

Fatigue Crack Path and Life Predictions in 2D Structures

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ABSTRACT

A methodology to predict the fatigue crack path in generic 2D structures and its propagation life under variable-amplitude (VA) loading considering crack retardation effects is presented. It uses specially developed self-adaptive finite elements to calculate the generally curved fatigue crack path and the associated mixed-mode stress intensity factors K_I and K_{II} at each propagation step, assuming constant-amplitude loading and fixed crack increments. This process requires only a few remeshing steps and is computationally efficient. Then, the calculated K_I values are fitted by an analytical expression, which is used in a local-approach fatigue design program to predict crack propagation lives under VA loading, considering load interaction effects such as crack retardation or arrest after overloads. This methodology is experimentally validated by fatigue crack growth tests on special specimens, whose standard geometry was modified with holes positioned to attract or to deflect the cracks.

INTRODUCTION

The prediction of fatigue crack propagation lives under variable-amplitude (VA) loading in complex two-dimensional (2D) structural components is a practical problem which still presents some interesting and challenging questions to the fatigue analyst. Since in these structures the crack path is generally curved, the first question is how to efficiently predict this path and to obtain the associated stress intensity factors (SIF) K_I and K_{II} . A finite element (FE) global discretization of the structural component using an appropriate mesh with specialized crack tip elements can be used to predict the crack path and to calculate K_I and K_{II} under constant-amplitude (CA) loading, but to be computationally efficient it must include appropriate automatic remeshing procedures in the code [1-2].

But even the best remeshing algorithm can not turn such a global method into an efficient calculation tool to predict fatigue lives under VA loading, which can have a great number of significant events. The time-consuming remeshing procedures and FE recalculations of the entire structure stress/strain field after each load event, counted e.g. by the sequential rain-flow method, requires such a large computer effort that this global approach is simply not a practical solution for this problem. Moreover, the FE modeling of crack retardation effects is, at best, only a partially solved question, and still cannot be reliably used in practical fatigue life predictions under VA loading [3].

On the other hand, predictions of fatigue crack growth (FCG) life can be efficiently made by the local approach, based on the direct integration of an appropriate $da/dN = f(\Delta K, R, K_C, K_{op}, \Delta K_{th}, \dots)$ modification of the Paris equation, where da/dN is the crack propagation rate, $\Delta K = K_{max} - K_{min}$ is the SIF range, $R = K_{min}/K_{max}$, K_C is the material toughness, K_{op} is the crack opening load and ΔK_{th} is the FCG threshold. The idea is to calculate the crack increment caused by each VA load event considering, if required, crack growth retardation or acceleration effects using semi-empirical design rules. However, this approach requires the SIF expression for the crack, which is simply not available for most real components. In these cases, the errors involved in using approximate handbook-type SIF expressions increase as the real (curved) crack deviates from the tabulated one, making the local approach accuracy at least questionable and generally unacceptable.

Since the advantages of these two approaches are complementary, this not so trivial prediction problem can be successfully divided into two tasks. First, the (generally curved) fatigue crack path and its SIF are calculated in a specialized FE program, supposing CA loading and using pre-fixed small crack increments and automatic remeshing schemes. Appropriated numerical methods are used to calculate the crack propagation path, based on the computation of the crack incremental direction using the SIF K_I and K_{II} generated by the FE program. Then, an analytical expression $K_I(\mathbf{a})$ is fitted to the mode I SIF calculated at each crack step, where \mathbf{a} is the length along the crack path. In the sequence, this $K_I(\mathbf{a})$ expression is used as an input to a general purpose fatigue design program based on the local approach, where the actual VA loading is efficiently treated by the integration of the crack propagation equation, considering load interaction effects such as crack retardation or arrest after overloads, if required.

This methodology has been experimentally validated through crack growth under CA loading experiments on modified compact tension C(T) specimens, in which holes were machined to curve the crack propagation path. FE predictions indicated that the fatigue crack is always attracted by the hole, but it can either curve its path and grow toward the hole (“sink in the hole” behavior) or just be deflected by the hole and continue to propagate after missing it (“miss the hole” behavior). The transition point between the two behaviors was identified, and modified C(T) specimens were machined with the hole just half a millimeter above or below the transition point. Several crack retardation models were then calibrated by testing regular C(T) specimens under VA loading, and the calibrated parameters were used to predict the fatigue lives of the modified C(T) specimens under similar but not identical loading conditions.

NUMERICAL COMPUTATION OF THE STRESS-INTENSITY FACTORS AND CRACK INCREMENT DIRECTION

The most used methods to compute the SIF along the (generally curved) crack path under 2D mixed-mode loading are the displacement correlation technique [4], the potential energy release rate computed by means of a modified crack-closure integral technique [5-6], and the J-integral computed by means of the equivalent domain integral (EDI) together with a mode decomposition scheme [7]. But since Bittencourt et al. [8] showed that for sufficiently refined FE meshes all three methods predict essentially the same results, it is not really important which one is used in practice. However, the FE code used in this work, named **Quebra2D** (meaning 2D fracture in Portuguese), allows the user to choose any one of them [1-3].

