SHORT CRACK EQUATIONS TO PREDICT STRESS GRADIENT EFFECTS IN FATIGUE

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ABSTRACT

In this work, short cracks emanating from circular holes are studied. For several combinations of notch dimensions, the smallest stress range necessary to both initiate and propagate a crack is calculated, resulting in expressions for the fatigue stress concentration factor K_f and therefore the notch sensitivity q. A generalization of El Haddad-Topper-Smith's parameter, which better correlates with experimental crack propagation data from the literature, is presented.

Introduction

It is well known that the notch sensitivity factor q can be associated with the presence of non-propagating cracks. Such cracks are present when the nominal stress range $\Delta\sigma_n$ is between $\Delta\sigma_0/K_t$ and $\Delta\sigma_0/K_f$, where $\Delta\sigma_0$ is the fatigue limit, K_t is the geometric and K_f the fatigue stress concentration factors of the notch. Therefore, in principle it is possible to obtain expressions for q if the propagation behaviour of small cracks emanating from notches is known.

Several expressions have been proposed to model the dependency between the threshold value ΔK_{th} of the stress intensity range and the crack size a for very small cracks [1]. Most of these expressions are based on length parameters such as El Haddad-Topper-Smith's a_0 [2], estimated from ΔK_{th} and $\Delta\sigma_0$, resulting in a modified stress intensity range

$$\Delta K_{\mathrm{I}} = \Delta\sigma\sqrt{\pi(a+a_0)}\,, \quad a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{\mathrm{th}}}{\Delta\sigma_0}\right)^2 \tag{1}$$

which is able to reproduce most of the behaviour shown in the Kitagawa-Takahashi plot [3]. Yu *et al.* [4] and Atzori *et al.* [5] have also used a geometry factor α to generalize the above equation to any specimen, resulting in

$$\Delta K_{\mathrm{I}} = \alpha\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\,, \quad a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{\mathrm{th}}}{\alpha\cdot\Delta\sigma_0}\right)^2 \tag{2}$$

Alternatively, the stress intensity range can retain its original equation, while the threshold expression is modified by a function of the crack length a, namely $\Delta K_{th}(a)$, resulting in

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \sqrt{\frac{a}{a+a_0}} \tag{3}$$

where ΔK_0 is the threshold stress intensity factor for a long crack. Several expressions have been proposed to model this crack size dependence [6-8]. Peterson-like expressions are then calibrated to q based on these crack propagation estimates. However, such q calibration is found to be extremely sensitive to the choice of $\Delta K_{th}(a)$ estimate.

In the following section, a generalization of El Haddad-Topper-Smith's equation is proposed to better model the crack size dependence of ΔK_{th}. This expression is then applied to a single crack emanating from a circular hole, resulting in improved estimates of q.

Analytical Development

A new expression for the threshold stress intensity factor of short cracks is proposed, based on El Haddad-Topper-Smith's equation:

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \left[1+\left(\frac{a_0}{a}\right)^{n/2}\right]^{-1/n} \tag{4}$$

In the above equation, n is typically found to be between 1.5 and 8.0. Clearly, Eqs. (1), (2) and (3) are obtained from Eq. (4) when $n = 2.0$. Also, the classical bi-linear estimate is obtained as n tends to infinity. The adjustable parameter n allows the ΔK_{th} estimates to better correlate with experimental crack propagation data collected from Tanaka et al. [9] and Livieri and Tovo [10], see Fig. 1.

