A DAMAGE ACCUMULATION MODEL TO PREDICT FATIGUE CRACK GROWTH UNDER VARIABLE AMPLITUDE LOADING USING εN PARAMETERS

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Analytical models based on damage accumulation by cyclic plasticity have been developed to predict the fatigue crack growth **da/dN** vs. **ΔK** curve using **εN** parameters. The strain singularity of the idealized crack is avoided by modeling the crack as a notch and by shifting the origin of the HRR field from the crack tip to a point inside the crack, which is localized by matching the HRR strain at the crack tip with the strain predicted at that point by a strain concentration rule. The idea that the crack growth is caused by the sequential failure of volume elements ahead of the crack tip is extended to deal with the variable amplitude loading case. A good agreement between the crack growth predictions (by the direct integration of a damage function based solely on εN parameters) and the experiments was obtained for one structural material under variable amplitude load histories. Moreover, an Elber-type opening load concept can be introduced into the model, to separate the fatigue damage from the closure contributions to the crack growth process.

INTRODUCTION

Various paths can be followed to explain and to predict the fatigue crack growth (FCG) process using solid mechanics-based theoretical tools and basic mechanical properties. Probably the most successful one correlates the stress intensity range (**ΔK**) controlled FCG with the strain range (**Δε**) controlled fatigue crack initiation process. Following this line of thought, various analytical models based on damage accumulation by cyclic plasticity have been developed to predict the crack growth curve **da/dN** vs. **ΔK** (obtained under constant amplitude loading). These models use **εN** parameters and expressions of the HRR type to represent the elastic-plastic strain range inside the plastic zone ahead of the crack tip, which is modeled as a sharp notch with a very small but finite tip radius to remove its singularity. The origin of the HRR field was shifted from the crack tip to a point inside the crack, lo-

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cated by matching the (now finite) HRR strain at the crack tip with the strain predicted at that point by a strain concentration rule, such as Neuber, Glinka, or the linear rule (Durán et al. [1]). A very reasonable agreement between the predictions and the experiments was obtained for three structural materials (SAE1020 and API 5L X-60 steels, and 7075 T-6 aluminum alloy) [1], using the calculated crack growth constant in McEvily rule (Schwalbe [2]) to predict the **da/dN** vs. **ΔK** curve.

The idea that the FCG is caused by the sequential failure of volume elements ahead of the crack tip is extended here to deal with the variable amplitude loading case, which has idiosyncrasies that must be treated appropriately. First, the volume elements must have variable width, which should be calculated at every load cycle by locating the point ahead of the crack tip where the accumulated damage reaches 1.0, assuming that the damage is caused solely by the cyclic plastic deformations induced by the loading. In this case, the load sequence effects, such as overload-induced crack growth retardation, are associated only to the (weak) mean load effect on the εN curve. However, an Elber-type opening load concept can be introduced into the model, to separate the damage from the closure contributions (which are both plasticity-induced) to the crack growth process. Experiments with variable amplitude load histories are used to validate the proposed models, using the powerful numerical tools in the **ViDa** software (Meggiolaro & Castro [3]).

MODELING THE da/dN vs. ΔK *CURVE USING* εN *PARAMETERS*

FCG is supposed to be caused by the sequential fracturing of small volume elements ahead of the crack tip (Figure 1). Under constant amplitude loading, the width of these volume elements (which may be viewed as small **εN** specimens) is also constant and equal to the crack increment per cycle.

In every load cycle, each one of these volume elements is submitted to elastic-plastic hysteresis loops of increasing amplitude as the crack tip approaches it, suffering a damage that is a function of the loop amplitude in that cycle (which depends on the distance **r** between the volume element and the fatigue crack tip). The fracture of the volume element at the crack tip (which causes the fatigue crack propagation) occurs when its accumulated damage reaches a critical value, quantified by some damage accumulation rule, e.g., Miner's rule:

$$\sum \frac{\mathbf{n_i}}{\mathbf{N_i}} = \mathbf{1} \tag{1}$$

where $\mathbf{n_i}$ is the number of cycles of the **i**-th load event and $\mathbf{N_i}$ is the number of cycles that the piece would last if loaded solely by that event.