And the three most used criteria for numerical computation of crack incremental growth direction in the linear-elastic regime are the Maximum Circumferential Stress ($\sigma_{\theta\max}$), the Maximum Potential Energy Release Rate ($G_{\theta\max}$), and the Minimum Strain Energy Density ($U_{\theta\min}$). In the first criterion, Erdogan and Sih [9] considered that the crack extension should occur in the direction that maximizes the circumferential stress in the region close to the crack tip. In the second, Hussain et al. [10] have suggested that the crack extension occurs in the direction that causes the maximum fracturing energy release rate. And in the last criterion, Sih [11] assumed that the crack growth direction is determined by the minimum strain energy density value near the crack tip. Bittencourt et al. [8] have also shown that if the crack orientation is allowed to change during the automatic simulation of the crack growth, these three criteria provide basically the same results for sufficiently refined FE meshes. The $\sigma_{\theta\max}$ criterion is the simplest, even presenting a closed form solution:

$$\theta = 2 \arctan \left(\frac{1}{4} \frac{K_I}{K_{II}} \pm \frac{1}{4} \sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8} \right) \quad (1)$$

where θ is the angle between the crack extension direction and the crack front, with a sign opposite to the sign of K_{II} . The **Quebra2D** program used in this work also allows the user to choose the crack incremental direction criterion, but all predictions presented here were made using the EDI method to calculate $K_I(\mathbf{a})$ and the $\sigma_{\theta\max}$ criterion to obtain the crack path.

LOAD INTERACTION EFFECTS ON FATIGUE CRACK PROPAGATION

In VA fatigue problems, the sequence of the loading events can have a very important effect on the crack propagation life. Normally, tensile overloads can retard or arrest the subsequent crack growth, and compressive underloads can accelerate it. Neglecting these sequence effects in fatigue life calculations can completely invalidate the predictions. However, the generation of a universal algorithm to quantify these sequence effects in FCG is particularly difficult, due to the number and to the complexity of the mechanisms involved, such as plasticity-induced crack closure, blunting and/or bifurcation of the crack tip, residual stresses and strains, incompatible crack front orientation, strain-hardening, crack face roughness, and oxidation of the crack faces, e.g. Besides, depending on the case, several of these mechanisms may act concomitantly or competitively, as a function of factors such as crack size, material microstructure, dominant stress-state, and environment [3].

Elber’s plasticity-induced fatigue crack closure generated by the plastic wake that surrounds the crack faces has long been used to explain the phenomenology of load interaction effects in FCG [12], despite a few important limitations [13]. According to Elber, only after completely opening the crack at a load K_{op} would the crack tip be stressed, and the so-called effective stress intensity range $\Delta K_{eff} = K_{max} - K_{op}$ instead of ΔK would be the FCG rate controlling parameter:

$$\frac{da}{dN} = A \cdot (K_{max} - K_{op})^m = A \cdot (\Delta K_{eff})^m \quad (2)$$

where A and m are material constants, which should be experimentally measured. The Elber mechanism can be used to justify experimentally observed FCG retardation after tensile overloads (OL) by the increase they can cause in the crack closure level. In fact, neglecting crack closure in many fatigue life calculations under VA loading can result in overly conservative predictions. There are several models to account for load sequence effects based on OL-induced changes in the crack plastic envelope, which can be subdivided into three main categories: (i) yield zone models, which account for retardation by comparing the OL and the current plastic zone sizes, Z_{oi} and Z_i (which could capture retardation caused by either crack closure or residual stress fields); (ii) crack closure models, which estimate the crack opening loads from experimental data; and (iii) strip-yield models, which numerically calculate the crack closure relations based on Dugdale’s model [3].

Perhaps the best-known yield zone models are those developed by Wheeler and by Willenborg et al. [12]. Both use the same idea to decide whether the crack growth is retarded or not: under VA loading, FCG retardation is predicted when the plastic zone Z_i of the i -th load event after an OL is embedded within the plastic zone Z_{ol} induced by that (previous) OL. The amount of retardation, as compared to the FCG rate that would be obtained at the i -th load cycle if the OL had no effect, is then assumed dependent on the distance from the border of Z_{ol} to the frontier of the i -th crack plastic zone Z_i . Wheeler uses a crack-growth reduction factor bounded by zero and unity, which is calculated for each load cycle after the OL to predict retardation as long as the current plastic zone Z_i is contained within a previously OL-induced plastic zone Z_{ol} . The retardation is maximum just after the OL (therefore it neglects delayed retardation effects), and stops when the border of Z_i touches the border of Z_{ol} .