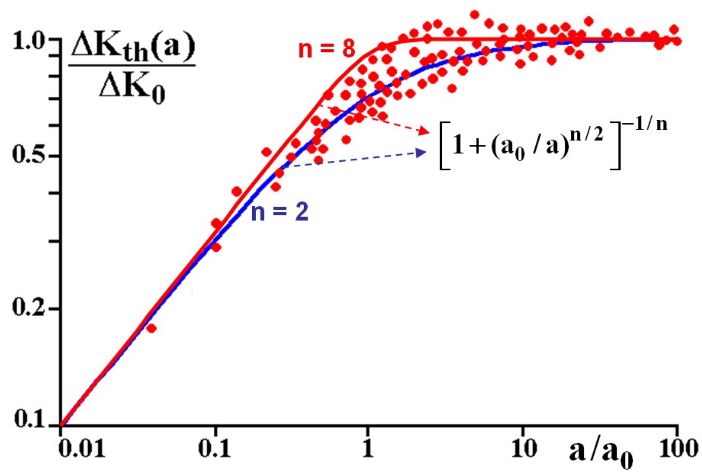

Figure 1. Ratio between short and long crack propagation thresholds as a function of a/a_0.

Equation (4) is now used to evaluate the behaviour of short cracks emanating from circular holes. The stress intensity range of a single crack with length a emanating from a circular hole with radius r is expressed, within 1%, by [11]

$$\Delta K_I = 1.1215 \cdot \Delta\sigma\sqrt{\pi a} \cdot f\left(\frac{a}{\rho}\right) \tag{5}$$

where

$$f\left(\frac{a}{\rho}\right) \equiv f(x) = \left(1+\frac{0.2}{(1+x)}+\frac{0.3}{(1+x)^6}\right)\cdot\left(2-2.354\frac{x}{1+x}+1.2056\left(\frac{x}{1+x}\right)^2-0.2211\left(\frac{x}{1+x}\right)^3\right), \quad x \equiv \frac{a}{\rho} \tag{6}$$

Note that, when the crack size a tends to zero, Eq. (5) becomes

$$\lim_{a\to 0} \Delta K_I = 1.1215 \cdot \Delta\sigma\sqrt{\pi a} \cdot 3 \tag{7}$$

as expected, since the above equation combines the solution for an edge crack in a semi-infinite plate with the stress concentration factor of a circular hole, K_t equal to 3. Note also that the other limit, when a tends to infinity, results in

$$\lim_{a\to\infty} \Delta K_I = \Delta\sigma\sqrt{\pi a/2} \tag{8}$$

which is the solution for a crack with length a in an infinite plate, where one of its edges is far enough from the circular hole not to suffer its influence in the stress field (in fact, the equivalent crack length would be $a+\rho$, however as a tends to infinity the ρ value disappears from the equation). Therefore, it follows that for a circular hole $f(x=0) = 3$ and $f(x\to\infty) = 1/1.1215\sqrt{2} \cong 0.63$.

From Eqs. (4-6), it follows that the crack will propagate when

$$\Delta K_I = 1.1215 \cdot \Delta\sigma\sqrt{\pi a} \cdot f\left(\frac{a}{\rho}\right) > \Delta K_{th} = \Delta K_0 \cdot \left[1 + \left(\frac{a_0}{a}\right)^{n/2}\right]^{-1/n} \tag{9}$$

Using α = 1.1215 and $\Delta K_{th} \equiv \Delta K_0$ for a long crack, then the crack length parameter from the above equation is

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{1.1215 \cdot \Delta\sigma_0}\right)^2 \tag{10}$$

Equations (9) and (10) result in a crack propagation criterion based on the adimensional functions *f* and *g*:

$$f\left(\frac{a}{\rho}\right) > \frac{\left(\frac{\Delta K_0}{\Delta\sigma_0\sqrt{\rho}}\right) \cdot \left(\frac{\Delta\sigma_0}{\Delta\sigma}\right)}{\left[\left(1.1215\sqrt{\frac{\pi a}{\rho}}\right)^n + \left(\frac{\Delta K_0}{\Delta\sigma_0\sqrt{\rho}}\right)^n\right]^{1/n}} \equiv g\left(\frac{a}{\rho}, \frac{\Delta\sigma_0}{\Delta\sigma}, \frac{\Delta K_0}{\Delta\sigma_0\sqrt{\rho}}, n\right) \tag{11}$$

