PROPAGATION OF 2D FATIGUE CRACKS UNDER COMPLEX LOADING

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Summary

The modeling of mode I fatigue propagation under complex loading of 2D cracks, which grow in two dimensions changing shape at each cycle, is discussed in this paper. Sequence effects, such as overload-induced crack retardation or arrest are also considered, based on the crack closure idea. A description and a taxonomy of these load interaction models are presented in a companion paper. The numerical implementation of these models is discussed, in particular their application to the **ViDa** software, developed to automate the fatigue dimensioning process under complex loading by all the traditional methods used in mechanical design.
Key-Words: fatigue, crack propagation, sequence effects.

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1. Introduction

Two-dimensional (2D) fatigue crack propagation usually is non-homologous: the crack tends to change form from cycle to cycle, because **ΔK** varies from point to point along the crack front. Moreover, load cycle interactions can have a very significant effect on fatigue crack growth. Tensile overloads can retard or arrest the crack, substantially affecting the rate of subsequent crack propagation [1-3].

However, fatigue life **prediction** of 2D cracks **considering** overload-induced retardation can be a challenging task: the changing form of the crack front may alter its stress state, which in turn changes the shape of the plastic zone ahead of the crack and the associated retardation effect. In addition, such retardation effects may vary along the crack front, e.g. causing crack arrest at its width direction but not at its depth. Such example is deceiving in inspections, since the trace of a surface crack can remain constant when in fact it is growing toward the inside of the piece.

In this work, the modeling of fatigue propagation of 2D cracks under complex loading is discussed. Load interaction effects, based on crack closure, are reviewed and introduced in the formulation. The numerical implementation of the calculation routines is presented, in particular their application to **ViDa**, a powerful software developed to automate the fatigue dimensioning process by **all** the traditional methods used in mechanical design.

2. Propagation of 2D Cracks

To calculate 2D crack propagation, stress intensity factor expressions must be obtained. There are analytical expressions for the stress intensity factor of some 2D cracks, however most solutions assume that the crack fronts have an elliptical shape. If the cracks have ellipsoidal fronts, and if they are built in a plate of width **w** or **2w** and thickness **t**, then **ΔK** is a function of Δσ, **a**, **a/c**, **a/t**, **c/w** and θ [4-6], where **a** and **c** are the ellipsis semi-axes, and θ is defined in Figure 1.