Thus, if a_{ol} and a_i are the crack sizes at the instant of the OL and at the (later) i -th cycle, and $(da/dN)_{ret,i}$ and $(da/dN)_i$ are the retarded crack growth rate and the corresponding non-retarded rate (at which the crack would be growing in the i -th cycle if the OL had not occurred), then, according to Wheeler

$$\left(\frac{da}{dN}\right)_{ret,i} = \left(\frac{da}{dN}\right)_i \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i}\right)^\beta, \quad a_i + Z_i < a_{ol} + Z_{ol} \quad (3)$$

where β is an experimentally adjustable constant. However, this model cannot predict OL-induced crack arrest because the resulting $(da/dN)_{ret,i}$ is always positive. A simple and effective modification can be used to predict both crack retardation and arrest in a continuous way, using a Wheeler-like parameter to multiply ΔK instead of da/dN after the OL:

$$\Delta K_{ret}(a_i) = \Delta K(a_i) \cdot \left(\frac{Z_i}{Z_{ol} + a_{ol} - a_i}\right)^\gamma, \quad a_i + Z_i < a_{ol} + Z_{ol} \quad (4)$$

where $\Delta K_{ret}(a_i)$ and $\Delta K(a_i)$ are the values of the stress intensity ranges that would be acting at a_i with and without retardation due to the OL, and γ is an experimentally adjustable constant, in general different from the original Wheeler model exponent β . This simple modification can be used with any of the crack propagation equations that recognize ΔK_{th} to predict both retardation and arrest of fatigue cracks after an OL, the arrest occurring if $\Delta K_{ret}(a_i) \leq \Delta K_{th}$.

Willenborg et al. assumed that both K_{max} and K_{min} at the i -th cycle after an OL are reduced by a residual stress intensity K_{RW} , arbitrarily calculated from the difference between the SIF required to produce a plastic zone that would reach the OL zone border (distant $Z_{ol} + a_{ol} - a_i$ from the crack tip at that cycle) and the i -th maximum SIF $K_{max,i}$. Since the range ΔK is unchanged by this reduction, the retardation effect would be caused only by the change in the effective load ratio R_{eff} . An important drawback of the Willenborg model is to predict crack arrest immediately after any OL $\geq 100\%$, independently of the material properties, stress level, or load spectrum. Several modifications have been proposed to improve the original model [12], however the assumption regarding the OL-induced residual compressive stresses through K_{RW} is at least very doubtful [3].

Probably the simplest crack closure model is the Constant Closure [14], based on the observation that for some flight load spectra the closure stresses do not deviate significantly from a certain stabilized value, assumed to be constant. The opening load K_{op} used in VA FCG calculations is generally estimated between 20% and 50% of the maximum OL or the load spectrum peak $K_{ol,max}$, $0.2 \cdot K_{ol,max} < K_{op} < 0.5 \cdot K_{ol,max}$. The main limitation of this model is that it can only be applied to loading histories with "frequent controlling overloads," because it does not model the decreasing retardation effects experimentally observed as the crack tip cuts through a single OL plastic zone. In other words, by keeping K_{op} constant, this model assumes that a new overload plastic zone is formed often enough to act before the crack can significantly propagate through the previous OL-induced plastic zone, and that secondary plasticity effects can be neglected in the intervals between OLs.

Newman [15] concluded from FE calculations that crack closure depends not only on the load ratio R , but also on the ratio between the maximum stress level σ_{max} and the material flow strength S_{fl} (defined as the average between the material yielding and ultimate strengths), and on a stress-state (plane stress/plane strain) constraint factor α . This stress-state constraint typically ranges from $\alpha = 1$ for pure plane-stress (but a value $\alpha = 1.15$ appears to have a better agreement with experimental results) to $\alpha = 1/(1 - 2\nu)$ for pure plane-strain, where ν is Poisson's ratio. Considering

$$f = \frac{K_{op}}{K_{max}} = \begin{cases} \max(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3), & R \geq 0 \\ A_0 + A_1 R, & -2 \leq R < 0 \end{cases} \quad (5)$$

where

$$\begin{cases} A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2) \cdot [\cos(\pi\sigma_{max} / 2S_{fl})]^{1/\alpha} \\ A_1 = (0.415 - 0.071\alpha) \cdot \sigma_{max} / S_{fl} \\ A_2 = 1 - A_0 - A_1 - A_3 \\ A_3 = 2A_0 + A_1 - 1 \end{cases} \quad (6)$$

then, according to Newman, the effective stress intensity range ΔK_{eff} can be expressed as:

$$\Delta K_{\text{eff}} = (1-f) \cdot K_{\text{max}} = \frac{1-f}{1-R} \Delta K \quad (7)$$

However, closure may not be the dominant crack retardation or arrest mechanism in plane-strain FCG. Moreover, fatigue cracks can be OL-retarded or arrested at high **R**-ratios, when there is no crack closure [13, 16]. Clearly, OL-induced changes in **K_{op}** cannot be used to justify the observed load sequence effects in these cases. Therefore, it should be emphasized that despite the crack closure concept popularity, it cannot be used to justify the entire FCG behavior observed under VA loading. In practice this means that, as recommended by Broek [17], retardation models should be calibrated by experimental data fitting. This is no surprise, since single equations are too simplistic to model all the mechanisms that can induce sequence effects in FCG (even **da/dN** × **ΔK** curves, that are much simpler, still need to be measured for design purposes).

