CRACK GROWTH PREDICTIONS UNDER VARIABLE AMPLITUDE LOADING BASED ON LOW CYCLE FATIGUE DATA

J.A.R. Durán, M.G. Souza, J.T.P. Castro and M.A. Meggiolaro
Dept. Mechanical Eng., Pontifical Catholic University of Rio de Janeiro (PUC-Rio)
Rua Marquês de São Vicente 225 - Rio de Janeiro - RJ - zip 22453-900 - Brazil

ABSTRACT

The εN method can be combined with fracture mechanics concepts to predict crack growth behavior, assuming that crack propagation is caused by the cyclic elastic-plastic deformations ahead of the crack tip. Therefore, the crack growth rate under constant $\Delta \mathbf{K}$ loading is assumed due to the sequential failure of fixed width volume elements ahead of the crack tip, which can be calculated through damage accumulation concepts. In this work, the critical damage approach is extended to the variable amplitude loading case, considering load interaction effects. The cyclic deformation at a given point in the crack path increases as the crack tip approaches its volume element, which fails when a critical damage value is reached. Therefore, under VA conditions, the crack growth at each cycle is equal to the region ahead of the crack tip that experiences damage beyond its critical value. However, the usual (singular) modeling of the crack tip stress field would invalidate any attempts to correlate εN and da/dN parameters. To remove this singularity, the crack is modeled as a notch with a small but finite radius ρ, using the method developed by Creager and Paris [1]. The strain distribution ahead of the crack tip is modeled using a modified HRR field. The now finite deformations at the notch root are calculated using a strain concentration rule such as Neuber or Glinka [2]. The proposed approach is validated through experiments on API-5L-X52 steel CT specimens.

KEYWORDS

Variable amplitude loading; low cycle fatigue; critical damage model; crack growth prediction

INTRODUCTION

Several theoretical models have been proposed to correlate the oligocyclic fatigue crack initiation process, controlled by the strain range $\Delta\varepsilon$, with fatigue crack propagation rates, controlled by the stress intensity range $\Delta \mathbf{K}$. These models consider that the cyclic plastic zone $\mathbf{r}_{Yc}$ ahead of the crack tip is composed by a sequence of very small volume elements, each one under a different strain range, which are being broken sequentially as the crack propagates. Each of these volume elements will be submitted to elastic-plastic hysteresis loops of increasing amplitude as the crack tip approaches it. Any given volume element suffers damage in each load cycle, caused by the amplitude of the loop acting in that cycle, which in turn depends on the distance **r** between the volume element and the fatigue crack tip. Fracturing of the volume element at the crack tip (which causes fatigue crack growth) occurs when its accumulated damage reaches a critical value. This critical value will logically be due to the sum of the damage suffered in each cycle, and a damage accumulation rule is required to quantify it. The linear damage accumulation rule may be used in this case to reach this objective.

Most of the proposed critical damage models consider the width of the volume element in the crack propagation direction as being the distance that the fatigue crack propagates on each cycle **da** [3, 4]. Others consider the fatigue crack propagation rate as being the element width divided by the number of cycles that the crack would need to cross it [2]. The theoretical models based on the low-cycle fatigue (LCF) process predict Paris' constants using the different cyclic properties of the material, and can only work in stage II of the fatigue crack propagation curve, without taking into account other factors that may influence it. However, all three stages of the da/dN curve can be modeled by modifying Paris' equation using semi-empirical relations such as McEvily's or Schwalbe's equations [5].

However, most models in the literature do not properly deal with the stress field singularity at the crack tip. As the stresses during the last cycle of each volume element would approach infinity according to linear elastic Fracture Mechanics (LEFM), all damage would be caused by this very last event. To avoid this, it has been proposed to simply stop the calculations before the last loading cycle, solving the singularity problem but still not properly modeling the actual elastic-plastic stresses at the crack tip.

Recently, an improved model that deals with the actual elastic-plastic stresses at the crack tip has been proposed [6-7]. This model uses εN parameters and expressions of the HRR type to represent the elastic-plastic strain range inside the plastic zone ahead of the crack tip. In this formulation, the crack tip is modeled as a sharp notch with a very small but finite tip radius to remove the singularity issues. The origin of the HRR field is shifted from the crack tip to a point inside the crack, located by matching the (now finite) HRR strain at the crack tip with the strain predicted at that point by a strain concentration rule, such as Neuber, Glinka, or the linear rule [2]. A very reasonable agreement between the predictions and the experiments has been obtained for three structural materials - SAE1020 and API 5L X-60 steels, and 7075 T-6 aluminum alloy - using the calculated crack growth constant in McEvily rule to predict the da/dN vs. ΔK curve [5-7].