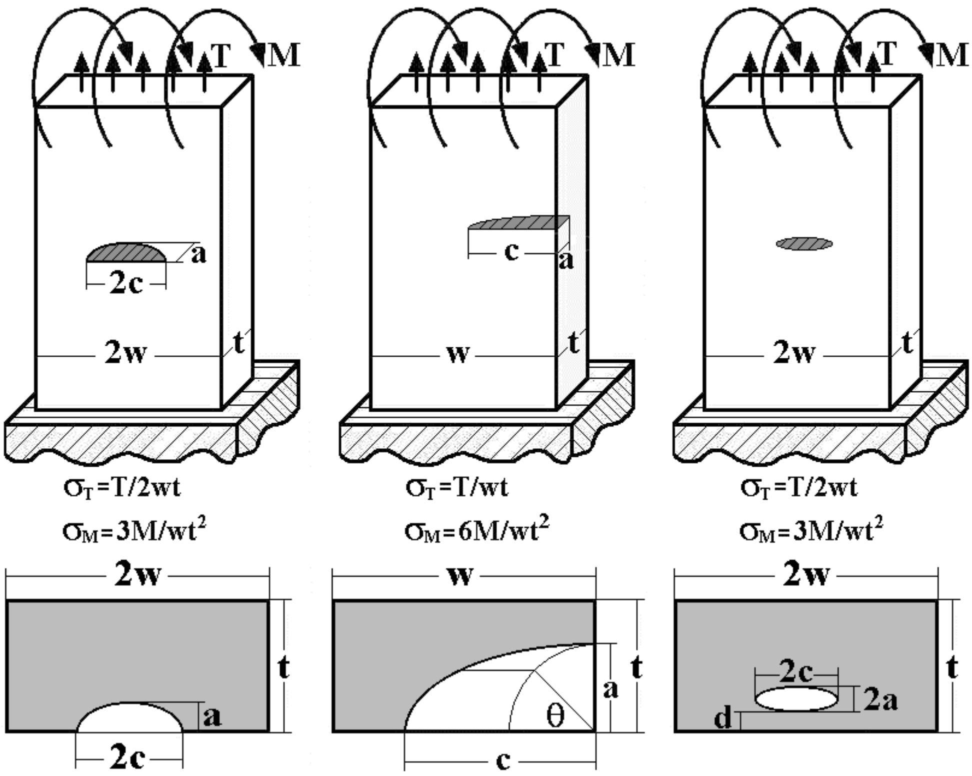

Figure 1 – Surface semi-elliptical, corner quart-elliptical and internal elliptical cracks

On the other hand, the 2D ellipsoidal crack propagation problem is a reasonable approximation for many actual surface, corner or internal cracks. Fractographic observations indicate that the successive fronts of those cracks tend to achieve an elliptical form, see Figure 2, and to stay approximately elliptic during their fatigue propagation, even when the initial crack shape is far from an ellipsis [7]. Therefore, it can be quite reasonable to assume in the modeling that the fatigue propagation just changes the shape of the 2D cracks (given by the ratio **a/c** between the ellipsis semi-axes), but preserves their basic ellipsoidal geometry.

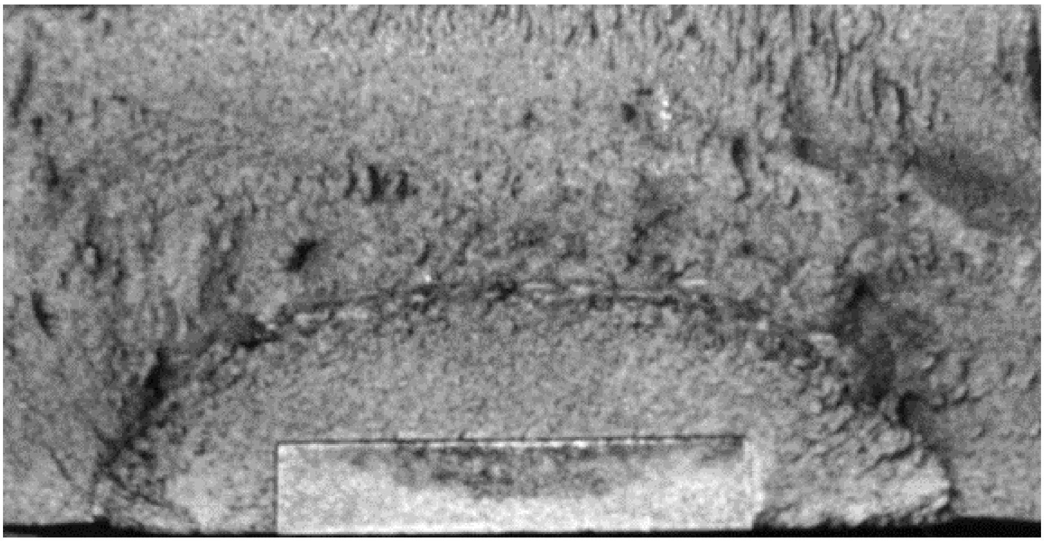

Figure 2 – A surface fatigue crack that started from a sharp rectangular notch and grew with an approximately semi-elliptical front

As an ellipsis is completely defined by its two semi-axes, to predict the growth of 2D (elliptical) cracks, including their shape changes, it is enough to calculate at each load cycle the lengths of the ellipsis axes **a** and **c**, **jointly** solving the **da/dN** and the **dc/dN** propagation problems. In addition, since these cracks have different values for **ΔK(a)** and **ΔK(c)**, there are three distinct 2D propagation cases under **simple loading** (assuming **ΔK(a)** > **ΔK(c)** to start with):

- **ΔK(a)** and **ΔK(c) > ΔK_{th}**: the crack spreads in both directions, changing shape at each **i**-th load cycle depending on the ratio **ΔK(a)/ΔK(c)**.
- **ΔK(a) > ΔK_{th}** and **ΔK(c) < ΔK_{th}**: the crack grows only in the **a** direction, until its size is big enough to make **ΔK(c) > ΔK_{th}**, when the problem reverts to case 1 (there are, however, pathological cases where **ΔK(a) decreases** with **a**; in these cases a crack can start spreading to later on stop if it reaches **ΔK(a) < ΔK_{th}**).
- **ΔK(a)** and **ΔK(c) < ΔK_{th}**: the crack does not propagate.

