ratio R_{eff}, calculated by reducing the maximum and minimum stress intensity factors acting on the crack tip, however not necessarily changing the value of ΔK; and (iv) K_{op} models, such as the strip yield model, which use estimates of the opening stress intensity factor K_{op} to directly account for Elber-type crack closure.

The **da/dN** load interaction models are discussed next.

3.1. da/dN Interaction Models

The **da/dN** interaction models use retardation functions to directly reduce the calculated crack propagation rate **da/dN**. ***Wheeler*** is the most popular of such models [7]. Wheeler introduced a crack-growth reduction factor, bounded by zero and unity, which is calculated for each cycle and is used as a multiplying factor on the crack growth increment for each cycle. There is retardation as long as the current plastic zone is contained within a previously overload-induced plastic zone. The retardation is maximum just after the overload, and stops when the border of Z_i touches the border of Z_{ol}, see Figure 1. Therefore, if a_{ol} and a_i are the crack sizes at the instant of the overload and at the *i*-th cycle, and $(da/dN)_{eff,i}$ and $(da/dN)_i$ are the effective (retarded) and the non-retarded crack growth rate (at which the crack would be growing if the overload had not occurred), then, according to Wheeler:

$$\left(\frac{da}{dN}\right)_{eff,i} = \left(\frac{da}{dN}\right)_i \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i}\right)^{\beta}, \ a_i + Z_i < a_{ol} + Z_{ol} \qquad (3)$$

where β is an experimentally adjustable constant.

The exponent β is obtained by selecting the closest match among predicted crack growth curves (using several β-values) with an experimental curve obtained under spectrum loading [9]. Wheeler found experimentally that the shaping exponent, β, was material dependent, having values of 1.43 for a steel and 3.4 for the titanium alloy Ti-6AL-4V. Broek [2] suggests that other typical values for β are between 0 and 2. However, flight-by-flight crack propagation tests performed by Sippel et al. [10] have shown that the exponent β is dependent not only on the material, but also on crack shape, stress level, as well as type of load spectrum. Finney [11] found experimentally that the calibration β-value depends on the maximum stress in the spectrum and on the crack shape parameter **Q** (defined as $\mathbf{Q = 1{+}1.464{\cdot}[a/c]^{1.65}}$ for surface cracks with depth **a** and width **2c**, and **Q** = **1** for through cracks). Therefore, life predictions based on limited amounts of supporting test data, or with spectra radically different from those for which the exponent β was derived, can lead to inaccurate and unconservative results.

In summary, the selection of proper values for the Wheeler exponent β usually yields adequate crack-growth predictions. In fact, one of the earlier advantages of the Wheeler model is that the exponent β can be tailored to allow for reasonably accurate life predictions. However, the Wheeler model cannot predict the observed phenomenon of crack **arrest**. As $\mathbf{Z_i \approx (K_{max}/S_y)^2}$, the lowest value of the predicted retardation factor happens immediately after the overload, and is equal to $\mathbf{(K_{max}/K_{ol})^{2\beta}}$, where $\mathbf{K_{max}}$ is the maximum stress intensity factor in the cycle just after the overload, and $\mathbf{K_{ol}}$ is the overload stress intensity factor. Therefore, the phenomenology of the load cycle interaction problem is not completely reproducible by the Wheeler model, since such retardation factor is always different than zero. To consider crack arrest, a modification of the Wheeler original model is presented next.

3.2. ΔK Interaction Models

The $\mathbf{\Delta K}$ interaction models use retardation functions to directly reduce the value of the stress intensity range $\mathbf{\Delta K}$. Meggiolaro and Castro [12] proposed a simple but effective modification to the original Wheeler model in order to predict crack arrest. This modified approach, called ***Modified Wheeler*** model, uses a Wheeler-like parameter to multiply $\mathbf{\Delta K}$ instead of **da/dN** after the overload:

$$\Delta K_{eff}(a_i) = \Delta K(a_i) \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i} \right)^{\gamma}, \; a_i + Z_i < a_{ol} + Z_{ol} \qquad (4)$$

where $\mathbf{\Delta K_{eff}(a_i)}$ and $\mathbf{\Delta K(a_i)}$ are the values of the stress intensity ranges that would be acting at $\mathbf{a_i}$ with and without retardation due to the overload, and γ is an experimentally adjustable constant, in general different from the original Wheeler model exponent β. This simple modification can be used with any of the propagation rules that recognize $\mathbf{\Delta K_{th}}$ to predict both the retardation and the arrest of fatigue cracks after an overload (the arrest occurring if $\mathbf{\Delta K_{eff}(a_i) \le \Delta K_{th}}$).

