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Prediction of Fatigue Life and Crack Path in Complex 2D Structural Components Under Variable Amplitude Loading

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Abstract: Fatigue crack propagation in complex two-dimensional structural components under constant and variable amplitude loading is numerically predicted and experimentally verified. Cracks are fatigue propagated under constant and variable amplitude loading in standard CT specimens with holes specially positioned to attract or to deflect the crack. Therefore, the cracks do not follow a straight-line path, but curve toward the hole reaching it or not, depending on the hole positioning. A reliable and cost effective two-phase methodology is used in two pieces of software to numerically predict the fatigue crack propagation. First, the fatigue crack path and its stress intensity factor are calculated in a specialized finite-element software, using small crack increments. Numerical methods are used to calculate the crack propagation path, based on the computation of the crack incremental direction, and the stress-intensity factors $\mathbf{K_I}$, from the finite element response. Then, an analytical expression is fitted to the calculated $\mathbf{K_I(a)}$ values, where a is the length along the crack path. This $\mathbf{K_I(a)}$ expression is used as an input to a powerful general purpose fatigue design software based in the local approach, developed to predict both initiation and propagation fatigue lives under variable amplitude loading, considering interaction effects such as crack retardation or acceleration after overloads.

Keywords: fatigue crack propagation, finite elements, load interaction effects

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Introduction

Fatigue crack propagation life prediction in intricate 2D structural components is a challenging problem, involving the calculation of the crack path, the associated stress intensity factors, and the crack propagation rates at each step [1, 2, 3]. The solution of this problem is achieved and experimentally validated by dividing it into two complementary global and local approaches, as described in detail in [4, 5]. First, self-adaptive finite elements are used to calculate, by means of three different methods, the (generally curved) fatigue crack path and the stress-intensity factors $\mathbf{K_I(a)}$ and $\mathbf{K_{II}(a)}$ under *simple* loading along the crack length **a**. This global method alone is not computationally efficient under variable amplitude loading, because it requires remeshing procedures and FE recalculations of the *whole* structure's stress/strain field at each load event. Instead, an analytical expression can be fitted to the discrete $\mathbf{K_I(a)}$ values calculated under *simple* loading and exported to a local approach program, where the direct integration of the crack growth rate equation can be efficiently used to calculate crack propagation under variable amplitude loading at each load event.

Two complementary software have been developed to implement this two-step hybrid methodology. The first one, named **Quebra2D**, is an interactive graphical program for simulating two-dimensional fracture processes based on a finite-element (FE) self-adaptive mesh-generation strategy. In this program, the crack increment direction and the stress-intensity factors are calculated using three different FE methods. The second program, named **ViDa**, is a general-purpose fatigue design software developed to predict both initiation and propagation fatigue lives under variable loading by all classical design methods, including load interaction effects. In particular, its crack propagation module accepts any stress-intensity factor expression, including the ones generated by the finite-element software.

In this work, the introduced two-step methodology is extended to include load interaction effects, such as crack retardation after tensile overloads and crack acceleration due to compressive underloads [6]. Elber-type crack retardation models are calibrated using regular compact tension (CT) specimens under variable amplitude loading, and the calibrated parameters are used to predict the fatigue lives of modified CT specimens, in which holes were machined to curve the crack propagation path (Figure 1).

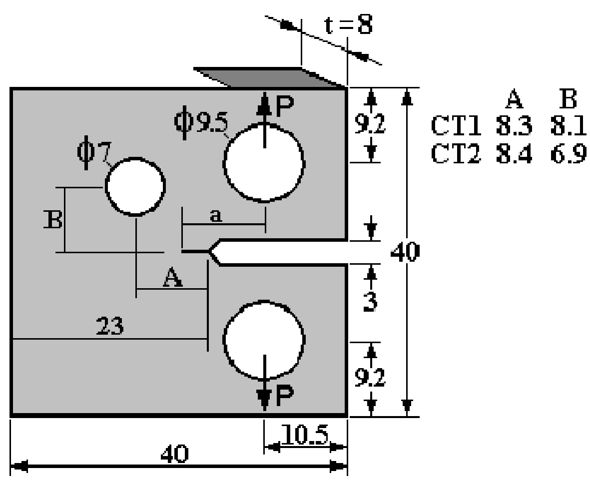


Figure 1 – *Details of the modified CT specimens (mm).*

Analytical and Experimental Background

This section describes the numerical modeling and testing procedures for studying fatigue crack propagation in modified compact tension (CT) test specimens (Figure 1).

The tested material is a cold rolled SAE 1020 steel, with the analyzed weight percent composition: C 0.19, Mn 0.46, Si 0.14, Ni 0.052, Cr 0.045, Mo 0.007, Cu 0.11, Nb 0.002, Ti 0.002, Fe balance. The Young modulus is $\mathbf{E = 205GPa}$, yield strength $\mathbf{S_Y = 285MPa}$, the ultimate strength is $\mathbf{S_U = 491MPa}$, and the reduction in area is **RA = 53.7%**. These properties were measured according to the ASTM E 8M-99 standard. The **da/dN** vs. **ΔK** data, obtained under two stress ratios, **R = 0.1** and **R = 0.7**, were measured following ASTM E 647-99 procedures, and were fitted by a modified Priddle **da/dN** equation (in m/cycle),

$$\frac{da}{dN} = 4.29 \cdot 10^{-5} \left(\frac{\Delta K - \Delta K_0 (1 - 0.55 \cdot R)}{K_C - K_{max}} \right)^{2.3} \quad (1)$$

where $\mathbf{\Delta K_0 = 12.2\ MPa\sqrt{m}}$ is the propagation threshold under **R = 0**, and the fracture toughness is $\mathbf{K_C = 280\ MPa\sqrt{m}}$.