However, when complex loadings are introduced, several other propagation cases must be considered. To effectively calculate the crack propagation behavior under complex loading, load interaction effects must be taken into account. The next section reviews the main load interaction models and their application to 2D crack propagation calculations.

3. Load Interaction Models

Several mathematical models have been developed to account for load interaction in crack propagation based on Elber's crack closure idea [8]. In these methods, the retardation mechanism is only considered within the plastic zone situated in front of the crack tip. According to these procedures, a larger plastic zone is created by means of an overload. When the overload is removed, an increased compressive stress state is set up in the volume of its plastic zone, reducing crack propagation under a smaller succeeding load cycle.

A review of the main load interaction models in fatigue crack growth is found in a companion paper [9], including proposed modifications to better model such effects as crack arrest and crack acceleration due to compressive underloads. A taxonomy of these models has been presented [9], dividing them in 4 categories: (i) **da/dN** models, such as the Wheeler model, which use retardation functions to directly reduce the calculated crack propagation rate **da/dN**; (ii) **ΔK** models, which use retardation functions to reduce the value of the stress intensity factor range **ΔK**; (iii) $\mathbf{R_{eff}}$ models, such as the Willenborg model, which introduce an effective stress ratio $\mathbf{R_{eff}}$, calculated by reducing the maximum and minimum stress intensity factors acting on the crack tip, however not necessarily changing the value of **ΔK**; and (iv) $\mathbf{K_{op}}$ models, such as the strip yield model, which use estimates of the opening stress intensity factor $\mathbf{K_{op}}$ to directly account for Elber-type crack closure. The main load interaction models and their equations - the variables definition is found in [9] - are shown in Table 1.

type	model	equations
$\mathbf{\frac{da}{dN}}$	Wheeler	$\left(\frac{da}{dN}\right)_{eff,i} = \left(\frac{da}{dN}\right)_i \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i}\right)^{\beta}$
ΔK	Modified Wheeler	$\Delta K_{eff}(a_i) = \Delta K(a_i) \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i}\right)^{\gamma}, \ R_{eff} = 1 - \frac{\Delta K_{eff}(a_i)}{K_{max}}$
	General. Wheeler	$\Delta K_{eff} = \Delta K \cdot \left(\frac{1-f}{1-R}\right)\left(\frac{Z_i}{(1+\bar{R}_{ul})Z_{ol} + a_{ol} - a_i}\right)^{\gamma'}, \bar{R}_{ul} = \begin{cases} 0 \ (R_{ul} \geq 0) \\ R_{ul} \ (R^- < R_{ul} < 0) \\ R^- \ (R_{ul} \leq R^-) \end{cases}$
$\mathbf{R_{eff}}$	Willenb.	$R_{eff} = \frac{K_{min} - K_R{}^W}{K_{max} - K_R{}^W}, \quad K_R{}^W = K_{ol}\sqrt{(Z_{ol} + a_{ol} - a_i)/Z_{ol}} - K_{max}$
	General. Willenb. (GW)	$R_{eff} = \frac{K_{min} - K_R}{K_{max} - K_R}, \ K_R = \frac{1 - \Delta K_{th}/\Delta K}{R_{so} - 1} K_R{}^W$
$\mathbf{K_{op}}$	Constant Closure	$K_{op} = f \cdot K_{max}$