The Modified Wheeler model predicts both crack retardation and crack **arrest**, however it does not model the reduction of retardation effects due to underloads. An underload stress, defined as the lowest compressive or tensile stress subsequent to the last overload cycle, can reduce the overload-induced retardation effects (also referred to as crack acceleration). Several mechanisms can be used to explain this crack acceleration phenomenon.

Chang et al. [13] proposed the concept of an effective overload plastic zone to model crack acceleration. In Chang's crack acceleration concept, the overload plastic zone $\mathbf{Z_{ol}}$ is reduced to an effective value $\mathbf{(Z_{ol})_{eff}}$ after a compressive underload, reducing the crack retardation effects by increasing the retardation parameter from equations (3) and (4),

$$(\mathbf{Z_{ol}})_{\mathbf{eff}} = (\mathbf{1} + \overline{\mathbf{R}}_{\mathbf{ul}}) \cdot \mathbf{Z_{ol}}, \text{ where } \overline{\mathbf{R}}_{\mathbf{ul}} = \begin{cases} \mathbf{0}, \text{ if } \mathbf{R_{ul}} \geq \mathbf{0} \\ \mathbf{max}(\mathbf{R}^{-}, \mathbf{R_{ul}}), \text{ if } \mathbf{R_{ul}} < \mathbf{0} \end{cases} \quad (5)$$

where $\mathbf{R_{ul}}$ is the underload stress ratio $\boldsymbol{\sigma_{ul} / \sigma_{ol}}$, $\boldsymbol{\sigma_{ul}}$ is the lowest underload stress after the most recent overload $\boldsymbol{\sigma_{ol}}$, and $\mathbf{R^{-}}$ is a cutoff value for negative stress ratios (with $-\mathbf{1} < \mathbf{R^{-}} < \mathbf{0}$). The overload plastic zone $\mathbf{Z_{ol}}$ is then replaced in equation (4) by its effective value $\mathbf{(Z_{ol})_{eff}}$ to calculate the (increased) value of $\mathbf{\Delta K_{eff}}$.

Equations (4-5), when applied to a crack propagation law that recognizes $\mathbf{\Delta K_{th}}$, can effectively predict crack retardation, arrest, and even acceleration. However, for a constant amplitude loading history, all Wheeler and Willenborg-type models predict **no** retardation, since the current plastic zone is always on or slightly ahead of the previous ones, never embedded. Even though these load interaction models are indirectly based on Elber's crack closure idea, none of them can predict the reduction in crack growth due to closure under constant amplitude. The reason for this idiosyncrasy is that most load interaction models only consider the retardation effects due to **secondary** plasticity, not **primary**. Primary plasticity refers to a plastic zone that causes yielding on previously virgin material, while secondary plasticity is termed for a plastic zone fully contained within a previous plastic zone generated by an overload, see Figure 1. Both primary and secondary plasticities cause a rise in the crack opening stress, decreasing the crack growth rate, however only secondary plasticity is considered in such load interaction models.

To consider the retardation effect due to **both** primary and secondary plasticities, a load interaction model called ***Generalized Wheeler*** is proposed here. Based on the Modified Wheeler model and on Chang's crack acceleration concept, the Generalized Wheeler model calculates the effective stress intensity range by

$$\mathbf{\Delta K_{eff}} = \mathbf{\Delta K} \cdot \left(\frac{\mathbf{1-f}}{\mathbf{1-R}}\right)\left(\frac{\mathbf{Z_i}}{(\mathbf{1}+\overline{\mathbf{R}}_{\mathbf{ul}})\mathbf{Z_{ol}} + \mathbf{a_{ol}} - \mathbf{a_i}}\right)^{\gamma'}, \quad \mathbf{a_i} + \mathbf{Z_i} < \mathbf{a_{ol}} + \mathbf{Z_{ol}} \quad (6)$$

where $\overline{\mathbf{R}}_{\mathbf{ul}}$ is defined in equation (5), $\mathbf{f}$ is the Newman closure function [14], defined as the ratio $\mathbf{K_{op}/K_{max}}$ between the crack opening and the maximum stress intensity factors at each cycle, and $\boldsymbol{\gamma'}$ is an experimentally adjustable constant. This proposed model recognizes crack retardation and arrest due to overloads, crack acceleration (reduction in retardation) due to underloads, and even retardation due to crack closure under constant amplitude loading. Another advantage of the Generalized Wheeler model is that it can be applied to **any da/dN** equation, in contrast with the Willenborg model, which can only be applied to **da/dN** equations that explicitly model the stress ratio **R**, as explained below.