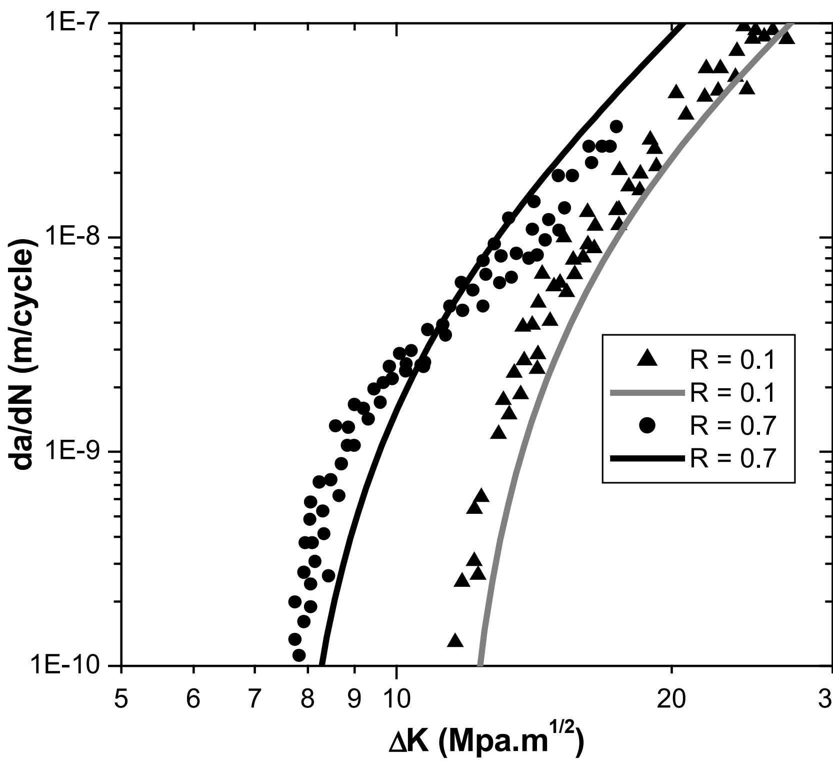


Figure 2 – *Modified Priddle* ***da/dN*** *equation fitted to the SAE 1020 steel.*

Prior to the experiments, the hole-modified CT specimens were FE modeled using the **Quebra2D** software [7, 8], where the position of a hole with 7mm diameter was varied in the models to obtain the most interesting curved crack path by a simple trial-and-error process. The chosen specimen geometries were machined, measured, and FE remodeled, to account for small deviations in the manufacturing process (Figure 1). In this way, it could be assured that the numerical models used in the predictions reproduced the real geometry of the tested specimens.

The predictions indicated that the fatigue crack was always attracted by the hole, but it could either curve its path and grow toward the hole ("sink in the hole" behavior) or just be deflected by the hole and continue to propagate after missing it ("miss the hole" behavior). To test the accuracy of the FE modeling, the transition point between these two crack growth behaviors was identified. Then, two borderline specimens were designed: the specimen CT1, with the hole just half a millimeter above that point (presenting the "miss the hole" behavior), and the specimen CT2, with the hole half a millimeter below it (with the "sink in the hole" behavior), see Figure 3. Due to machining tolerances, the actual difference between the vertical position of the holes in specimens CT1 and CT2 turned out to be 1.2mm instead. These specimens were then remodeled to predict the actual crack path. The initial meshes in the FE models have about 1300 elements and 2300 nodes, and the final ones after the simulated crack propagation (as shown in Figure 3) have about 2200 elements and 5500 nodes.

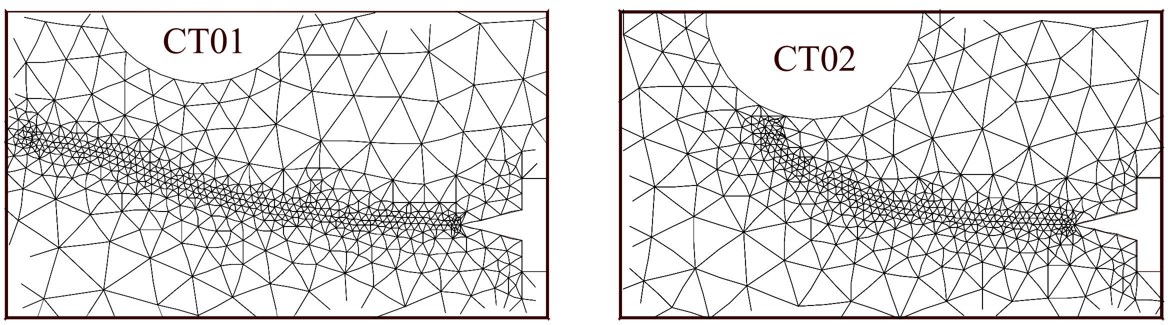


Figure 3 – *FE mesh automatically generated for the modified CT specimens.*

The FE automatic calculation procedure is performed in 4 steps: (i) the FE model of the holed specimen is solved to obtain the $\mathbf{K_I}$ and $\mathbf{K_{II}}$ stress-intensity factors (computed e.g. by the Modified Crack Closure integral technique [9, 10]) and the corresponding propagation direction (calculated e.g. by the $\boldsymbol{\sigma_{\theta max}}$ method [11]); (ii) the crack is increased in the growth direction by the (small) required step; (iii) the model is remeshed to account for the new crack size; and (iv) the process is iterated until rupture or until a specified crack size is reached. As a result, a list of $\mathbf{K_I}$ and $\mathbf{K_{II}}$ values is generated at short but discrete intervals along the predicted crack paths. Figure 4 shows a screen output of the **Quebra2D** software with a final meshed model. Figure 5 shows the calculated $\mathbf{K_I}$ values for the regular and the modified CT specimens, which can be easily exported to the **ViDa** software to predict the fatigue life including load interaction effects.