	Strip Yield	$$\frac{da}{dN}=\begin{cases} 0, \quad K_{max}<K_{op}+\Delta K_{th} \\ A\cdot(K_{max}-K_{op})^n\left[1-\left(\frac{\Delta K_{th}}{K_{max}-K_{op}}\right)^p\right], \quad K_{op}+\Delta K_{th}\leq K_{max}\leq K_* \\ A\cdot(K_*-K_{op})^n\left[1-\left(\frac{\Delta K_{th}}{K_*-K_{op}}\right)^p\right]+A_p[K_{max}^{\ m}-K_*^{\ m}], \quad K_*<K_{max}<K_C \end{cases}$$

Table 1 – Review of the load interaction model equations

A qualitative comparison of the load interaction models, showing their main advantages and disadvantages, is presented in Table 2.

	$\frac{da}{dN}$	ΔK models		R_{eff} models				K_{op} models	
	Wheeler	Modif.Wheeler	Gener.Wheeler	Willenborg	Gener.Willenb.	Mod.Gen.Will.	Walker-Chang Willenborg	Const. Closure	Strip Yield
models crack retardation	√	√	√	√	√	√	√	√	√
models crack arrest		√	√	√	√	√	√	√	√
models crack acceleration			√			√	√	√	√
works for any load spectrum	√	√	√	√	√	√	√		√
works with any da/dN equation	√	√	√					√	
models primary plasticity			√						√
number of experimental param.	1	1	1-2	0	1	1	1-2	1	5

Table 2 – Qualitative comparison of the load interaction models

The load interaction models presented above have been originally developed for planar (1D) cracks, but many times it is necessary to study surface, corner or internal cracks, which spread in 2D. Using these concepts, it is not particularly difficult to model load interaction effects in 2D crack propagation. The idea is to maintain the fundamental hypothesis of the ellipsoidal geometry preservation, accounting for the coupled (retarded) growth of the semi-axes **a** and **c**.

However, as the sizes of the plastic zones depend on the value of ΔK, and as in general $\Delta K(a) \neq \Delta K(c)$, the retardation effects in 2D growth can be different in each direction. For instance, in surface part-through cracks it is recommended to use an equation for the plastic zone size under **plane strain** to compute retardation effects at the maximum depth (**a**), while a **plane stress** equation should be used at the width direction (**2c**). Thus, it is necessary to apply the load interaction models **independently** to both crack propagation directions. The next section shows how to numerically implement the above models in 2D crack propagation calculations.

4. Numerical Implementation of 2D Crack Propagation

The numerical implementation of 2D fatigue crack propagation under complex loading by the **local** approach is discussed below. Sequence effects, such as overload-induced crack retardation or arrest are also considered. The local approach is so called because it does not require the global solution of the structure's stress field, since it is based on the direct integration of the fatigue crack propagation rule of the material, $\mathbf{da/dN = F(\Delta K, R, \Delta K_{th}, K_C, ...)}$, where $\mathbf{\Delta K}$ is the stress intensity range, $\mathbf{R = K_{min}/K_{max}}$ is a measure of the mean load, $\mathbf{\Delta K_{th}}$ is the fatigue crack propagation threshold and $\mathbf{K_C}$ is the toughness of the structure. An appropriate stress intensity factor expression for $\mathbf{\Delta K}$ and a good **da/dN** rule must be used to obtain satisfactory predictions. Therefore, neither the $\mathbf{\Delta K}$ expression nor the type of crack propagation rule should have their accuracy compromised when using this approach.

In the sequence of this text, first the main features of the software **ViDa** are concisely described. This software has been developed to automate all the traditional local approach methods used in fatigue design [10-12], including the **SN**, the **IIW** (for welded structures) and the $\boldsymbol{\varepsilon}$**N** for crack initiation, and the **da/dN** for crack propagation. Then the 2D cycle-by-cycle method is discussed, including the modeling of load sequence effects.