3.3. $\mathbf{R_{eff}}$ Interaction Models

In the $\mathbf{R_{eff}}$ models, an effective stress ratio $\mathbf{R_{eff}}$ is introduced, calculated by reducing the maximum and minimum stress intensity factors acting on the crack tip. The best-known $\mathbf{R_{eff}}$ model is the ***Willenborg*** model [8]. As in the Wheeler model, the retardation for a given applied cycle depends on the loading and the extent of crack growth into the overload plastic zone. Willenborg et al. assumed that the maximum stress intensity factor $\mathbf{K_{max}}$ occurring at the current crack length $\mathbf{a_i}$ will be reduced by a residual stress intensity $\mathbf{K_R}^W$. The value of $\mathbf{K_R}^W$ is calculated, more or less arbitrarily, from the difference between the stress intensity required to produce a plastic zone that would reach the overload zone border (distant $\mathbf{Z_{ol}} + \mathbf{a_{ol}} - \mathbf{a_i}$ from the current crack tip) and the current maximum applied stress intensity $\mathbf{K_{max}}$,

$$K_R{}^W = K_{ol}\sqrt{(Z_{ol} + a_{ol} - a_i)/Z_{ol}} - K_{max} \tag{7}$$

where $\mathbf{K_{ol}}$ is the maximum stress intensity of the overload, $\mathbf{Z_{ol}}$ is the overload plastic zone size, and $\mathbf{a_{ol}}$ is the crack size at the ocurrence of the overload (see Figure 1).

Willenborg et al. expect that both stress intensity factors $\mathbf{K_{max}}$ and $\mathbf{K_{min}}$ at the current *i*-th cycle are effectively reduced by an amount $\mathbf{K_R}^W$. Since the stress-intensity range $\Delta\mathbf{K}$ is unchanged by the uniform reduction, the retardation effect is only sensed by the change in the effective stress ratio $\mathbf{R_{eff}}$ calculated by

$$R_{eff} = \frac{K_{eff,min}}{K_{eff,max}} = \frac{K_{min} - K_R{}^W}{K_{max} - K_R{}^W} \tag{8}$$

As a result, crack propagation rules that do not model explicitly the effects of the stress ratio **R** ***cannot*** be used with the Willenborg retardation model. For instance, if the Paris law is used for crack propagation, the Willenborg model will ***not*** predict crack retardation after overloads, since the value of $\Delta\mathbf{K}$ remains unchanged (and thus the value of **da/dN** as well). This is a limitation of the original Willenborg formulation, not present in the Wheeler model.

A problem in the original Willenborg model is the prediction that $\mathbf{K_{eff,max}} = \mathbf{0}$ (and therefore crack arrest) immediately after overload if $\mathbf{K_{ol}} \geq \mathbf{2}\ \mathbf{K_{max}}$. That is, if the overload is twice as large as (or larger than) the following loads, the Willenborg model implies that the crack always arrests.

To account for the observations of continuing crack propagation after overloads larger than a factor of two or more (i.e. shut-off ratios larger than 2), Gallagher [15] generalized Willenborg's original development [8] by introducing an empirical (spectra/material) constant into the calculations. In Gallagher's ***Generalized Willenborg*** model, a modified residual stress intensity $\mathbf{K_R} = \Phi\cdot\mathbf{K_R}^W$ is used, instead of Willenborg's original $\mathbf{K_R}^W$, where Φ is given by

$$\Phi = \frac{1 - \Delta K_{th}/\Delta K}{R_{so} - 1} \tag{9}$$

and $\mathbf{R_{so}}$ is a constant defined as the overload (shut-off) ratio required to cause crack arrest. Using this constant it is possible to model shut-off ratios different than 2 for $\mathbf{K_{ol}/K_{max}}$, compensating for this Willenborg's original model limitation. Typical values for $\mathbf{R_{so}}$ are 3.5 for steel and nickel alloys, and 2.3 for aluminum and titanium alloys. The value of the shut-off ratio $\mathbf{R_{so}}$ is not only material-dependent, but it is also affected by the stress level and the frequency of overload cycle occurrence. Note that in general the GW model predicts larger $\mathbf{R_{eff}}$ values (and thus **less** retardation) than Willenborg's original model. However, in the Generalized Willenborg model, no special consideration is given to multiple overloads or stress levels, and their effect is taken to be the same as that for a single overload. Also, this model cannot predict the observed reduction in crack retardation after underloads.