NUMERICAL CALCULATIONS

Two pieces of software have been developed to solve the curved FCG problem. The already mentioned **Quebra2D** is an efficient interactive graphical program for simulating two-dimensional fracture processes using adaptive FE analyses. Its graphical interfaces are flexible and friendly, and its powerful automatic remeshing schemes work both for regions with no cracks or with one or multiple cracks, which may be either embedded, surface breaking or branched, using an adaptation of an algorithm previously proposed for generating unstructured meshes for arbitrarily shaped three-dimensional regions [18]. The 2D algorithm has been designed to meet four specific requirements.

First, it should produce well-shaped elements, avoiding elements with poor aspect ratio. While it does not guarantee bounds on element-aspect ratios, empirical observations show that the algorithm is largely successful in this task [1]. Second, the generated mesh should conform to an existing discretization of the boundary. This is important to simulate crack growth, since it allows local remeshing near a growing crack. The algorithm, however, is not restricted to small regions near cracks and is relatively fast. In the examples shown in this work, the entire mesh was regenerated at each crack propagation step in a time interval of less than ten seconds using a Pentium 650 MHz PC with 128 MB of RAM. Another advantage of this strategy is that the boundary curves are discretized independently from the model's domain, thus resulting in a more regular boundary discretization. Third, the algorithm should shift smoothly between regions with elements of highly varying size, because in crack analysis it is not uncommon for the elements near the crack tip to be two or three orders of magnitude smaller than the other elements, specially when dealing with branched fatigue cracks. Fourth, the algorithm should have specific capabilities for modeling cracks, which are usually idealized without volume. That is, the surfaces representing the two sides of a crack face are distinct, but geometrically coincident. This means that nodes on opposite sides of crack faces may have identical coordinates, and the algorithm must be able to discriminate between the nodes and to select the one on the proper crack side.

In this way, the algorithm implemented in **Quebra2D** incorporates well-known meshing procedures and introduces a few original steps. It includes an advancing front technique along with a quadtree procedure to develop local guidelines for the generated elements' size with the best possible shape. To enhance the quality of the mesh element's shape, an *a posteriori* local mesh improvement procedure is used. Another innovation is the generation of internal nodes simultaneously with the elements, using a quadtree only as a node-spacing function. This approach tends to give a better control over the generated mesh quality and to decrease the amount of heuristic cleaning-up procedures. Moreover, it specifically handles discontinuities in the domain or boundary of the model, such as the evolving crack examples that will be shown below.

The input data is a polygonal description of the boundary of the region to be meshed, given by lists of nodes defined by their coordinates and of boundary segments (or edges) defined by their node connectivities, which can represent geometries of any shape, including holes or cracks. From the boundary segments, a background auxiliary quadtree structure is created to control the sizes of the FE generated by the advancing front technique. The given boundary edges form the initial front that advances as the algorithm progresses. At each step of this meshing procedure, a new triangle is generated for each front base edge. The front advances replacing the base edge with new triangle edges. Consequently, the domain region is contracted, possibly into several regions. The process stops when all the contracted regions result in single triangles. For further details see [1-3].

Fatigue life is predicted by a powerful program named **ViDa** (which means "life" in Portuguese, but also stands for Visual Damagemeter). This program has been developed to automate, in a friendly environment, all the calculations required to predict fatigue life under VA loading. It includes all classical fatigue design routines based on the local approach, giving the user total control over the calculation procedures by the **SN**, the **IIW** (for welded structures) the **eN** or the **da/dN** methods. In particular its FCG routines include several load interaction models and over thirty **da/dN** models, but it accepts any other by means of its equation interpreter, reflecting its open philosophy. **ViDa** runs on PCs under Windows 95/NT or better operating systems, and includes all necessary tools to perform the predictions, such as an intuitive and friendly graphical interface in six languages; intelligent databases for stress concentration and stress intensity factors, crack propagation and retardation models, mechanical and other properties of more than 13000 materials, and more; traditional and sequential rain-flow counters; graphical output for all computed results, including elastic-plastic hysteresis loops and 2D crack fronts, e.g.; automatic adjustment of crack initiation and propagation experimental data; the equation interpreter mentioned above, etc. Crack growth can be calculated considering any propagation model and any **ΔK** expression that can be typed in a BASIC syntax (making it an ideal companion to the **Quebra2D** software, which can be used to generate the **ΔK(a)** expression for any 2D geometry).

Moreover, the software has reliable safety features to automatically stop the calculations if, during any loading event, it detects that: (i) $K_{max} = K_C$; (ii) the crack has reached its maximum specified size; (iii) the stress in the residual ligament reaches the rupture strength of the material S_u ; (iv) da/dN reaches **0.1mm/cycle** (for most engineering alloys, above this rate the problem is fracturing, not fatigue cracking); or else if (v) one of the borders of the piece is reached by the crack front, in the part-through crack propagation case (however, for some geometries, the software is able to model the transition from part-through to through cracks). It also informs the user when there is yielding in the residual ligament before the maximum specified crack size or number of load cycles is reached. In this way, the computed values can be used with the guarantee that the validity limits of the mathematical models are never exceeded.