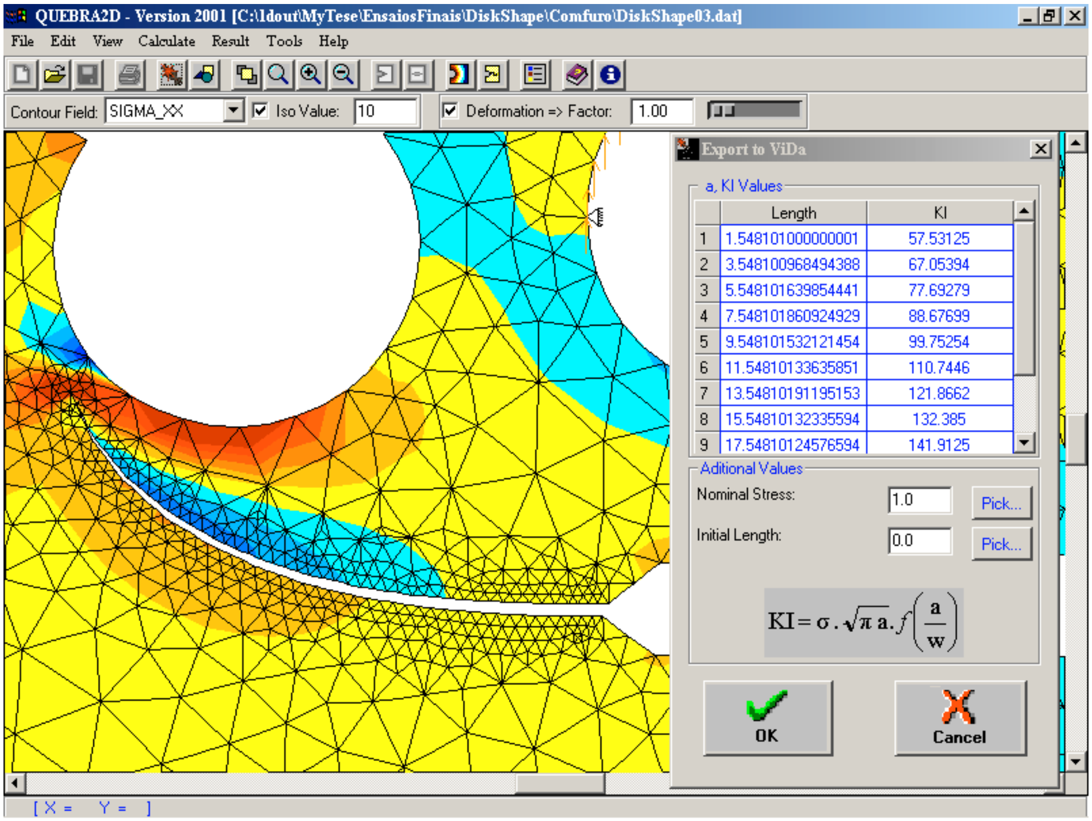


Figure 4 – *Screen output of the* ***Quebra2D*** *software.*

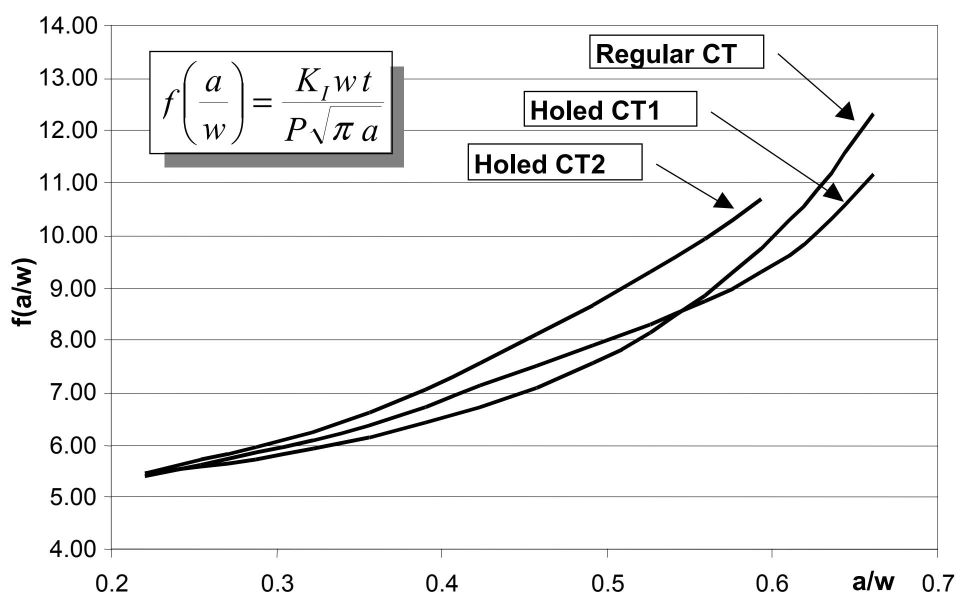


Figure 5 – ***f(a/w)*** *curves for the standard and for the modified CT specimens.*

All crack growth tests were performed at 20 and 30 Hz frequencies in a 250kN computer-controlled servo-hydraulic testing machine. The loads were regularly tuned to keep the specified stress intensity factors. In addition, a digital camera was used with an image analysis program to measure the crack size *and* path. This is a quite precise and economical option to automate those measurements, but its details are considered beyond the scope of this paper. After the tests the measured loads were inserted into the **ViDa** software [12, 13], which used the FE calculated $\mathbf{K_I}$ values to predict the specimen life. Figure 6 shows a screen output of the **ViDa** software, which includes choices of several editable **da/dN** curves, $\mathbf{K_I}$ and $\mathbf{K_{II}}$ equations, and load interaction models. Note that the loading history can be represented by a sequential list of peaks and valleys ($\boldsymbol{\sigma_{max}}$, $\boldsymbol{\sigma_{min}}$), or else by the equivalent sequence of alternate and mean stresses and number of reversions ($\boldsymbol{\sigma_a}$, $\boldsymbol{\sigma_m}$, **2N**).