Under constant **ΔK** loading, in every load cycle the crack advances a distance **da**. Thus, neglecting the damage accumulated outside the cyclic plastic zone $\mathbf{r_{Yc}}$, there are $\mathbf{r_{Yc}/da}$ elements ahead of the crack tip at any instant. Since the plastic zone advances with the crack, each new load cycle breaks the element adjacent to the crack tip, induces an increased loop amplitude in all other unbroken elements (because the crack tip approaches them by **da**), and adds a new element to the damage zone. Therefore, the number of cycles per growth increment is $\mathbf{n_i = 1}$ and, since the elements are considered as small **εN** specimens, they break when:

$$\sum_{i=0}^{r_{Yc}/da} \frac{1}{N(r_{Yc} - i \cdot da)} = \sum_{r_i=0}^{r_{Yc}} \frac{1}{N(r_i)} = 1 \tag{2}$$

where $\mathbf{N(r_i) = N(r_{Yc} - i \cdot da)}$ is the fatigue life corresponding to the strain range $\mathbf{\Delta\varepsilon(r_i)}$ acting at $\mathbf{r_i}$ from the crack tip. If ε'_f is the coefficient and **c** is the exponent of the plastic part of Coffin-Manson's rule, and if the elastic damage is neglected,

$$N(r_i) = \frac{1}{2}\left(\frac{\Delta\varepsilon_p(r_i)}{2\varepsilon'_f}\right)^{1/c} \tag{3}$$

If **n'** is the Ramberg-Osgood cyclic strain hardening exponent and $\mathbf{S_{Yc}}$ is the cyclic yield strength, the strain range inside the cyclic plastic zone can be described by Schwalbe's [2] modification of the HRR field:

$$\Delta\varepsilon_p(r_i) = \frac{2S_{Yc}}{E} \cdot \left(\frac{r_{Yc}}{r_i}\right)^{1/1+n'} \tag{4}$$

Considering the width of volume elements **da** as a differential distance **dr** ahead of the crack tip, and approximating the Miner's summation by an integral:

$$\frac{da}{dN} = \int_0^{r_{Yc}} \frac{dr}{N(r)} \tag{5}$$

The HRR field used to describe the stress and strain fields inside the plastic zone ahead of the idealized crack tip is singular for $\mathbf{r = 0}$. Thus, $\mathbf{N(r) \to 0}$ when $\mathbf{r \to 0}$, what is not physically reasonable. However, no real crack has zero radius tip, and it is possible to eliminate the strain singularity by shifting the HRR coordinate system origin into the crack by a distance **X**, following Creager's idea (Creager & Paris [4])

$$\frac{da}{dN} = \int_0^{r_{Yc}} \frac{dr}{N(r + X)} \tag{6}$$

To determine **X** and **N(r + X)** two paths can be followed, as illustrated in Figure 2. The first considers, as Creager did, $\mathbf{X} = \rho/2$, ρ being the actual crack tip radius, which can be estimated by $\rho = \mathbf{CTOD/2}$. The second determines **X** by first calculating the plastic strain range $\Delta\varepsilon_p(X)$ acting at the crack tip, using a strain concentration rule and the crack linear elastic stress concentration factor K_t. For a detailed explanation and the experimental validation of these models see [1].

da/dN *MODELS FOR VARIABLE AMPLITUDE LOADING*

For variable amplitude (VA) loading, the FCG ***cannot*** be assumed constant because ΔK_i can vary at each load cycle. The models developed above can be indirectly used to calculate FCG under VA loading by integrating the predicted **da/dN** curve using the cycle by cycle method. However, the idea here is to ***directly*** quantify the fatigue damage induced by the VA loading considering the crack growth as the result of the sequential fracturing of small variable size volume elements inside the cyclic plastic zone ahead of the crack tip.