FCG can be calculated by the cycle-by-cycle integration method, which in principle associates to each load reversal the growth the crack would have if that 1/2 cycle was the only one to load the piece. With this assumption, it is easy to write a general expression for the incremental crack growth using any FCG rule $da/dN = F(\Delta K, R, \Delta K_{th}, K_C, \dots)$. If a_i , $\Delta\sigma_i$ and R_i are the crack length, stress range and load ratio in the i -th 1/2 cycle of the loading, then the crack will grow by δa_i given by:

$$\delta a_i = \frac{1}{2} \cdot F(\Delta K(\Delta\sigma_i, a_i), R(\Delta\sigma_i, \sigma_{max_i}), \Delta K_{th}, K_C, \dots) \quad (8)$$

The total growth of the crack is quantified by $\Sigma(\delta a_i)$. Therefore, the cycle-by-cycle method is similar in concept to the linear damage accumulation rule used in the **SN** and **εN** fatigue design methods. And, as in Miner's rule, it requires that *all* events that cause fatigue damage be recognized before the calculation, e.g. by rain-flow counting the loading. But, since it must be applied sequentially, load interaction effects can be recognized. However, the traditional rain-flow counting algorithm alters the loading order, and this can cause serious problems in the predictions, since loading order effects in crack propagation are of two different natures: (i) delayed effects, which can retard or stop the subsequent crack growth due, for instance, to plasticity-induced Elber-type crack closure or to crack tip bifurcation (these interaction effects among the loading cycles usually increase crack life and, if neglected, may induce excessively conservative predictions); and (ii) instantaneous fracture, which occurs in the first load peak where $K_{max} \geq K_C$, an event which must, of course, be precisely predicted.

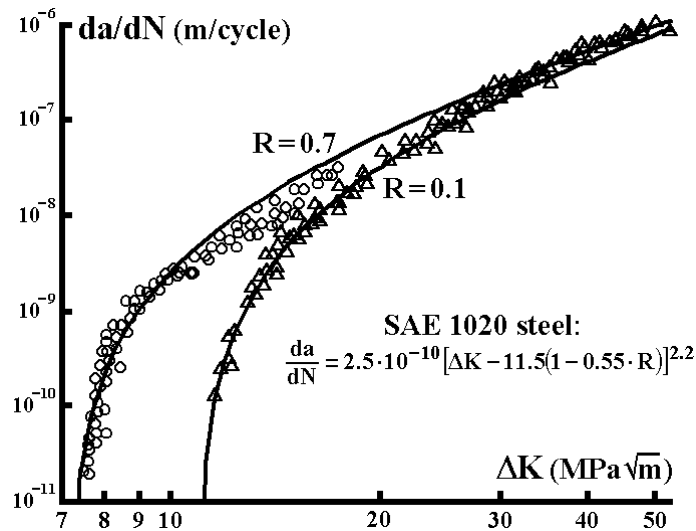
Since the **ViDa** loading input can preserve the time order information, a sequential rain-flow counting option was introduced in that software [1-2]. With such technique, the effect of each large loading event is counted when it actually happens, and not *before* its occurrence, as in the traditional rain-flow method. Therefore, the sequential rain-flow option avoids the anticipation of overload-induced effects, which can cause *non-conservative* crack propagation life predictions: as $K(\sigma, a)$ usually grows with the crack, a given overload applied when the crack is large can be much more harmful than when the crack is small. Sequential rain-flow avoids most sequencing problems caused by the traditional method, and it is certainly an advisable option since it presents advantages over the original algorithm while maintaining its main features without increasing its difficulty. Finally, it must be mentioned that the numerical implementation of retardation models in a cycle-by-cycle algorithm is not conceptually difficult, but it requires a considerable programming effort, see [1-3] for further details. All load interaction models presented above have been implemented in the **ViDa** software.

EXPERIMENTAL RESULTS

To verify the modeling techniques used to predict curved crack paths, tests were performed under CA fatigue loading on four-point bending single-edge notch SE(B) and on compact tension C(T) specimens of 1020 steel, both modified with holes machined to curve the crack propagation path. The analyzed (w%) composition was 0.19 **C**, 0.46 **Mn**, 0.14 **Si**, 0.052 **Ni**, 0.045**Cr**, 0.007 **Mo**, 0.11 **Cu**, 0.002 **Nb**, 0.002 **Ti**, **Fe** balance. The yield strength $S_Y = 285\text{MPa}$, ultimate strength $S_U = 491\text{MPa}$, Young modulus $E = 205\text{GPa}$, and area reduction $RA = 53.7\%$, were measured according to ASTM E 8M-99 standard. The $da/dN \times \Delta K$ data from 16 standard C(T) specimens were measured following ASTM E 647-99 procedures and fitted by a modified McEvily equation (in m/cycle), as shown in Fig. 1. Here, $\Delta K_0 = \Delta K_{th}(R = 0) = 11.5 \text{ MPa}\sqrt{\text{m}}$, and $K_C = 280 \text{ MPa}\sqrt{\text{m}}$.