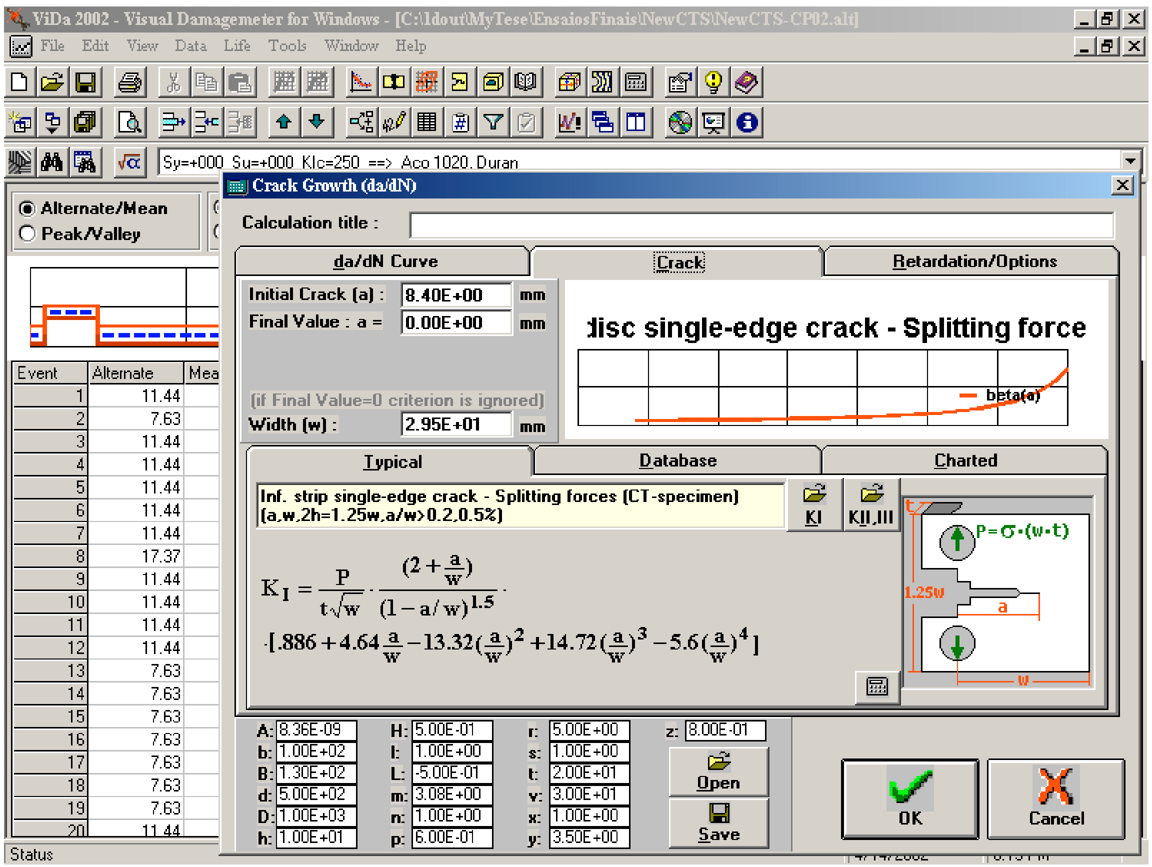


Figure 6 – *Screen output of the* **ViDa** *software.*

Several load interaction models are included in the **ViDa** software [14-17]. Wheeler is perhaps the most popular of such models [14], introducing a crack-growth reduction factor bounded by zero and unity. This factor is calculated for each cycle to predict retardation as long as the current plastic zone $\mathbf{Z_i}$ is contained within a previously overload-induced plastic zone $\mathbf{Z_{ol}}$. The retardation is maximum just after the overload, and stops when the border of $\mathbf{Z_i}$ touches the border of $\mathbf{Z_{ol}}$ (Figure 7).

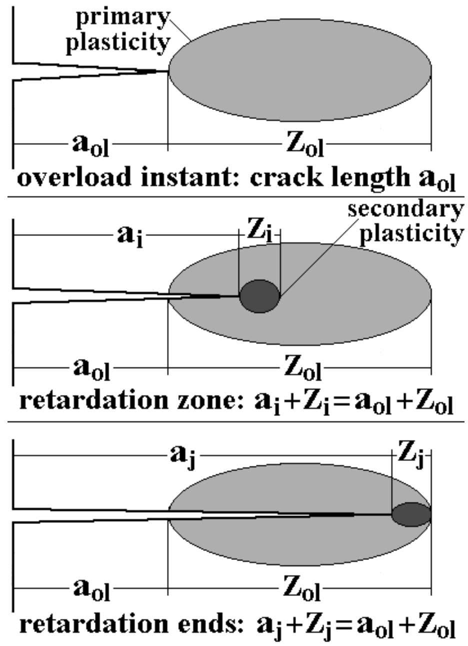


Figure 7 – *Yield zone crack growth retardation region used by Wheeler.*

Therefore, if $\mathbf{a_{ol}}$ and $\mathbf{a_i}$ are the crack sizes at the instant of the overload and at the (later) **i**-th cycle, and $\mathbf{(da/dN)_{ret,i}}$ and $\mathbf{(da/dN)_i}$ are the retarded and the corresponding non-retarded crack growth rate (at which the crack would be growing in the **i**-th cycle if the overload had not occurred), then, according to Wheeler