Since the model based on the Linear strain concentration rule resulted in the best predictions in [1] (because the fatigue crack propagation data were obtained under dominant plane strain conditions), it is the only one used below. And since load interaction effects can have a significant importance in FCG, they can also be introduced in the model, e.g., considering mean load σ_m effects by:

$$N(r+X) = \frac{1}{2}\left(\frac{\Delta\varepsilon_p(r+X)}{2\varepsilon'_f}\left(1-\frac{\sigma_m}{\sigma'_f}\right)^{-c/b}\right)^{1/c} \tag{7}$$

where σ'_f is the coefficient and **b** is the exponent of the elastic part of the Morrow elastic-plastic εN rule. And to separate the damage and the closure contributions to FCG (considering crack closure as the only crack retardation mechanism), an Elber-type opening load concept can be easily used to filter the (**R > 0**) loading by using:

$$\Delta K_{eff} = \frac{\Delta K - \Delta K_{th}}{1-R} \tag{8}$$

The damage function is again a function of **r**:

$$d_i(r+X_i) = \frac{n_i}{N_i(r+X_i)} \tag{9}$$

If the piece is virgin, the crack increment caused by the first load event is the **r** value that makes the equation (9) equal to 1: $da_1 = r_1$, where $d_1(r_1 + X_1) = 1$.

In all subsequent events, the crack increments take into account the damage accumulated by the previous loading, in the same way it was done for the constant loading case. But as the coordinate system moves with the crack, a coordinate transformation of preceding damage functions is necessary:

$$\mathbf{D_i} = \sum_{\mathbf{j=1}}^{\mathbf{i}} \mathbf{d_j}\left(\mathbf{r} + \sum_{\mathbf{p=j}}^{\mathbf{i-1}} \mathbf{da_p}\right) \tag{10}$$

Since $\mathbf{r_i}$, the distance where the accumulated damage equals 1 in the **i**-th event is a variable that depends on $\mathbf{\Delta K_i}$ (or $\mathbf{\Delta K_{eff_i}}$) and on the previous loading history, elements of different widths may be broken by this model, as shown in Figure 3.

EXPERIMENTAL RESULTS

FCG under variable amplitude loading was tested using AISI 1020 steel CT specimens, 50 mm wide by 10 mm thick. Pre-cracking was made under constant amplitude loading with an initial $\mathbf{\Delta K = 20\ MN/m^{3/2}}$ until reaching $\mathbf{a/w = 0.26}$. FCG occurred under LEFM conditions. Testing was conducted in a 100 kN computer-controlled servo-hydraulic machine. Crack size was monitored within a 20μm accuracy by the Back Face Strain technique [5, 6], using a 5mm 120Ω strain gage. The VA load history was a series of blocks containing 101 peaks and valleys, as shown in Figure 4, with a duration of 2 seconds each.

The loading history was counted by the sequential rain-flow method, and the corresponding hysteresis loops were obtained using the **ViDa** software [3]. The damage calculation was made using a specially developed code based on Equations (3-10). Figure 5 compares the predictions with the experimentally obtained data.

CONCLUSIONS

A new damage accumulation model, entirely based on εN cyclic properties, was proposed for predicting fatigue crack propagation under variable amplitude loading. The main features of this model are realistic considering the finite strain at the actual fatigue crack tips, which are treated as sharp notches with a point radius equal to half its CTOD. The HRR field is then modified using any strain concentration rule, such as Neuber, Glinka, or the linear rule, and damage accumulation is explicitly calculated at each load cycle.