Before the tests, the hole-modified specimens were FE modeled following the procedures described in the previous sections. Then the hole position was varied in the (numerical) models to obtain the most interesting prediction for the curved crack path, by means of a simple trial-and-error process. After that, the chosen specimen geometries were machined, measured and FE remodeled, to account for small deviations in the manufacturing process. In this way, it could be assured that the numerical models used in the predictions reproduced the real geometry of the tested specimens.

Even though the curved crack path geometry is 2D, once it is calculated the crack itself can be described by its (one-dimensional) length a measured along the crack path (since there is no K_{III} nor warping of the crack plane). Hence, its K_I expression can be written as a function of a , $K_I(a) = \sigma\sqrt{\pi a} \cdot f(a/w)$. The discrete values of the geometry factors $f(a/w)$ calculated for each crack step analyzed by **Quebra2D** were then exported to **ViDa**, where they were automatically fitted by an appropriate continuous analytical function. Using this $K_I(a)$ expressions and the da/dN crack propagation curve measured under pure Mode-I loading, the load program that would be applied during the test was calculated to maintain a quasi-constant stress-intensity range around $\Delta K_I \approx 20\text{MPa}\sqrt{\text{m}}$, with $R = 0.1$. These values are well within stage-II fatigue crack growth (Paris regime) in the 1020 steel da/dN vs. ΔK curve, see Fig. 1.



I.
Fig. 1. Modified McEvilly da/dN equation fitted to the SAE 1020 steel data.

The experimental procedures used during the tests were very similar to those in the standard measurement of da/dN vs. ΔK curves. All the tests were run at 20Hz frequency in a 250kN computer-controlled servo-hydraulic testing machine. The loads were regularly adjusted to maintain the specified quasi-constant ΔK . The only major difference was the use of a digital camera and an image-analysis program to measure the crack size and path. This is a quite precise and economical option to automate these measurements, but its details are considered beyond the scope of this paper.

Cracks were fatigue propagated in SEN specimens with a hole slightly to the side of the starting notch line (created using a 0.3mm jeweler's saw). Fig. 2 shows a picture of a typical crack path after the test and the FE crack path prediction (the line that connects the open dots in the figure) made *before* the test. This crack path modeling is indeed quite satisfactory.

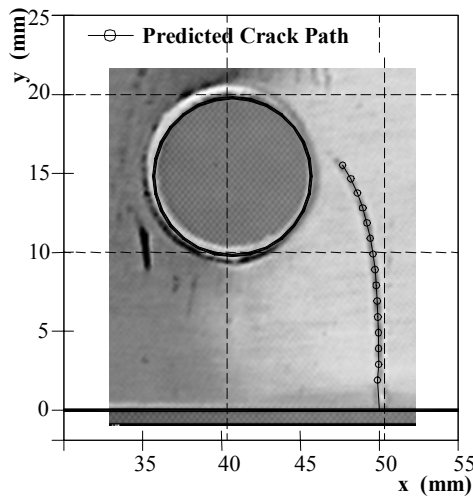


Fig. 2: Predicted and measured crack path [2].

Three of the modified C(T) specimens were tested, each with a 7mm diameter hole positioned at a slightly different horizontal distance **A** and vertical distance **B** from the notch root, see Fig. 3. This odd configuration was chosen because two non-trivial and unexpected crack growth behaviors had been predicted by the FE modeling of the holed C(T) specimens, depending on the hole position. The predictions indicated that the fatigue crack was always attracted by the hole, but it could either curve its path and grow toward the hole or just be deflected by the hole and continue to propagate after missing it.

To test the accuracy of the FE modeling, the transition point between the “sink in the hole” and the “miss the hole” crack growth behaviors was identified. Then, two borderline specimens were dimensioned: one with the hole only half a millimeter below that point and the other with the hole half a millimeter above it. Due to machining tolerances, the actual difference between the vertical position of the holes in specimens turn out to be slightly different. These specimens were then remodeled to predict the actual crack path. The measured and the predicted crack paths are compared in Fig. 4.

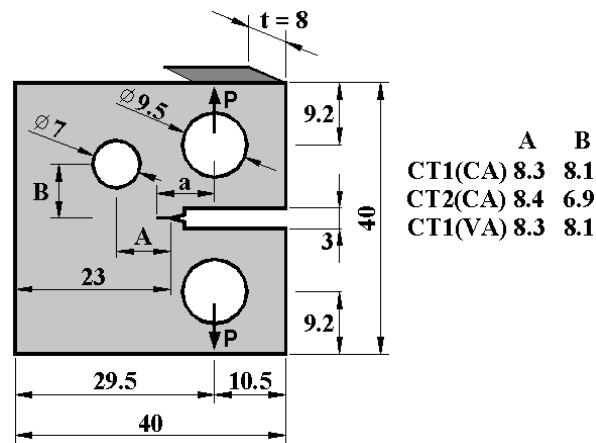


Fig. 3. Measured dimensions of the hole-modified C(T) specimens (mm).

