ON THE NUMERICAL IMPLEMENTATION OF THE εN METHODOLOGY UNDER VARIABLE AMPLITUDE LOADING

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ABSTRACT

The εN method has been widely used to design against fatigue crack initiation, especially under low cycle conditions. However, its practical implementation in fatigue design software under variable amplitude (VA) loading requires a careful and detailed approach to avoid completely wrong life predictions. Particularly when dealing with variable amplitude loads, it is not possible to predict physically admissible strain ranges at the critical point (generally a notch root) without recognizing load order. Since plasticity generates memory, sequence effects must be accounted for when accurately modeling elastic-plastic hysteresis loops (HL). These calculations must be performed even if the piece is virgin, if the residual stress and strain state is zero, and if the cyclical hardening or softening transient can be neglected. In this work, several procedures are discussed to calculate physically admissible HL that reproduce experimental observations, verified through VA experiments on 4340 and API S-135 steel specimens.

KEYWORDS

Variable amplitude; εN Method; hysteresis loop modeling; Neuber rule; fatigue automation

INTRODUCTION

The εN is a modern design method, corroborated by traditional institutions such as the SAE [1-4]. A brief review of the main equations used in this methodology is presented next. Ramberg-Osgood is one of many empirical relations that can be used to model the cyclic response of the materials. Its main limitation is not to recognize a purely elastic behavior, and its main advantage is its mathematical simplicity. It is used to describe the stresses and strains at the notch root by

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \tag{1}$$

where **E** is the Young's modulus, **K'** is the hardening coefficient and **n'** is the hardening exponent of the cyclically stabilized $\Delta\sigma\Delta\varepsilon$ curve.

Neuber is the most used equation to correlate the nominal stress $\Delta\sigma_n$ and strain $\Delta\varepsilon_n$ ranges with the stress $\Delta\sigma$ and strain $\Delta\varepsilon$ ranges they induce at a notch root. The Neuber equation states that the product between the stress concentration factor K_σ (defined as $\Delta\sigma/\Delta\sigma_n$) and the strain concentration factor K_ε (defined as $\Delta\varepsilon/\Delta\varepsilon_n$) is constant and equal to the square of the geometric stress concentration factor K_t, thus

$$K_t^2 = \frac{\Delta\sigma \cdot \Delta\varepsilon}{\Delta\sigma_n \cdot \Delta\varepsilon_n} \tag{2}$$

When the nominal stresses are lower than S_{Yc}, the cyclic yielding strength, it is common practice to model them as Hookean and, therefore, to assume elastic nominal loads through a simplified form of Neuber's equation.

Given $\Delta\sigma_n$, the material properties **E**, **K'** and **n'**, and the elastic stress concentration factor K_t, the notch root stress and strain ranges $\Delta\sigma$ and $\Delta\varepsilon$ are then calculated by an appropriate numerical algorithm. The relationship between the critical point stress range $\Delta\varepsilon$ and its fatigue initiation life **N** is usually given by the classical Coffin-Manson rule

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N)^b + \varepsilon_f'(2N)^c \tag{3}$$

where σ_f', ε_f', **b** and **c** are material constants, which are normally measured in fully alternated tension-compression fatigue tests. The effect of a mean stress σ_m at the critical point is usually calculated by other equations, such as Morrow's and Smith-Watson-Topper's.

There is vast experimental support to justify the use of these εN equations to predict fatigue crack initiation under simple loads. However, when using this method under variable amplitude loading, it is common to neglect loading order effects and to simply calculate the damage caused by the i-th load event as if it was independent of all others. Hence, the classical idea is to rain-flow count the nominal loads $\Delta\sigma_{ni}$, to calculate the corresponding notch root strain range $\Delta\sigma_i$ by

$$(K_t \Delta\sigma_{n_i})^2 = \Delta\sigma_i \cdot \left(\Delta\sigma_i + 2E \cdot \left(\frac{\Delta\sigma_i}{2K'} \right)^{1/n'} \right) \tag{4}$$

and to obtain the respective strain range $\Delta\varepsilon_i$ and damage d_i using

$$\frac{\Delta\sigma_i}{E} + 2 \cdot \left(\frac{\Delta\sigma_i}{2K'}\right)^{\frac{1}{n'}} = \Delta\varepsilon_i = \frac{2\sigma_f'}{E}(2N_i)^b + 2\varepsilon_f'(2N_i)^c \Rightarrow d_i = \frac{n_i}{2N_i} \tag{5}$$

Despite its many shortcomings, most fatigue designers use the linear damage accumulation (or the Palmgren-Miner's) rule, $d = \Sigma d_i$, and predict failure when $\Sigma d_i = \beta$, with $\beta = 1$ being the most used value. However, the traditional εN procedure based on rain-flow counting of the loading followed by Neuber, Ramberg-Osgood, Coffin-Manson and Miner rules does not guarantee the prediction of physically admissible hysteresis loops at notches under VA loading, as discussed next.