$$\left(\frac{da}{dN}\right)_{ret,i} = \left(\frac{da}{dN}\right)_i \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i}\right)^{\beta}, \quad a_i + Z_i < a_{ol} + Z_{ol} \tag{2}$$

where β is an experimentally adjustable constant, obtained by selecting the closest match among predicted crack growth curves (using several β-values) with an experimental curve measured under spectrum loading. However, this model cannot predict crack arrest because the resulting $\mathbf{(da/dN)_{ret,i}}$ is always positive. Cut-off values have been proposed to include crack arrest in the original Wheeler model, however this approach results in discontinuous **da/dN** equations.

Meggiolaro and Castro [6] proposed a simple but effective modification to the original Wheeler model in order to predict both crack retardation and arrest. This approach, called the ***Modified Wheeler*** model, uses a Wheeler-like parameter to multiply $\mathbf{\Delta K}$ instead of **da/dN** after the overload

$$\Delta K_{ret}(a_i) = \Delta K(a_i) \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i}\right)^{\gamma}, \quad a_i + Z_i < a_{ol} + Z_{ol} \tag{3}$$

where $\mathbf{\Delta K_{ret}(a_i)}$ and $\mathbf{\Delta K(a_i)}$ are the values of the stress intensity ranges that would be acting at $\mathbf{a_i}$ with and without retardation due to the overload, and γ is an experimentally adjustable constant, in general different from the original Wheeler model exponent β. This simple modification can be used with any of the propagation rules that recognize $\mathbf{\Delta K_{th}}$ to predict both retardation and arrest of fatigue cracks after an overload, the arrest occurring if $\mathbf{\Delta K_{ret}(a_i) \le \Delta K_{th}}$.

Another crack retardation model included in **ViDa** is the Constant Closure model, originally developed at Northrop for use on their classified programs [15]. This load interaction model is based on the observation that for some load spectra the closure stress does not deviate significantly from a certain stabilized value. This stabilized value is determined by assuming that the spectrum has a "controlling overload" and a "controlling underload" that occur often enough to keep the residual stresses constant, and thus the closure level constant.

In the constant closure model, the opening stress intensity factor $\mathbf{K_{op}}$ is the only empirical parameter, with typical values estimated between 20% and 50% of the maximum overload stress intensity factor. The value of $\mathbf{K_{op}}$, calculated for the controlling overload event, is then applied to the following (smaller) loads to compute crack growth, recognizing crack retardation and even crack arrest (if $\mathbf{K_{max} \leq K_{op}}$).

The main limitation of the Constant Closure model is that it can only be applied to loading histories with "frequent controlling overloads," because it does not model the decreasing retardation effects as the crack tip cuts through the overload plastic zone. In this model, it is assumed that a new overload zone, with primary plasticity, is formed often enough before the crack can significantly propagate through the previous plastic zone, thus not modeling secondary plasticity effects by keeping $\mathbf{K_{op}}$ constant.

Experimental Results and Analysis

Simple Loading

CT1 and CT2 specimens were tested under simple (constant amplitude) loading. Using the calculated $\mathbf{K_I(a)}$, each load program was adjusted to maintain a quasi-constant stress-intensity range around $\mathbf{\Delta K_I \approx 20MPa\sqrt{m}}$, with $\mathbf{R = K_{min}/K_{max} = 0.1}$. These loading values induce a stage-II (Paris regime) crack growth in the 1020 steel **da/dN** curve.

The measured and the predicted curved crack paths present a good match (Figure 8). The fatigue lives of both specimens under simple loading are very well predicted in **ViDa** using **da/dN** crack propagation data measured in standard (straight cracked) CT specimens under pure Mode-I loading (Figures 9 and 10).

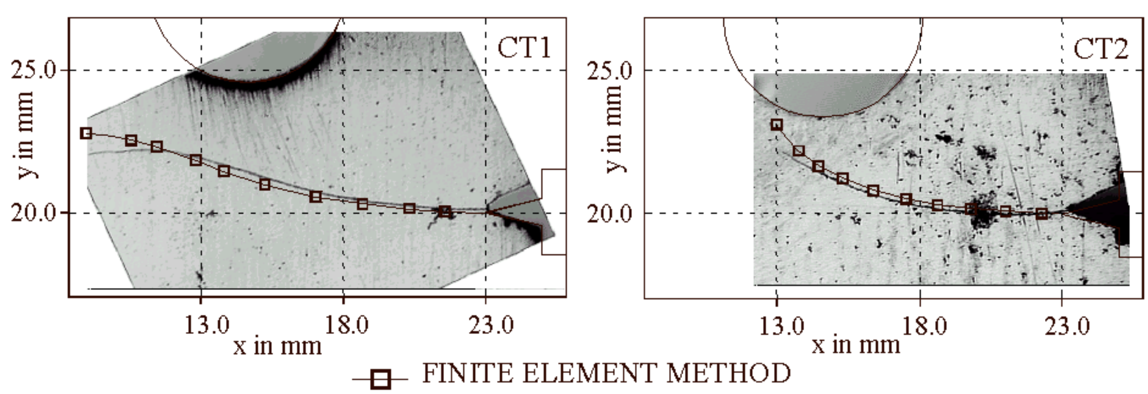


Figure 8 – *Predicted and measured crack paths for the modified CT specimens (mm).*