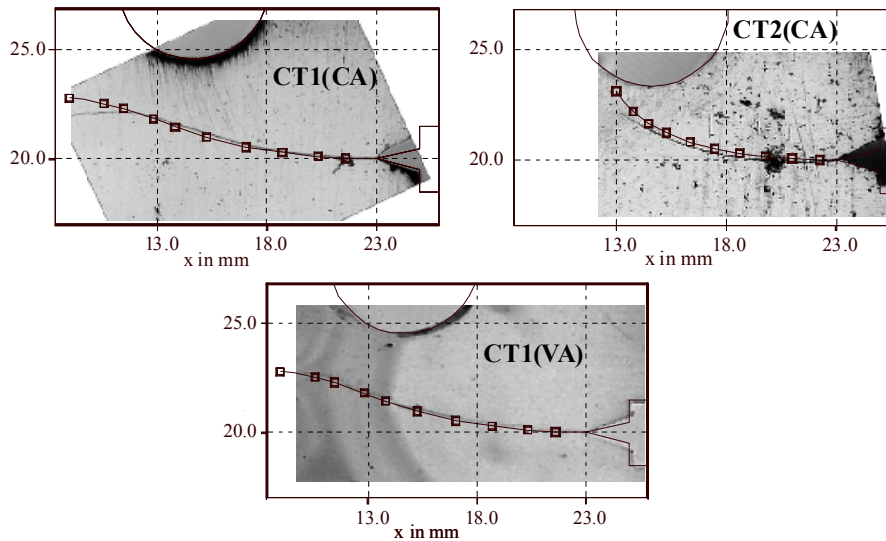


Fig. 4. Predicted and measured crack paths for the modified C(T) specimens (mm) [3].

The initial meshes in the FE models had, in average, about 1300 elements and 2300 nodes, and the final ones after the simulated crack propagation had about 2200 elements and 5500 nodes. Specimens CT1(CA) and CT2(CA) were tested under CA loading. Two other specimens were tested under VA loading: one standard C(T) specimen, and the holed specimen CT1(VA). The goals of this experiment were: (i) to check whether the curved crack paths predicted under CA loading would give good estimates of the measured paths under VA loading; and (ii) to verify whether load interaction models calibrated for straight cracks in the standard C(T) could be used to predict the fatigue life of the holed specimens, which present a curved crack path. The VA load histories applied to the tested specimens are shown in Fig. 5.

The predicted and measured crack paths for the three modified specimens tested under CA or VA loading, shown in Fig. 4, present a very good match. This suggests that the crack path under VA loading is the same as the one predicted under CA loading. Therefore, assuming that only the crack growth rate (but not its path) is influenced by load interaction effects, the discussed two-step methodology can be generalized to the VA loading case. Therefore, the SIF values calculated under CA loading along the crack path using the **Quebra2D** program were exported to the ViDa software to predict fatigue life, considering load interaction effects.

To evaluate whether load interaction models calibrated from straight-crack experiments could be applied to specimens with curved cracks, several crack retardation models were fitted to the data measured on the standard C(T) data under VA loading. The better results were obtained by the Constant Closure model, where K_{op} was calibrated as 26% of the maximum overload SIF, $K_{ol,max}$; by the Modified Wheeler model, where the exponent γ was estimated as 0.51; and by Newman's closure model (generalized for the VA loading case), where the stress-state constraint was fitted as $\alpha = 1.07$, a value suggesting dominant plane-stress FCG conditions. The measured and fitted growth behavior is shown in Fig. 6.

The fitted load interaction parameters were then used to predict the crack growth behavior under VA loading of the hole-modified CT1(VA) specimen, see Fig. 7. The significant retardation effects of the CT1(VA) specimen were very well predicted

using these three load interaction models in the **ViDa** program. In particular, the Modified Wheeler model results in very good predictions, possibly because its simplistic empirical yield-zone formulation can account for both closure and residual stress effects. These results suggest that load interaction models calibrated using straight cracks can be used to predict crack retardation behavior of curved cracks under VA loading.

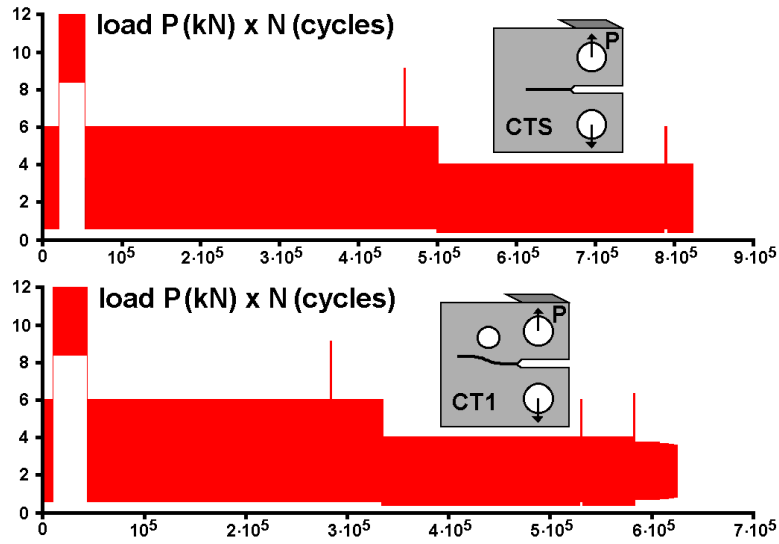


Fig. 5. Applied load history (in kN) for the standard C(T) and for the modified CT1(VA) specimens.