APPLICATION OF THE εN METHOD UNDER VARIABLE AMPLITUDE LOADING

When dealing with variable amplitude loads, it is not possible to predict physically acceptable strain ranges at the critical point without recognizing the load order. Since plasticity generates memory, sequence effects must be accounted for when accurately modeling elastic-plastic hysteresis loops. Some of these issues are discussed next.

Hysteresis Loop Calculation under VA Loading

To guarantee the quality of the predictions, it is indispensable to first assure that the calculation model reproduces the hysteresis loops at the critical point, for only then calculating the damage caused by the loops. Even if the piece is virgin, if the residual stress and strain state is zero, and if the cyclical hardening or softening transient can be neglected, the increments of plastic strain are dependent on the load history. Therefore, it is necessary to distinguish the first 1/2 cycle from the subsequent load events. In the idealized case, the first 1/2 cycle departs from the origin of the σε plane following the (cyclic) σε curve, giving

$$(K_t\sigma_{n_0})^2 = \sigma_0\cdot\left(\sigma_0 + E\cdot\left(\frac{\sigma_0}{K'}\right)^{1/n'}\right);\ \frac{\sigma_0}{E}+\left(\frac{\sigma_0}{K'}\right)^{\frac{1}{n'}} = \varepsilon_0 = \frac{2\sigma'_f}{E}(2N_0)^b + 2\varepsilon'_f(2N_0)^c \Rightarrow d_0 = \frac{1}{2N_0} \quad (6)$$

But distinguishing the first elastic-plastic event is still not enough to guarantee correct hysteresis loop predictions. It is also necessary to guarantee that all subsequent loops are bounded by the cyclic σε curve and by the wrapper of the hysteresis loops. Hence, the automation software should verify if and when the predicted hysteresis loops cross the cyclic σε curve or previously induced loops. At the crossing point, the hysteresis loop equation must be switched to follow the cyclic σε curve or the curve of a previously induced loop.

Figure 1 shows hysteresis loops associated with the strain history {0 → 5 → 3.5 → 8 → 5 → 6.5 → −1.5 → 2 → 0.5 → 8}×10^3μm/m, following the events 0 through 9. Three types of loop corrections can be noticed: (i) in the first half-cycle, the cyclic σε curve must be followed as the material is assumed to be virgin; (ii) between events 2 and 3 the HL must switch to the cyclic σε curve; and (iii) the HL starting from events 5 and 8 must switch to the outer wrapper of the loops to reach points 6 and 9, respectively. These corrections must be performed otherwise physically inadmissible loops are generated, resulting in potentially non-conservative predictions.

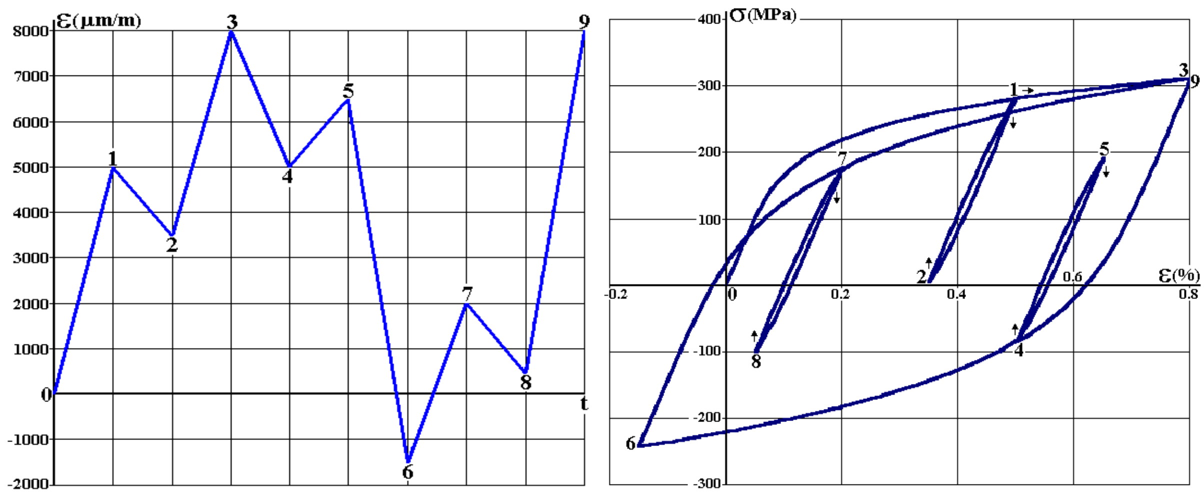

Figure 1: Variable amplitude deformation history and associated hysteresis loops.