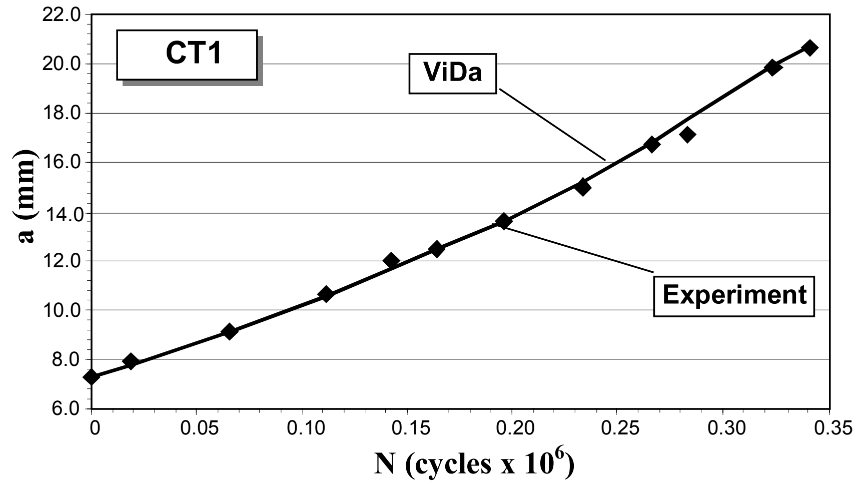


Figure 9 – *Predicted and measured fatigue crack growth for the CT1 specimen.*

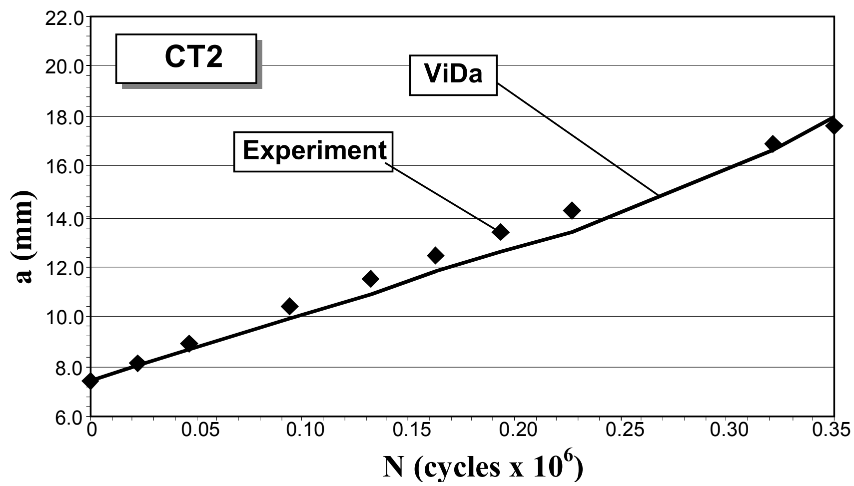
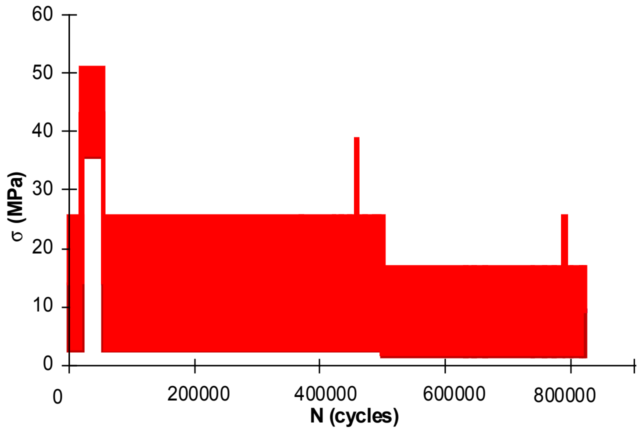


Figure 10 – *Predicted and measured fatigue crack growth for the CT2 specimen.*

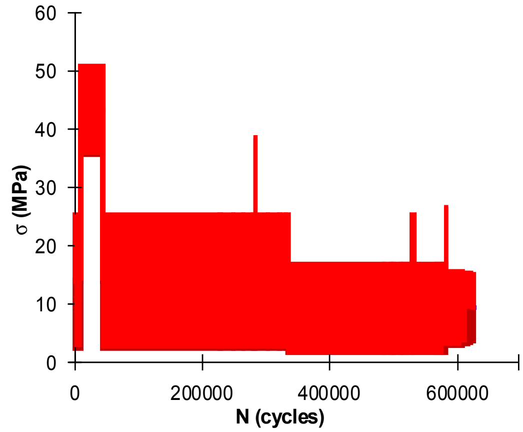
Therefore, it seems reasonable to use **da/dN** data measured from straight cracks to predict the propagation behavior of curved cracks under simple loading, using the presented two-step methodology. In addition, this approach can be generalized to variable amplitude loading including load interaction effects, such as overload-induced crack retardation or acceleration, as discussed next.

Variable Amplitude Loading

Two specimens were tested under variable amplitude loading: a regular CT specimen and a holed CT1-type specimen (Figure 3). The goal of this experiment is to verify if load interaction models calibrated for straight cracks (such as those in the regular CT specimen) can be used to predict the fatigue life of the CT1-type specimen, which presents a curved crack path. The load histories applied to the specimens are shown in Figure 11.