Several other $\mathbf{R_{eff}}$ models have been proposed, such as the ***Modified Generalized Willenborg*** and the ***Walker-Chang Willenborg*** [16, 17]. However, even with all proposed modifications to improve the original Willenborg model, the assumption regarding the residual compressive stresses through the residual stress intensity $\mathbf{K_R^W}$ is still very doubtful [2]. To better model the closure effects on crack retardation, some methods **directly** estimate the value of the opening stress intensity factor $\mathbf{K_{op}}$, instead of indirectly accounting for its effects through arbitrary parameters such as $\mathbf{K_R^W}$. These $\mathbf{K_{op}}$ load interaction models are presented next.

3.4. $\mathbf{K_{op}}$ Interaction Models

In the $\mathbf{K_{op}}$ models, the opening stress intensity factor $\mathbf{K_{op}}$ caused by an overload is directly computed and applied to the **subsequent** crack growth to account for Elber-type crack closure. Perhaps the simplest $\mathbf{K_{op}}$ model is the ***Constant Closure*** model, originally developed at Northrop for use on their classified programs [18]. This load interaction model is based on the observation that for some load spectra the closure stress does not deviate substantially from a certain stabilized value. This stabilized value is determined by assuming that the spectrum has a "controlling overload" and a "controlling underload" that occur often enough to keep the residual stresses constant, and thus the closure level constant.

In the constant closure model, the opening stress intensity factor $\mathbf{K_{op}}$ is the only empirical parameter, with estimates between 30% and 50% of the maximum overload stress intensity factor. The crack opening stress $\mathbf{K_{op}}$ can also be calculated, for instance, from Newman's closure function [14], using the stress ratio **R** between the controlling underload and overload stresses. The value of $\mathbf{K_{op}}$, calculated for the controlling overload event, is then applied to the following (smaller) loads to compute crack growth, recognizing crack retardation and even crack arrest (if $\mathbf{K_{max} \leq K_{op}}$).

The main limitation of the Constant Closure model is that it can only be applied to loading histories with "frequent controlling overloads", because it does **not** model the **decreasing** retardation effects as the crack tip cuts through the overload plastic zone, as shown in equations (3) and (7) for the Wheeler and Willenborg models. In the Constant Closure model, it is assumed that a new overload zone, with primary plasticity, is formed often enough before the crack can significantly propagate through the previous plastic zone, thus not modeling secondary plasticity effects by keeping $\mathbf{K_{op}}$ constant.

To account for crack retardation due to **both** primary and secondary plasticity, the European Space Agency and the National Aerospace Laboratory in the Netherlands, in cooperation with NASA, developed the ***DeKoning-Newman Strip Yield*** load interaction model [19, 20]. In this model, a crack growth law is described in an **incremental** way, modeling crack growth attributed to increments $\mathbf{\delta K}$ in the stress intensity factor as the crack changes from closed to fully open configurations. This incremental law is then integrated **at each cycle** from the minimum to the maximum stress intensity factors $\mathbf{K_{min}}$ and $\mathbf{K_{max}}$ to find the crack growth rate **da/dN**.

This incremental description also allows that a distinction be made between the part of a load range where secondary plastic flow is observed and the part where primary plastic flow starts developing under a monotonic increasing load. For each of these domains, a different incremental crack growth law can be formulated [1]. In this way, it is possible to effectively calculate fatigue crack growth considering retardation and arrest due to **both** primary and secondary plasticity. However, the penalty of this approach is the large number of required experimental constants, in addition to the numerical effort.

There are several other load interaction models in the literature [21, 22], but none of them has definitive advantages over the models discussed above. This is no surprise, since single equations are too simplistic to model all the several mechanisms that can induce retardation effects. Therefore, in the same way that a curve **da/dN** vs. **ΔK** must be experimentally measured, a load interaction model must be calibrated through experimental data, as recommended by Broek [2].