Note that if Coffin-Manson's equation was considered in the above example, it would not be necessary to draw the hysteresis loops, because the damage would be completely defined by the strain ranges $\Delta\varepsilon_i$ from the original history. However, when the mean stress effect is modeled through other strain-life curves (e.g. Morrow or Smith-Watson-Topper), then the actual stresses associated with each load reversal must be calculated, which can only be done by calculating the associated loops. Moreover, if the original history is given as a load or stress history (instead of a strain one), then the actual strains must be obtained drawing the corrected hysteresis loops. Therefore, strain-controlled εN results cannot be used to predict the behavior of stress-controlled components under variable amplitude loading, unless the hysteresis loops are calculated.

The numerical procedures to obtain the corrected hysteresis loops are, at least in theory, relatively easy to implement. They involve keeping track of the hierarchy of each hysteresis loop to correctly draw loops within others. The starting point of every new HL must be accounted for in a table. Then, every time a smaller loop is closed (such as the ones formed by events 1-2, 4-5 or 7-8 in Figure 1), its entry is removed from the table, switching curves to the previous HL in the list. The algorithm continues until the end of the history.

To reduce the computational complexity of the above algorithm, it has been recommended in the literature to reorder the loading history, placing the largest event in the beginning. In this simplification, all subsequent events would lie within the large loop associated with the first one, eliminating the need for many of the discussed corrections. However, this procedure is not adequate, because it alters the loading order, ignoring the memory effects associated to plasticity. This anticipation is only admissible when the largest event occurs relatively in the beginning of the global history, such as in the case of periodic loading. Otherwise, if there's any transient characteristic in the original history, this simplification should not be used.

Rain-flow Counting of Elastic-Plastic Events

Another issue in the classical εN approach is cycle counting. The usual practice is to apply the rain-flow method to the original loading history. This is perfectly admissible under linear elastic conditions, such as in the SN methodology. However, in the elastic-plastic case this procedure is not recommended since the rain-flow method changes the loading order. Even sequential rain-flow algorithms [5], which may be appropriate for crack growth calculations, cannot be used in the εN method prior to drawing the hysteresis loops. Only the original loading (without any cycle counting) can generate the experimentally observed hysteresis loops.

On the other hand, rain-flow counting is fundamental to account for the large amplitude events hidden in variable amplitude loading histories. The correct approach in these cases is to rain-flow count the calculated strains, which must be done only *after* having drawn the hysteresis loops. This is the only cycle counting procedure that results in the experimentally observed hysteresis loops, while preserving the load order. The other approaches tend to result in significantly non-conservative predictions, unless the variable amplitude history is well behaved, with frequent controlling overloads.

Elastic-Plastic Nominal Stresses

When dealing with stress concentration, it is common practice to model the nominal stresses as purely elastic, while using an elastic-plastic model such as Ramberg-Osgood to represent the behavior at the critical point. However, this approach is inconsistent since the material is the same at both regions. When the nominal stresses are not substantially smaller than the cyclic yielding strength **S_Y'**, the predicted hysteresis loops at the notch root can be significantly non-conservative. In fact, when the nominal stresses are in the order of **S_Y**, the Hookean model can predict stresses and strains at the notch root that are ***smaller*** than the nominal ones, a clear non-sense. A detailed study performed on measured properties of 517 different structural steels reveals that the Hookean modeling can lead to non-conservative life prediction errors above 100% even for nominal stress amplitudes as low as **$0.3 \cdot S_Y$'** [6].

To avoid these type of errors induced by the simplified Neuber approach, it is necessary to use the Ramberg-Osgood model to describe not only the stresses at the notch root, but also to describe the nominal stresses. In this case, given the nominal stress range $\Delta\sigma_n$, the stress range at the notch root $\Delta\sigma$ can be calculated from

$$K_t^2\left(\Delta\sigma_n^2 + \frac{2E\Delta\sigma_n^{(n'+1)/n'}}{(2K')^{1/n'}}\right) = \Delta\sigma^2 + \frac{2E\Delta\sigma^{(n'+1)/n'}}{(2K')^{1/n'}} \Rightarrow \Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \qquad (7)$$

An interesting result is that both K_σ and K_ε tend to constant values as the nominal stress amplitude is increased, resulting in lower and upper bounds for K_σ and K_ε

$$K_t^{2n'/(1+n')} \le K_\sigma \le K_t \le K_\varepsilon \le K_t^{2/(1+n')} \qquad (8)$$

It is worth emphasizing that these corrections are indispensable under penalty of generating predictions that are (i) physically inadmissible, and (ii) probably *non*-conservative. Only after applying all the required corrections it is possible to predict decent loops and, hence, the correct fatigue damage if the load amplitude is variable.