(a)



(b)

Figure 11 – *Applied load history: (a) regular CT and (b) holed CT1-type specimens.*

Figure 12 shows the predicted (curved) crack path for the CT1-type specimen (in mm) and the measured one under variable amplitude loading. As in the simple loading case, the predicted and measured crack paths show a very good match, suggesting that the crack path under variable amplitude loading is the same as under simple loading. In fact, it can be seen in Figure 12 that the significant overload plastic zone generated at about 550,000 cycles in the CT1-type specimen did not deviate significantly the (curved) crack path, which continued to propagate according to the predicted path under simple loading. Therefore, assuming that only the crack growth rate (but not its path) is influenced by load interaction effects, the presented two-step methodology can be generalized to the variable amplitude loading case. The crack path and associated stress intensity factors are then calculated under simple loading using FE software (such as **Quebra2D**), and the load interaction effects only need to be considered later in the life assessment program (such as **ViDa**).

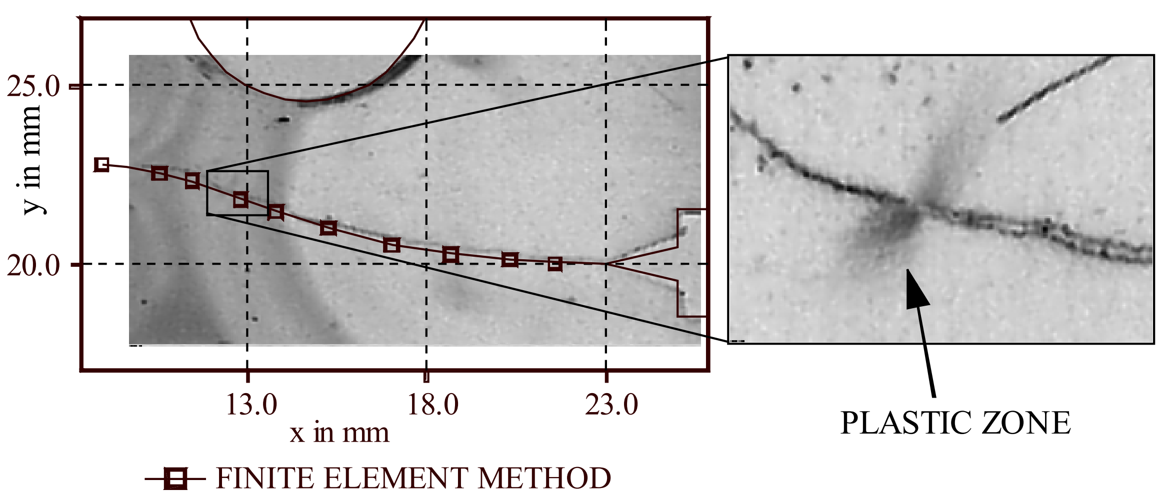


Figure 12 – *Predicted and measured crack paths for the modified CT1 specimens under variable amplitude loading (left) and detail of an overload plastic zone (right, 60x zoom).*

Figure 13 shows the measured crack sizes under variable amplitude loading on a standard CT and on a holed CT1-type specimen. In this case, the Constant Closure model has been used to consider crack retardation effects due to the tensile overloads. First, the standard CT was tested under variable amplitude loading, and then the opening stress intensity factor $\mathbf{K_{op}}$ was estimated using **ViDa** as 22% of the maximum overload stress intensity factor $\mathbf{K_{max}}$ to fit the experimental results. Finally, the curved crack growth in the holed CT1-type specimen was predicted using the 22% parameter obtained for straight cracks, showing a very good match. These results suggest that the load interaction models calibrated using straight cracks can be used to predict the crack retardation (or acceleration) behavior of generally curved cracks.