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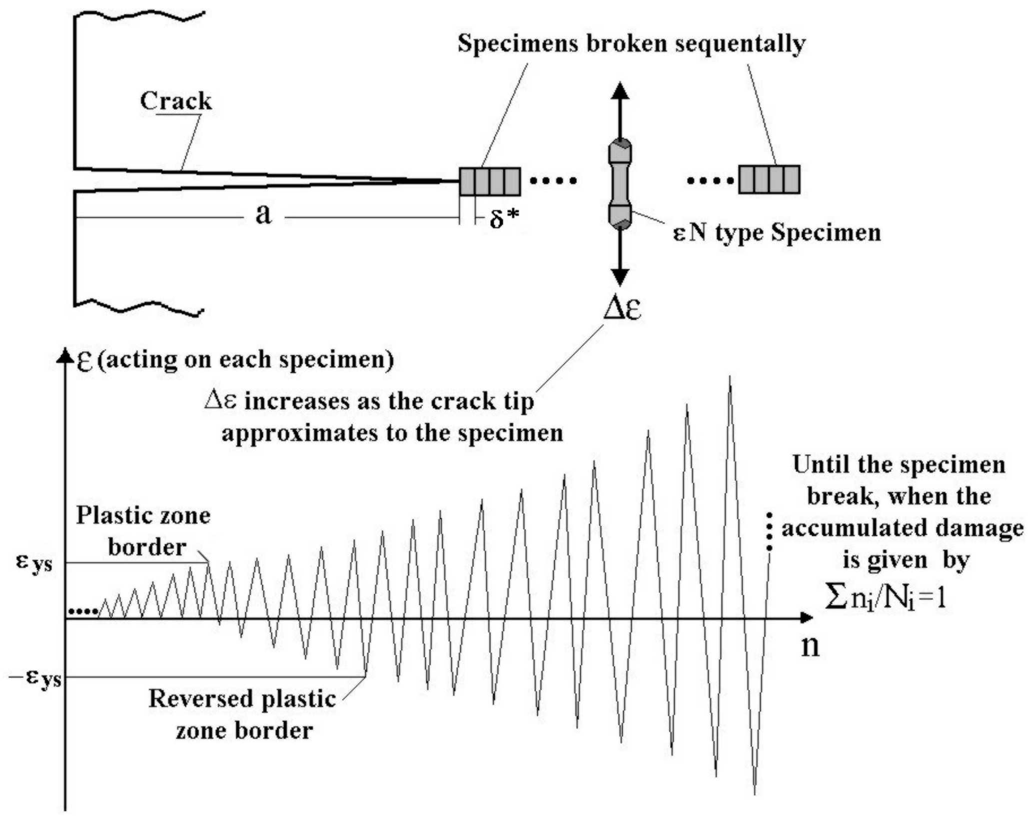

FIGURE 1 Fatigue crack growth caused by sequentially breaking **εN** specimens.