An algorithm to computationally implement these routines has been implemented and its efficiency verified through VA experiments on 4340 and API S-135 steel specimens [7], as shown in Figures (2) and (3).

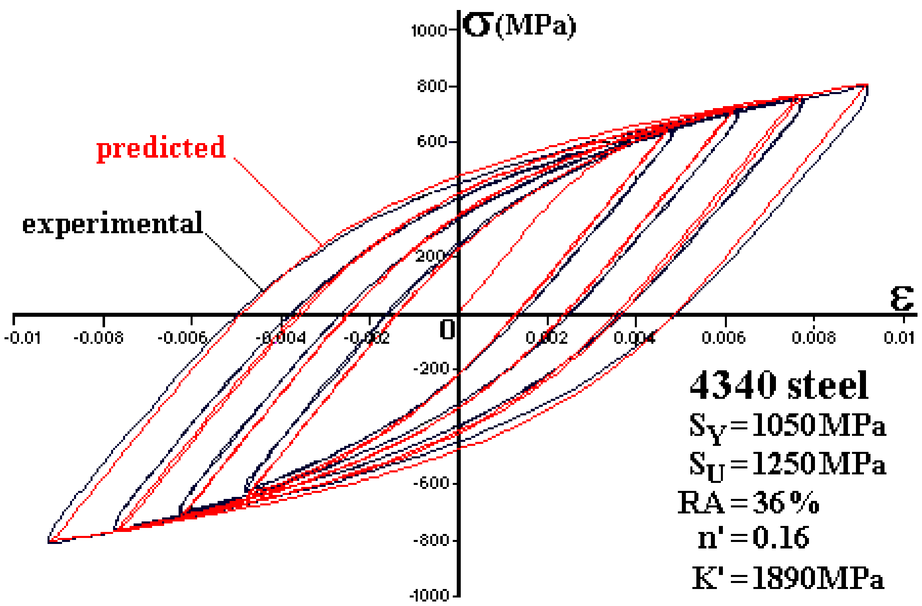

Figure 2: Measured and predicted hysteresis loops under increasing/decreasing amplitude loading in 4340 steel.

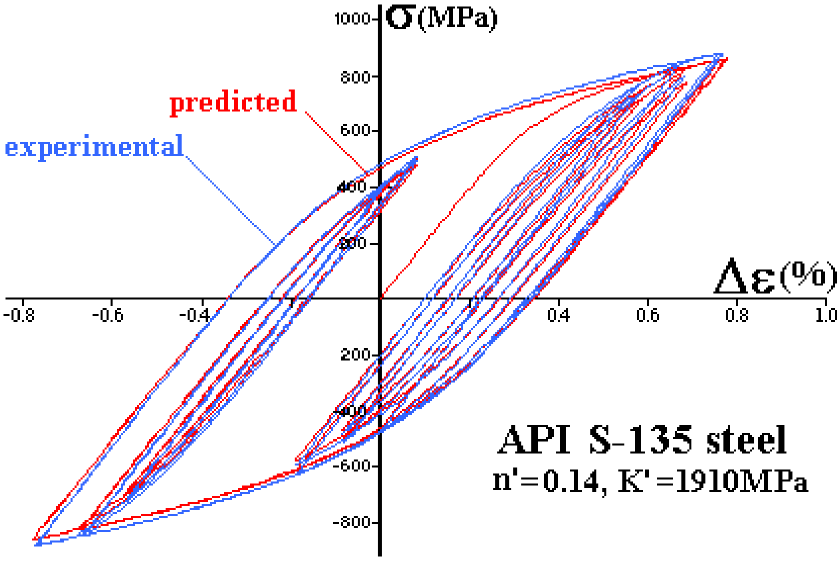

Figure 3: Measured and predicted loops under variable amplitude loading in API S-135 steel.

CONCLUSIONS

From this work it can be concluded that precise fatigue life predictions require an accurate description of the stress-strain history at the critical point. Therefore, the practical implementation of the εN methodology requires, in that order: (i) calculating the hysteresis loops $\Delta\sigma_i\Delta\varepsilon_i$ at the notch root; (ii) following the $\sigma\varepsilon$ cyclic curve at the first event; (iii) including all hysteresis loop corrections under variable amplitude loading; (iv) applying the linear strain concentration rule or Neuber's rule considering elastic-plastic nominal stresses; (v) rain-flow counting the resulting $\Delta\varepsilon_i$; and finally (vi) calculating damage according to Miner, using *measured* strain-life data and efficient numerical methods. Such predictions can only be made with the aid of appropriate automation software, since the numerical effort to sequentially solve the εN equations is quite heavy.

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