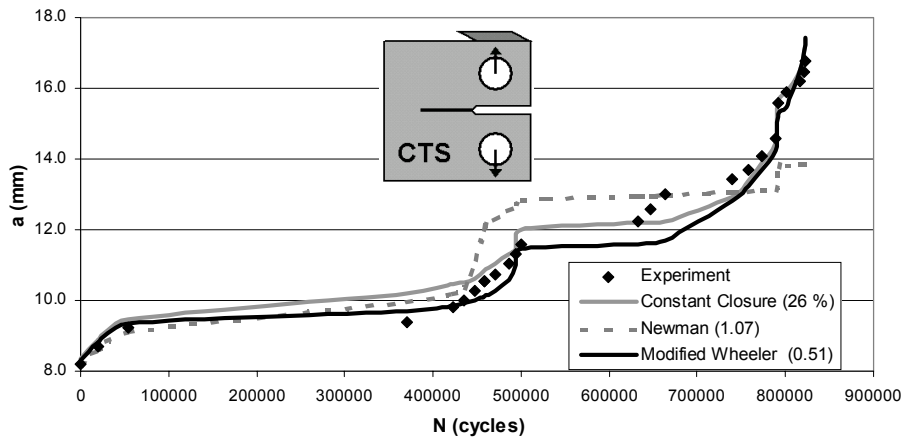


Fig. 6. Measured crack sizes and calibrated calculations on a standard C(T) under VA loading.

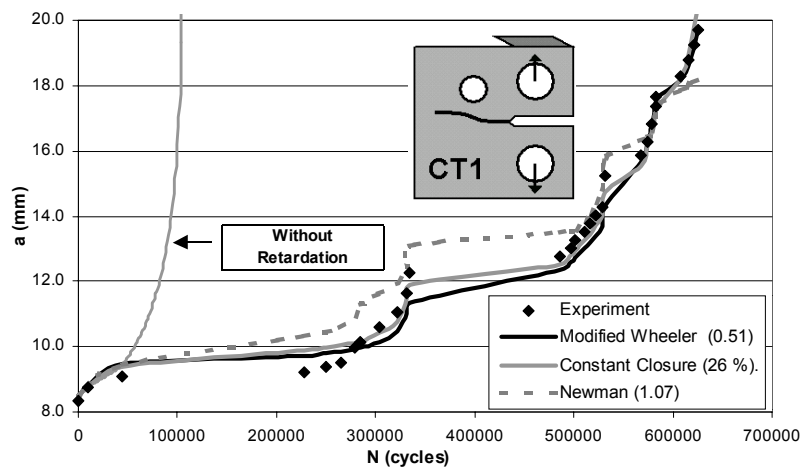


Fig. 7. Crack growth predictions (based on straight-crack calibrations) on a modified C(T) specimen under VA loading.

However, it must be pointed out that the VA histories in Fig. 5 have similar stress levels and OL ratios. This similarity might be one of the reasons why the same load interaction model parameters could be used for both VA cases. The load-spectrum dependency of the crack retardation model parameters might result in poor predictions if completely different VA histories are considered. In addition, the very high sensitivity of the crack growth predictions with the load interaction model parameters is another error source that cannot be ignored. This sensitivity is particularly high when the crack growth rates approach stage I values, as seen in the post-overload regions with almost horizontal slope in Figs. 6 and 7. In this threshold region, miscalculations of just a few percent for the effective SIF can be the difference between crack growth or crack arrest. Since most life cycles are spent during stage I growth, this is the dominant (and most important) region in fatigue design, where the crack growth rates and load interaction effects should be better modeled and measured. This point must be carefully considered when analyzing in the literature crack retardation experiments performed under the Paris regime, where the high sensitivity of fatigue life with load interaction model parameters is masked by the smaller effect of crack closure or residual stress fields.

CONCLUSIONS

A two-phase methodology to predict fatigue crack propagation in 2D structures was extended to variable amplitude loading histories, modeling crack retardation effects. First, self-adaptive finite elements were used to calculate the fatigue crack path and the stress intensity factors along the crack length, at each propagation step. The computed values were then used to predict the propagation fatigue life of the structure by the local approach, considering overload-induced crack retardation effects. Two complementary software products have been developed to implement this methodology. Experimental results validated the application of the proposed methodology to the variable amplitude loading case, suggesting that overloads do not significantly deviate the crack path predicted under constant amplitude loading. Moreover, the developed software could effectively predict the crack propagation path and fatigue life of an intricate two-dimensional specimen under variable amplitude loading.

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