4.1. The ViDa software

The objective of this software is to automate in a friendly environment all the calculations required to predict fatigue life under complex loading by the local approach. It runs on PCs under Windows 95 or better operating system, and it includes all the necessary tools to perform the predictions, such as intuitive and friendly graphical interfaces in multiple idioms; intelligent databases for stress concentration and intensity factors, crack propagation rules, material properties, etc.; traditional and sequential Rain-Flow counters, graphic generators of elastoplastic hysteresis loops and of 2D crack fronts; automatic adjustment of crack initiation and propagation experimental data; an equation interpreter, etc. The software calculates crack growth considering any propagation rule and any $\mathbf{\Delta K}$ expression that can be typed in.

Here, the stress intensity factor range is expressed as $\mathbf{\Delta K = \Delta\sigma\cdot[\sqrt{(\pi a)}\cdot f(a/w)]}$, where $\Delta\sigma$ is the nominal stress range (in relation to which the $\mathbf{\Delta K}$ expression is defined), **a** is the crack length, **f(a/w)** is a non-dimensional function of **a/w**, and **w** is a characteristic size of the structure. Therefore, $\Delta\sigma$ quantifies the influence of the loading and $\mathbf{\sqrt{(\pi a)}\cdot f(a/w)}$ quantifies the effect of the geometry of the piece and of the crack shape and size in $\mathbf{\Delta K}$.

The propagation is calculated at each load event. An event is defined by a block of simple load, in which the loading remains constant during **n** cycles, or at each variation of the load amplitude in the complex case. In any case, the software automatically stops the calculations, and indicates the value of the parameters that caused the stop, if during the loading it detects:

- fracture if $\mathbf{K_{max} = K_C}$, or if
- the crack reaches its maximum specified size, or if
- the stress in the residual ligament reaches the material rupture strength $\mathbf{S_U}$, or if
- the crack propagation rate **da/dN** reaches 0.1mm/cycle (above this rate the problem is fracturing, not fatigue cracking), or else if

- one of the borders of the piece is reached by the front of the crack, in the 2D crack propagation case. However, for some geometries, the software is able to calculate 2D crack propagation **after** the borders of the piece are reached, by modeling the stress intensity factors of the transition from part-through to through cracks.

Moreover, the software informs when there is yielding in the residual ligament before the maximum specified crack size or number of load cycles is reached. In this way, the calculated values can be used with the guarantee that the limit of validity of the mathematical models is never exceeded.

4.2. The 2D Cycle-by-Cycle Method

The basic idea of this method is to associate to each load reversion the growth that the crack would have if that 1/2 cycle was the only one to load the piece (this implicates in neglecting interaction effects among the several events of a complex loading, such as overload-induced retardation or stop in the crack growth).

If in the **i**-th loading event the ellipsoidal crack has semi-axes $\mathbf{a_i}$ and $\mathbf{c_i}$, under

$$\Delta\mathbf{K}(\mathbf{a_i}) = \Delta\sigma_i\cdot[\sqrt{(\pi a_i)}\cdot f_a(a_i/c_i,\ a_i/t,\ c_i/w)] \quad \text{and} \tag{1}$$

$$\Delta\mathbf{K}(\mathbf{c_i}) = \Delta\sigma_i\cdot[\sqrt{(\pi c_i)}\cdot f_c(a_i/c_i,\ a_i/t,\ c_i/w)] \tag{2}$$

stress intensity ranges, and if $\mathbf{da/dN = F(\Delta K,\ R,\ \Delta K_{th},\ K_C,...)}$ is the crack growth rule of the material, the stress range is $\Delta\sigma_i$, and the mean load causes **R**, then the crack increment in this **i**-th 1/2 cycle is given by:

$$\delta a_i = \frac{1}{2}\cdot F(\Delta K(\Delta\sigma_i, a_i, f_a), R(\Delta\sigma_i, \sigma_{max_i}), \Delta K_{th}, K_C, ...) \quad \text{and} \tag{3}$$

$$\delta c_i = \frac{1}{2}\cdot F(\Delta K(\Delta\sigma_i, c_i, f_c), R(\Delta\sigma_i, \sigma_{max_i}), \Delta K_{th}, K_C, ...) \tag{4}$$

The crack growth is calculated by the simultaneous solution of $\Sigma\delta\mathbf{a_i}$ and $\Sigma\delta\mathbf{c_i}$. Therefore, the cycle-by-cycle rule is similar in concept to the linear damage accumulation used in the **SN** and **εN** fatigue design methods. And, as in Miner's rule, it requires that **all** the events that cause fatigue damage be recognized before the calculation, by rain-flow counting the loading.