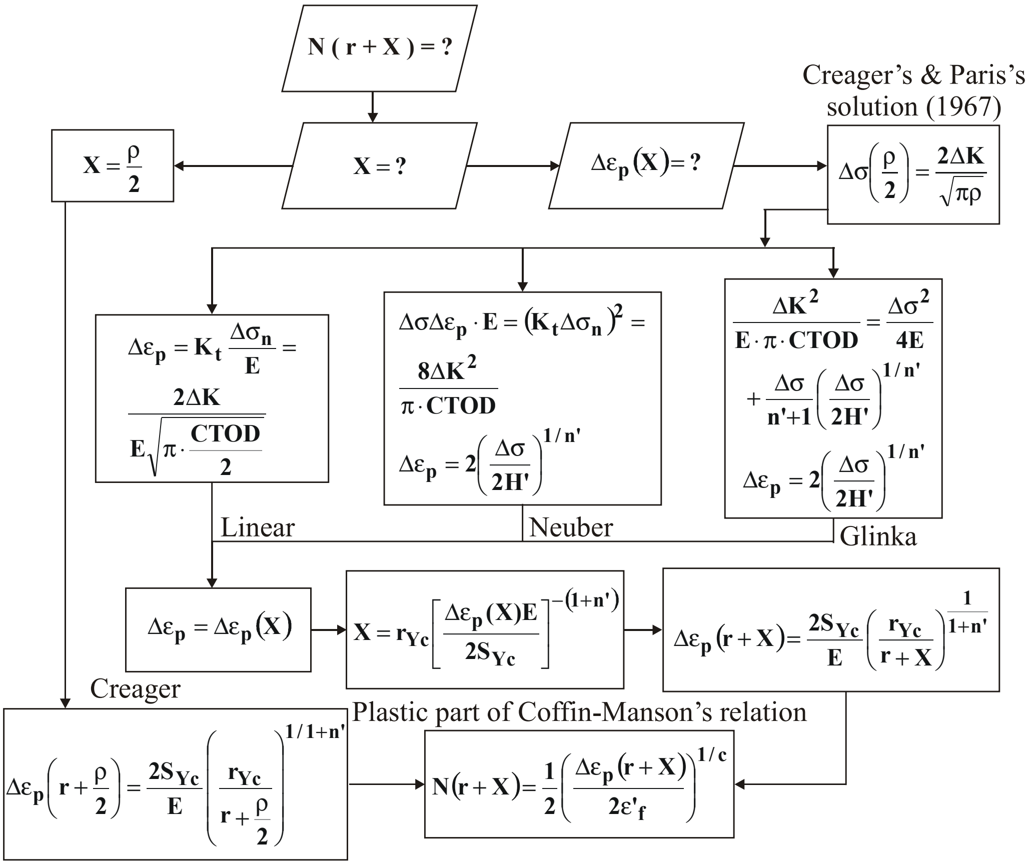

FIGURE 2 Flowchart of the different FCG models developed in [1].

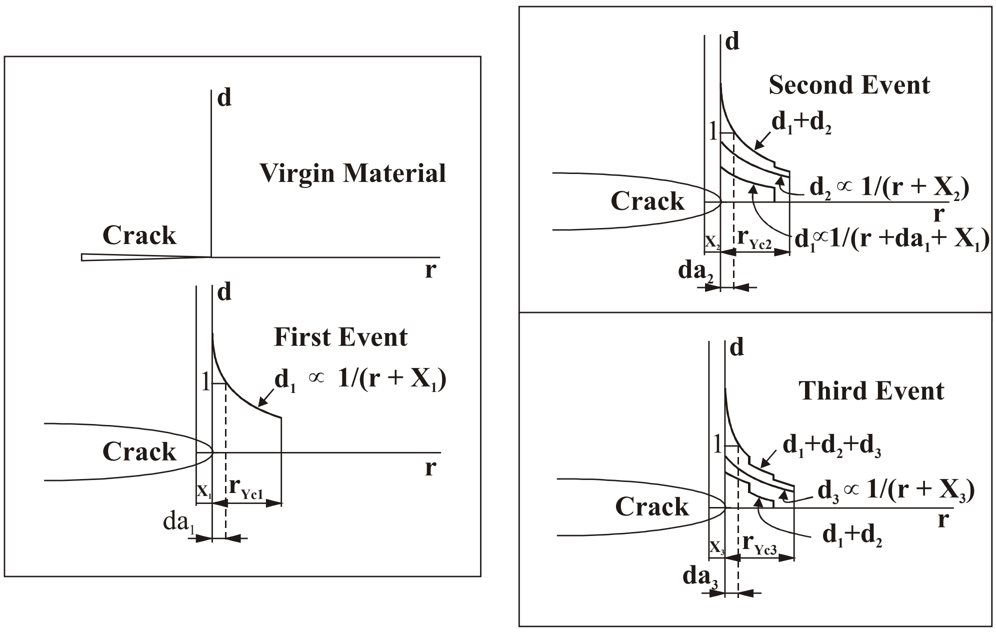

FIGURE 3 Scheme of FCG under **VA** loading.