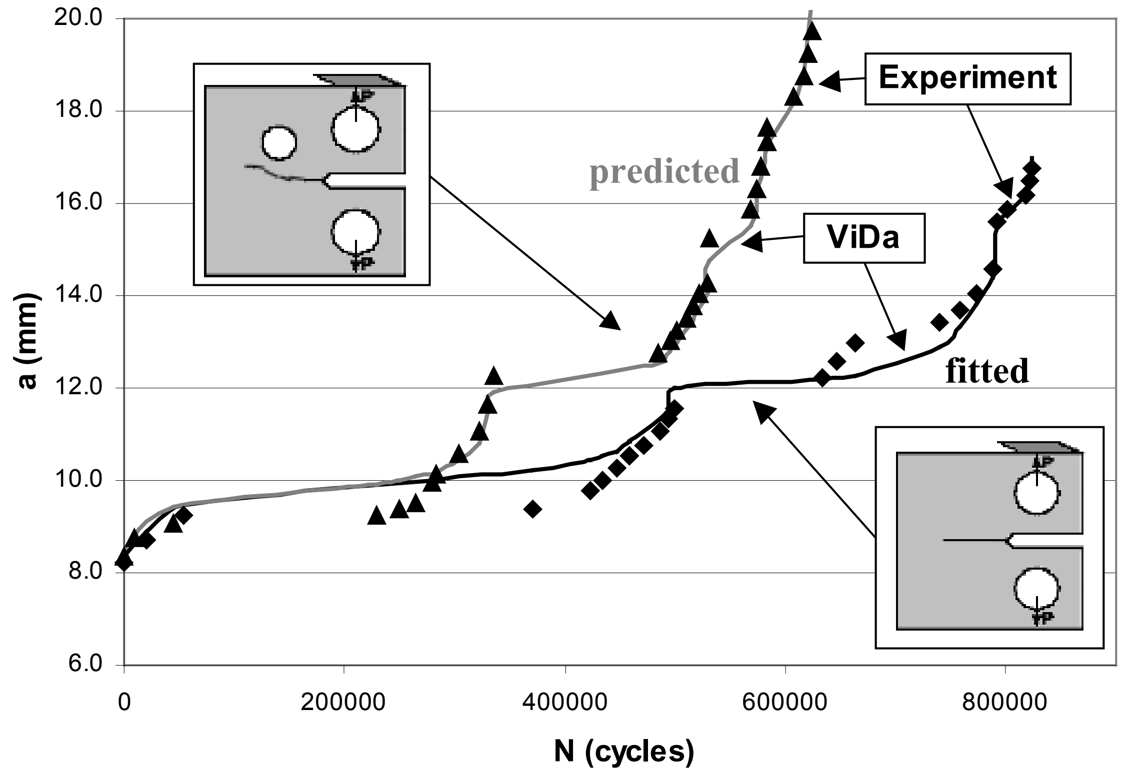


Figure 13 – *Predicted and measured crack sizes on a holed and on a standard CT.*

Several other load interaction models were similarly evaluated using the **ViDa** software, such as the Willenborg [16], Modified Willenborg [17], and Modified Wheeler [6] models. Willenborg always predicts crack arrest after a 100% overload because it assumes an overload shut-off ratio $\mathbf{R_{so}}$ of 2.0, therefore it cannot be applied to the holed specimen loading history (otherwise it would wrongfully predict crack arrest at about 50,000 cycles). The Modified Willenborg allows higher values of the shut-off ratio $\mathbf{R_{so}}$, however this model is limited to **da/dN** equations that explicitly model the effect of the load ratio **R**. For the considered **da/dN** equation, it was found that the predicted lives were very sensitive to the calibrated value of $\mathbf{R_{so}}$, making it difficult to obtain accurate results. On the other hand, the Modified Wheeler model resulted in very good predictions for curved crack growth, using a value of 0.4 for the exponent $\boldsymbol{\gamma}$ calibrated from straight crack experiments (Figure 14). Note that retardation effects cannot be neglected in this experiment, as it can be seen in the figure below.

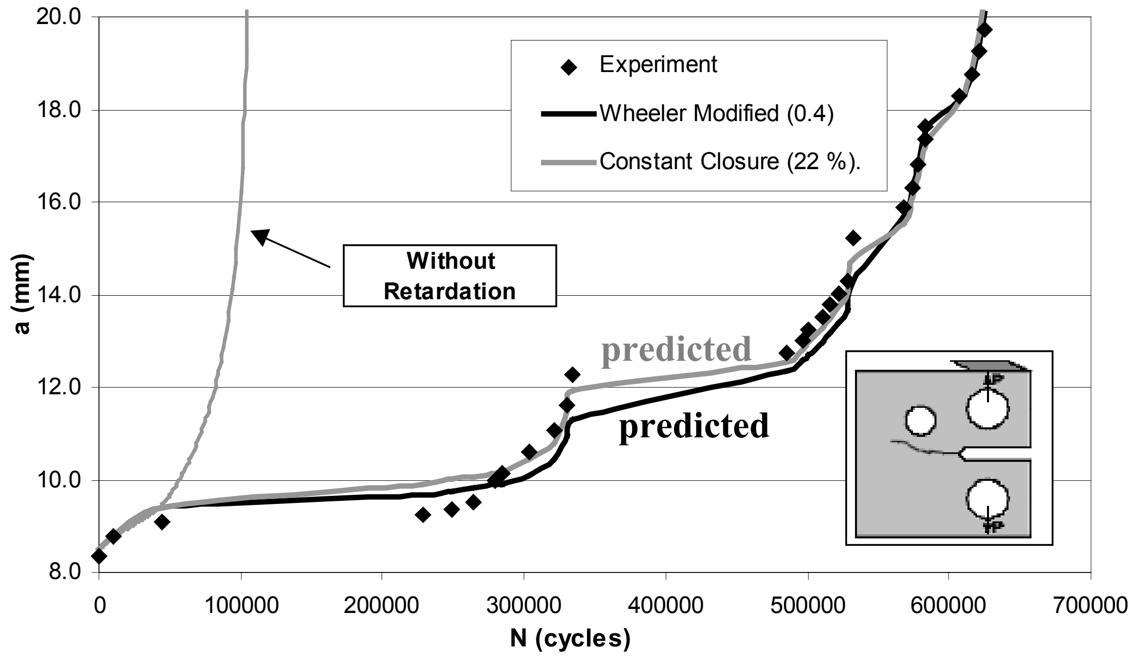


Figure 14 – *Predicted and measured crack growth on a holed CT using the Constant Closure and Modified Wheeler models.*

Conclusions

A two-phase methodology was presented to predict fatigue crack propagation in generic 2D structures. First, self-adaptive finite elements were used to calculate the fatigue crack path and the stress-intensity factors along the crack length $\mathbf{K_I(a)}$ and $\mathbf{K_{II}(a)}$, at each propagation step. The computed $\mathbf{K_I(a)}$ was then used to calculate the propagation fatigue life by the local approach, considering overload-induced crack retardation effects.

Two complementary software have been developed to implement this methodology. The first is an interactive graphical program for simulating two-dimensional fracture processes based on a finite-element adaptive mesh-generation strategy. The second is a general-purpose fatigue design software developed to predict both initiation and propagation fatigue lives under variable loading by all classical design methods. Particularly, its crack propagation module accepts any stress-intensity factor expression, including the ones generated by the finite-element software.

Experimental results validated the proposed methodology, in particular suggesting that overloads do not significantly deviate the crack path predicted under simple loading. Moreover, the developed software demonstrated that effective and economical predictions of crack propagation paths and fatigue lives can be obtained for arbitrary two-dimensional structural components under variable amplitude loading.

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