4. Comparison of the Load Interaction Models

All load interaction models presented in this work have been implemented in **ViDa**, a powerful software developed to automate the fatigue dimensioning process by **all** the traditional methods used in mechanical design [23-25]. This software has been developed to predict **both** initiation and propagation fatigue lives under complex loading by **all** classical design methods: **SN**, **IIW** (for welded structures) and **εN** to predict crack initiation, and **da/dN** for studying 1D and 2D crack propagation, **considering** retardation effects. Here, the presented load interaction models and the proposed modifications are compared, using the efficient numerical methods in **ViDa**, with experimental results from various load histories.

Using **ViDa**, the crack propagation life of an 8-mm-thick center-cracked tensile specimen, made of a 7475-T7351 aluminum alloy, is calculated using several block loading histories, including underload and overload events (see Table 1). The calculated lives are compared to experimental tests performed by Zhang [26, 27], who measured crack growth rates through scanning electron microscopy. Forman's da/dN equation proposed in [5] is used to compute crack growth (in mm), using $\mathbf{A = 6.9 \cdot 10^{-7}}$, $\mathbf{m = 2.212}$, $\mathbf{p = 0.5}$, $\mathbf{q = 1.0}$, $\Delta \mathbf{K_0} = \mathbf{3MPa\sqrt{m}}$, and $\mathbf{K_C} = \mathbf{73MPa\sqrt{m}}$. All models are calibrated using the first history (except for the Willenborg model, without parameters to be adjusted), and a comparison is made through the remaining loadings. Table 1 shows Zhang's test results and the percentage error of the predicted lives using several load interaction models.

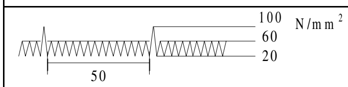
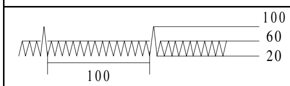
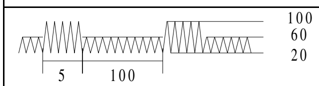
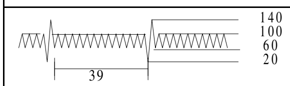
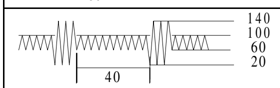
Loading history	Zhang's test life (cycles)	No Interaction	Wheeler	Gener. Wheeler	Willenborg	Gener.Willenb.	Modif.Gen.Will.	Const. Closure
	474,240	–17%	0%	0%	+293%	0%	0%	0%
	637,730	–36%	–22%	–22%	+269%	–22%	–22%	–22%
	409,620	–13%	+2.3%	+2.3%	+185%	+2.2%	+2.6%	+2.4%
	251,050	–27%	–18%	–18%	+27%	–20%	+24%	–75%
	149,890	–2.3%	+6.6%	+6.4%	+44%	+4.0%	+42%	–60%

Table 1 – Percentage errors in the test lives predicted by the load interaction models

As expected, the Willenborg model resulted in poor predictions, since this model cannot be calibrated. The remaining models performed similarly for the second and third histories. However, as the maximum load increased from 100 to 140 MPa in the last two histories, the Modified Generalized Willenborg and the

Constant Closure models showed increased errors. The Wheeler and Generalized Wheeler models performed very similarly, because the considered histories didn't include compressive underloads. These two models and the Generalized Willenborg model resulted in the best predictions for Zhang's histories. Unfortunately, these tests were performed under the second crack growth regimen (not influenced by the threshold ΔK_{th}), so the advantages of the Modified and Generalized Wheeler in modeling crack arrest could not be exemplified here.

5. Conclusions

In this work, load interaction effects on fatigue crack propagation were discussed. Overload-induced retardation effects on the crack growth rate were evaluated using several different models, and improvements to the traditional equations were proposed to recognize crack arrest and acceleration due to compressive underloads. The models have been implemented on a general-purpose fatigue design software named **ViDa**, developed to predict both initiation and propagation fatigue lives under complex loading by all classical design methods. Using this software, the presented load interaction models and the proposed modifications were compared with experimental results on center-cracked tensile specimens for various load spectra. In particular, the proposed modifications to the Wheeler model showed a good agreement with the experimental data.

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