The idea that FCG is caused by the sequential failure of volume elements ahead of the crack tip is extended here to deal with the variable amplitude loading case. Experiments with variable amplitude load histories are used to validate the proposed models, using powerful numerical tools in the **ViDa** software [8].

ANALYTICAL BACKGROUND

The proposed model assumes that FCG is caused by the sequential fracturing of small volume elements ahead of the crack tip. In every load cycle, each of these volume elements is submitted to elastic-plastic hysteresis loops of increasing amplitude as the crack tip approaches it, suffering damage that is a function of the loop amplitude in that cycle. The fracture of the volume element at the crack tip (which causes fatigue crack propagation) occurs when its accumulated damage reaches a critical value, quantified by some damage accumulation rule, e.g., Miner's rule.

Under constant $\mathbf{\Delta K}$ loading, in every load cycle the crack advances a distance **da**. Thus, neglecting the damage accumulated outside the cyclic plastic zone $\mathbf{r_{Yc}}$, there are $\mathbf{r_{Yc}/da}$ elements ahead of the crack tip at any instant. Since the plastic zone advances with the crack, each new load cycle breaks the element adjacent to the crack tip, induces an increased loop amplitude in all other unbroken elements (because the crack tip approaches them by **da**), and adds a new element to the damage zone. Therefore, the number of cycles per growth increment is $n_i = 1$ and, since the elements are considered as small **εN** specimens, they break when:

$$\sum_{i=0}^{r_{Yc}/da} \frac{1}{N(r_{Yc} - i \cdot da)} = \sum_{r_i=0}^{r_{Yc}} \frac{1}{N(r_i)} = 1 \tag{1}$$

where $N(r_i) = N(r_{Yc} - i \cdot da)$ is the fatigue life corresponding to the strain range $\Delta\varepsilon(r_i)$ acting at a distance r_i from the crack tip. If ε_f' is the coefficient and **c** is the exponent of the plastic part of Coffin-Manson's rule, and if the elastic damage is neglected,

$$N(r_i) = \frac{1}{2}\left[\frac{S_{Yc}}{E\varepsilon_f'} \cdot \left(\frac{r_{Yc}}{r_i}\right)^{\frac{1}{1+n'}}\right]^{1/c} \tag{2}$$

where **n'** is the Ramberg-Osgood cyclic strain hardening exponent and S_{Yc} is the cyclic yield strength [5]. In addition, if Morrow's elastic-plastic **εN** equation is considered above instead of Coffin-Manson's rule, then mean load σ_m effects can also be accounted for.

The HRR field used to describe the stress and strain fields inside the plastic zone ahead of the idealized crack tip is singular for $r_i = 0$. Thus, $N(r_i) \to 0$ when $r_i \to 0$, what is not physically reasonable. However, no real crack has zero radius tip, and it is possible to eliminate the strain singularity by shifting the HRR coordinate system origin into the crack by a distance **X**, following Creager's idea [1]. Considering the width of volume elements **da** as a differential distance **dr** ahead of the crack tip, Miner's summation can then be approximated by the integral

$$\frac{da}{dN} = \int_0^{r_{Yc}} \frac{dr}{N(r+X)} \tag{3}$$

To determine **X** and $N(r + X)$, two paths can be followed. The first considers, as Creager did, $X = \rho/2$, ρ being the actual crack tip radius, which can be estimated by $\rho = CTOD/2$. The second determines **X** by first calculating the plastic strain range $\Delta\varepsilon_p(X)$ acting at the crack tip, using a strain concentration rule and the crack linear elastic stress concentration factor K_t.

Under variable amplitude (VA) loading, FCG cannot be assumed constant because ΔK_i can vary at each load cycle. Crack growth is then calculated as the result of the sequential fracturing of small *variable* size volume elements inside the cyclic plastic zone ahead of the crack tip. If the piece is virgin, the crack increment caused by the first load event is the **r** value that makes $N(r + X)$ equal to 1, i.e. $da_1 = r_1$, where $N(r_1 + X_1) = 1$. In all subsequent events, the crack increments take into account the damage accumulated by the previous loading, in the same way it was done for the constant loading case. But as the coordinate system moves with the crack, a coordinate transformation of preceding damage functions is necessary. Therefore, since the distance r_i where the accumulated damage equals 1 in the **i**-th event is a variable that depends on ΔK_i and on the previous loading history, elements of different widths may be broken by this model at each load cycle.