However, this counting algorithm alters the **order** of the loading, as shown in Figure 3. This can cause serious problems in the predictions, because the loading order effects in crack propagation are of two different natures:

- delayed effects, that can retard or stop the subsequent growth of the crack due, e.g., to plasticity-induced Elber-type crack closure [13] or to crack tip bifurcation. These interaction effects among the loading cycles normally increase the crack life and, if neglected in the cycle-by-cycle calculation, may induce excessively conservative predictions.
- instantaneous fracture, that occurs when $\mathbf{K_{max} = K_C}$ in **one** event, which must be precisely predicted.

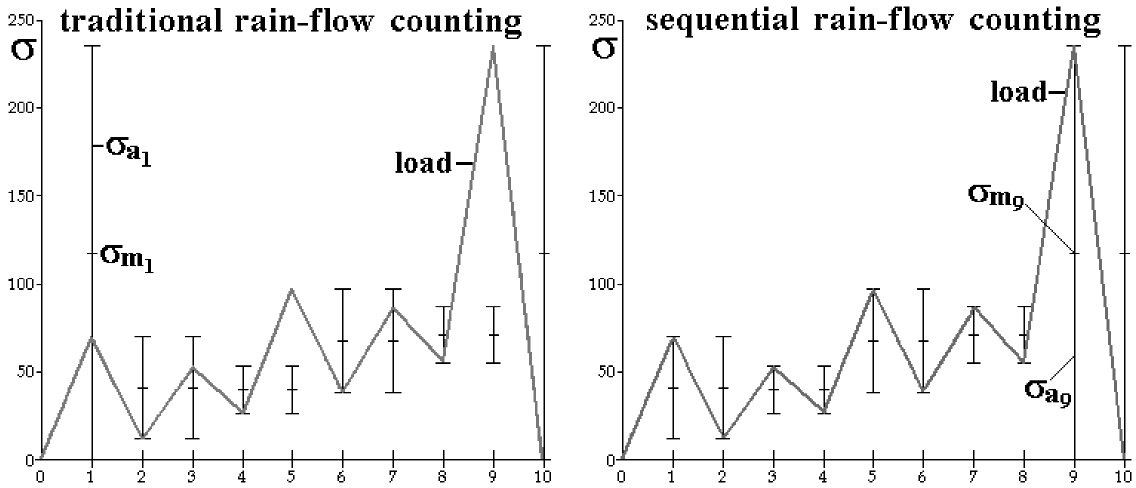

Figure 3 – Traditional rain-flow counting (anticipating the large load events) and Sequential rain-flow counting (preserving most of the loading order)

As already mentioned above, the loading input in the **ViDa** software is sequential, and preserves the time order information that is lost when histograms or any other loading statistics are generated. To take advantage of this feature, a **sequential** rain-flow counting option was introduced in that software. With this technique, the effect of each large loading event is counted when it happens (and not before its occurrence, as in the traditional rain-flow method). The idea is explained in Figure 3.

The main advantage of the sequential rain-flow counting algorithm is to avoid the premature calculation of the overload effects, which can cause **non**-conservative crack propagation life predictions (as $\mathbf{K(\sigma, a)}$ in general grows with the crack, a given overload applied when the crack is large can be much more harmful than applied when the crack is small). The sequential rain-flow does not eliminate all the sequencing problems caused by the traditional method, but it is certainly an advisable option because it presents advantages over the original algorithm, without increasing its difficulty.

Since the crack does not grow while closed, the compressive part of the loading should be discarded, that is, the negative peaks and valleys can be zeroed before the computations to decrease the numerical effort of the cycle-by-cycle method. And, in the same way, a range filtering option can be very useful to discard the small loads that cause no damage inducing $\mathbf{\Delta K_i < \Delta K_{th}(R_i)}$, following the ideas of the race-track method [14].