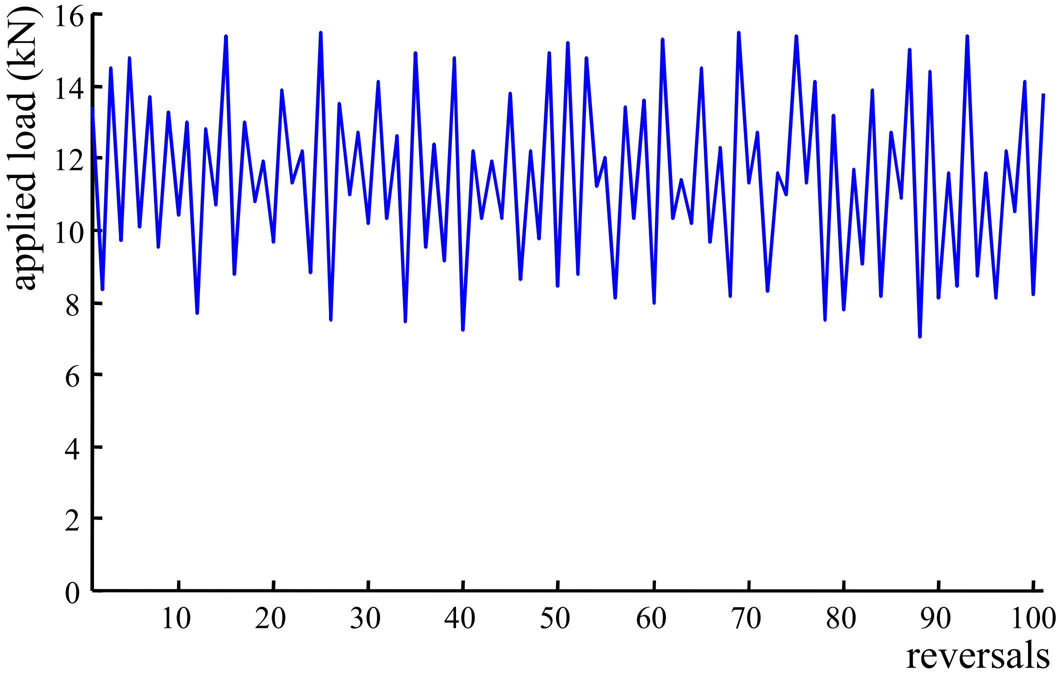

FIGURE 4 Load block applied to the CTS.

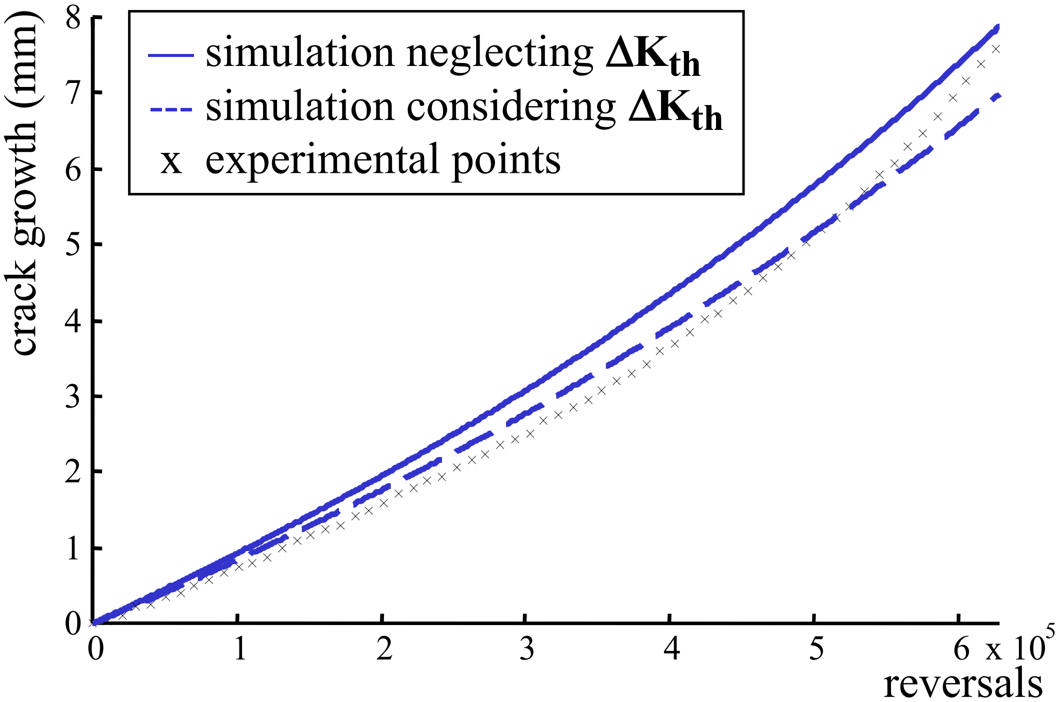

FIGURE 5 Crack growth simulation based on εN parameters and experimental data for AISI 1020 steel (**ΔK_{th}(R = 0) = 11MPa√m**).