RESULTS AND DISCUSSION

FCG experiments under variable amplitude loading were performed using API-5L-X52 steel CT specimens, 50 mm wide by 10 mm thick. Pre-cracking was made under constant amplitude loading with an initial $\mathbf{\Delta K = 20\ MPa/m^{1/2}}$ until reaching **a = 12.55mm** (**a/w = 0.25**). FCG occurred under LEFM conditions. Testing was conducted in a 100 kN computer-controlled servo-hydraulic machine. Crack size was monitored within a 20μm accuracy by the Back Face Strain technique [9], using a 5mm 120Ω strain gage.

Oligocyclic fatigue tests were carried out under axial strain control according to the ASTM E 606-92 specifications, using the same equipment described above. Two specimens were tested at each strain amplitude, and to obtain the **εN** curve fifty specimens were tested under deformation ratios varying from **R = –1** to **R = 0.8**, see Figure 1. The test frequency varied between 1 and 10 Hz, and the data acquisition system sampled a minimum of 500 points per cycle. The module method (ASTM E 606-92) was used to determine the steels fatigue life. The measured material properties are shown in the table below.

E [GPa]	S_u [MPa]	S_y [MPa]	S_y’ [MPa]	H’ [MPa]	n’
200	527	430	370	840	0.132

σ'_f [MPa]	ε'_f	b	c	ΔK_{th} (R=0.1) [$MPa\sqrt{m}$]	da/dN (R=0.1) [m/cycle]
720	0.31	–0.076	–0.53	8.0	$2{\cdot}10^{-10}{\cdot}(\Delta K{-}8)^{2.4}$

Table 1: Mechanical properties of the API 5L X52 steel

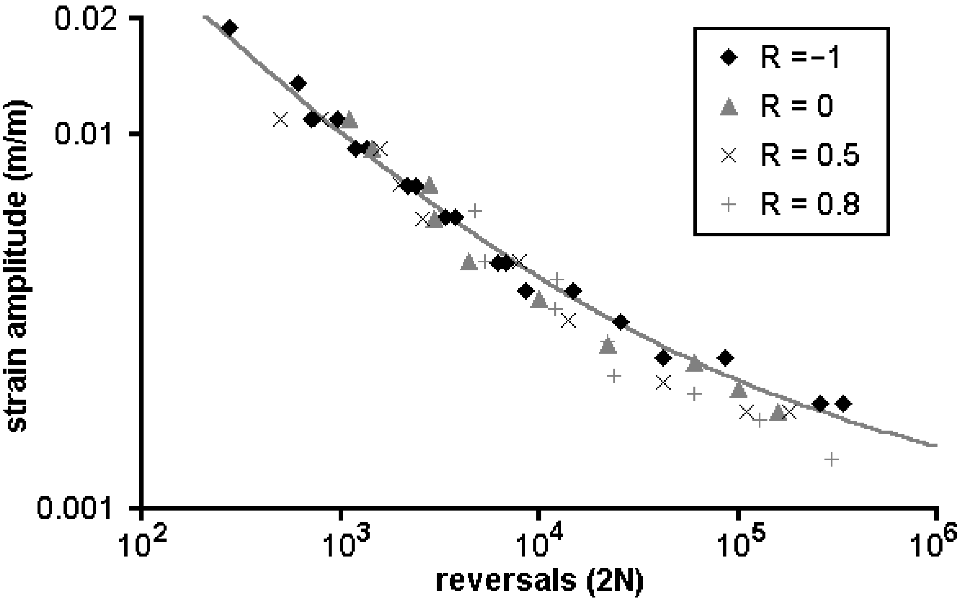

Figure 1: Measured and fitted strain-life data for the API 5L X52 steel

Note from the figure above that the studied material is almost insensitive to the deformation ratio, in special for short lives. Morrow’s strain-life equation, which includes the mean stress effect only in Coffin-Manson’s elastic term, was found to best fit the experimental data. Morrow’s fitted equation is plotted for **R = –1** in Figure 1.

Crack growth was then conducted at 25 Hz under a VA load history consisting of a series of 50,000 blocks containing 100 reversals (50 cycles) each, as shown in Figure 2. The high mean stress levels were chosen to avoid crack closure effects, since they were not yet included in the model (even though they can be easily accounted for when drawing the hysteresis loops). The load history was counted by the sequential rain-flow method [8]. The damage calculation was made using a specially developed code based on Equations (1-3) and the linear strain concentration rule. Figure 3 compares the predictions and experiments.