The computational implementation of equations (3-4), even with the pre-zeroing of the compressive peaks and valleys and with the range filtering of the loading, is still not numerically efficient. For this reason, an additional feature to reduce the computational time can be quite useful: the option of maintaining the geometrical part of $\mathbf{\Delta K}$ constant during small variations in crack size.

As $\mathbf{\Delta K_a = \Delta\sigma \cdot [\sqrt{(\pi a)} \cdot f_a]}$ and $\mathbf{\Delta K_c = \Delta\sigma \cdot [\sqrt{(\pi c)} \cdot f_c]}$, where $\mathbf{f_a}$ and $\mathbf{f_c}$ are non-dimensional functions (usually complicated) that depend only on the piece and crack geometry and not on the loading, it can be said that the range of the stress intensity factor at each load reversion depends on two variables of different nature:

- on the stress range $\Delta\sigma_i$ in that event, and
- on the crack dimensions $\mathbf{a_i}$ and $\mathbf{c_i}$ in that instant.

$\Delta\sigma_i$, of course, can vary significantly at each load reversion when the loading is complex, but fatigue cracks always grow very slowly. In fact, at least in structural metals, the largest observed rates of stable growth are of the order of μm/cycle, and during most of the life the crack growth rates are better measured in nm/cycle.

However, as in general the usually complicated $\mathbf{f_a}$ and $\mathbf{f_c}$ expressions do not present discontinuities, one can take advantage of the small changes in $\mathbf{f_a}$ and $\mathbf{f_c}$ during small increments in crack length. In this way, instead of calculating at each load cycle the value of $\Delta K_i = \Delta\sigma_i \cdot [\sqrt{(\pi a_i)} \cdot f_a]$, a task that demands great computational effort, it is more efficient to hold $\mathbf{f_a}$ (or $\mathbf{f_c}$) constant during a (small) percentage of crack increment **δa%** (or **δc%**), that should be specifiable by the calculation software user. A crack increment equivalent to 50μm, a number of the order of the resolution threshold of crack measurement methods in fatigue tests, can be a good choice both from the physical and numeric points of view.

4.3. Load Cycle Interaction Effects

The numeric implementation of load interaction models in a cycle-by-cycle algorithm is not conceptually difficult, but it requires a considerable programming effort. For instance, when applying the **Modified Wheeler** model [15] (which predicts crack retardation and arrest) to the 2D cycle-by-cycle algorithm, the effective values of the stress intensity ranges, $\Delta K_{eff}(a_i)$ and $\Delta K_{eff}(c_i)$, are calculated by:

$$\Delta K_{eff}(a_i) = \Delta K(a_i) \cdot \left(\frac{Z_i(a_i)}{Z_{ol}(a_{ol}) + a_{ol} - a_i} \right)^{\gamma}, \quad a_i + Z_i(a_i) < a_{ol} + Z_{ol}(a_{ol}) \text{ and} \tag{5}$$

$$\Delta K_{eff}(c_i) = \Delta K(c_i) \cdot \left(\frac{Z_i(c_i)}{Z_{ol}(c_{ol}) + c_{ol} - c_i} \right)^{\gamma}, \quad c_i + Z_i(c_i) < c_{ov} + Z_{ol}(c_{ol}) \tag{6}$$

From these values, it is easy to calculate crack growth in the two directions, replacing ΔK by ΔK_{eff} in equations (3) and (4).

Only the Modified Wheeler model is presented above, but it is trivial to write similar equations for the other load interaction models. All load interaction models presented in this paper have been successfully implemented in the **ViDa** software, predicting crack retardation, arrest, and even acceleration after compressive or tensile underloads. To illustrate the main ideas in the numerical implementation of these models, a simplified flow-chart of the calculation algorithm used in the **ViDa** software is shown in Figure 4.