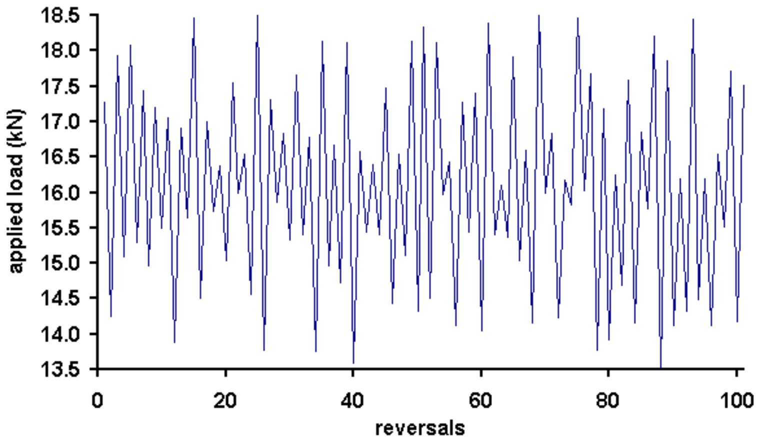

Figure 2: Load block applied to the CTS

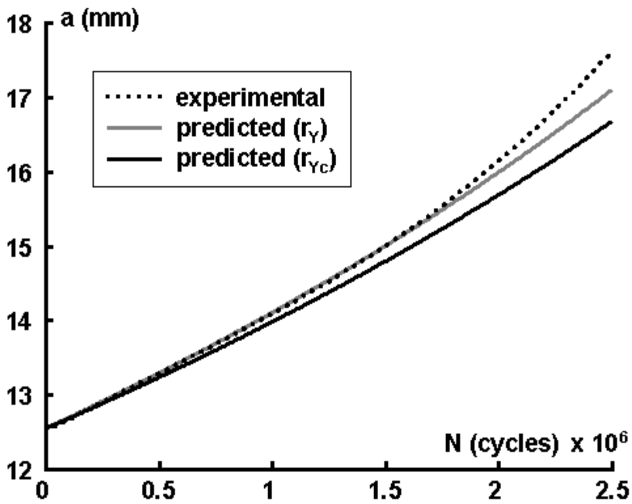

Figure 3: Comparison between crack growth measurements and εN-based predictions

As seen in the figure above, the crack growth predictions under variable amplitude loading based solely on εN parameters were very accurate. The prediction that assumed no damage outside the cyclic plastic zone $\mathbf{r_{Yc}}$ (solid black line in Figure 3) underestimated crack growth. However, when the small (but ***significant***) damage in the material between the cyclic and monotonic plastic zone borders is also included in the calculations, then an even better agreement is obtained (gray line in Figure 3).

Note that crack growth is slightly underestimated after $1.8{\cdot}10^6$ cycles, probably due to the simplification in Equation (2), which neglects the elastic damage and the mean stress effects. Perhaps after considering the elastic damage contribution and the high mean stresses from the studied history, the predicted crack rates would increase to the experimental levels.

Finally, the presented predictions assumed the linear strain concentration rule, however other models such as Neuber's or Glinka's [2] could be used, leading to different results.

CONCLUSIONS

An εN-based damage accumulation model has been proposed to predict fatigue crack propagation under variable amplitude loading. The stress field singularity is removed by modeling the crack as a sharp notch with a small but finite radius ρ. The geometric stress concentration factor of the notch is then estimated from ρ and the K_I expression of an equivalent cracked specimen using the method developed by Creager and Paris. The finite deformations at the notch root are calculated using a strain concentration rule such as Neuber or Glinka, and the strain distribution ahead of the crack tip is modeled using a modified HRR field, obtaining the hysteresis loops at each volume element ahead of the crack path. Due to the non-linearity of Coffin-Manson's εN curve, the damage at the volume elements beyond the current yield zone (or, less conservatively, beyond the reversed yield zone) may be neglected, simplifying the numerical calculations. Experimental results on API 5L X52 steel show a good agreement between measured crack growth under VA loading and the predictions based purely on εN data. This methodology can be complemented by strip-yield model calculations, which are used to predict the crack closure caused by the residual strains at the crack faces. Moreover, the effect of residual stress fields ahead of the crack tip can be directly accounted for when drawing the hysteresis loops, providing a powerful physical model to understand crack retardation effects based solely on εN concepts.

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Contact address: meggi@mec.puc-rio.br