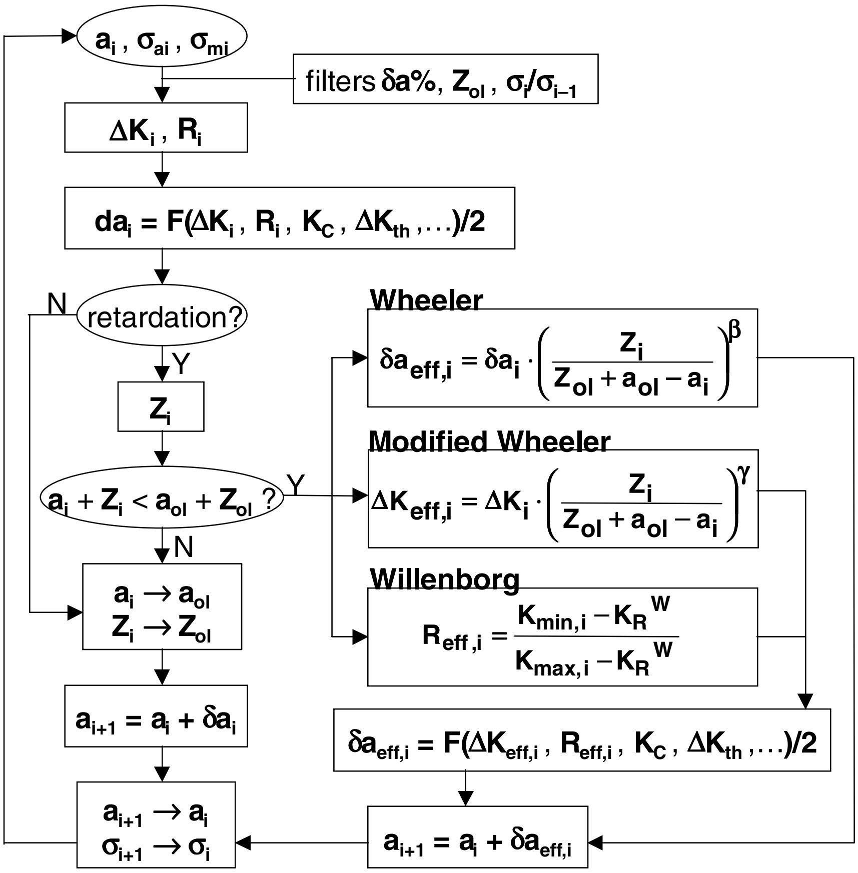

Figure 4 – Simplified flowchart of the calculation algorithm used in the **ViDa** software to predict fatigue crack propagation under complex loading

Some calculation details are worth mentioning. The first one refers to the use of a **varying** crack size step, instead of a constant one, for the integration of the **da/dN** curve. By choosing an integration step that is a percentage **δa%** of the crack size at each cycle, the numerical computation is much more efficient. However, such crack size increments may cause trouble when combined with load interaction models, since the plastic zone sizes can be very small compared to the current crack size. To account for this, **ViDa** compares **δa%** at each step with the size of the overload plastic zone $\mathbf{Z_{ol}}$, replacing (if necessary) the value **δa%** of the integration step by a percentage of $\mathbf{Z_{ol}}$.

A second detail can save a lot of computational time when the loading is complex. Small variations in the loading amplitude do not cause experimentally detectable crack retardation, and they should not be considered as overloads in the calculation model. Therefore, a numerical filter for overloads can be profitably introduced in the algorithm, specifying that there is no overload effect if $\sigma_i/\sigma_{i\_1} < \alpha_{ol}$, where σ_{i_1} and σ_i are consecutive loading peaks, and α_{ol} is an adjustable constant (which, in the absence of better information, can be chosen, e.g., as 1.25 or 1.3).

Finally, it is worthwhile to remind that the border of the plastic zone moves forward with the crack. Therefore, in the complex loading case, the controlling border of the load interaction zone advances and must be recomputed as new overloads induce primary plastic zones that cross the previous frontier.

5. Conclusions

In this work, the modeling and numerical implementation of fatigue propagation of bidimensional cracks were discussed, including load interaction effects. The calculation models were implemented on a general-purpose fatigue design software named **ViDa**, developed to predict both initiation and propagation fatigue lives under complex loading by all classical design methods.

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