Defining

$$x \equiv \frac{a}{\rho} \quad \text{and} \quad k \equiv \frac{\Delta K_0}{\Delta\sigma_0\sqrt{\rho}} \tag{12}$$

then the crack propagates whenever

$$f(x) > g\left(x, \frac{\Delta\sigma_0}{\Delta\sigma}, k, n\right) \tag{13}$$

Figure 2 plots *f* and *g*, assuming a material/notch combination with k = 1.5 and n = 6, as a function of the normalized crack length x. For a high applied $\Delta\sigma$, the ratio $\Delta\sigma_0/\Delta\sigma$ becomes small, and the function *g* is always below *f*, meaning that a crack of any length will propagate. The lower curve in Fig. 2 shows the function *g* obtained from a ratio $\Delta\sigma_0/\Delta\sigma$ = 1.4, never crossing *f*. On the other hand, for a $\Delta\sigma$ small enough such that $\Delta\sigma_0/\Delta\sigma \geq K_t$ = 3, then *g* is always above *f* and no crack will initiate nor propagate, as shown by the top curve in the figure.

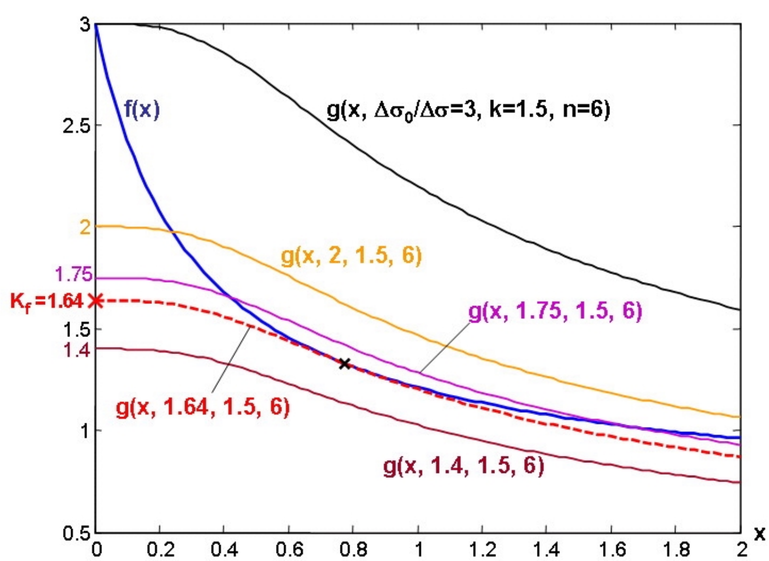

Figure 2. Calculation of the fatigue stress concentration factor K_f from the plots of the functions *f* and *g*.

Three other cases can be noted, as follows. In the first case, the *g* curve with $\Delta\sigma_0/\Delta\sigma$ = 2 in the figure above has only one intersection point with *f*. This means that such stress levels cause a crack to initiate at the notch, however it will only propagate until a size $a = x \cdot \rho$ obtained from the *x* value at the intersection point. Therefore, non-propagating cracks will appear at the notch root.

In the second case, the *g* curve with $\Delta\sigma_0/\Delta\sigma$ = 1.75 in the figure above has two intersection points with *f*. Therefore, non-propagating cracks will also appear, with maximum sizes obtained from the first intersection point (on the left). Interestingly, cracks longer than the value defined by the second intersection will re-start propagating until fracture. Crack growth between the two intersections would need to be caused by a different mechanism, e.g. corrosion or creep.

Finally, the third case can be seen in Figure 2 considering the *g* curve with $\Delta\sigma_0/\Delta\sigma$ = 1.64. In this case, both *f* and *g* functions are tangent and meet in a single point. This $\Delta\sigma_0/\Delta\sigma$ value is therefore associated with the smaller stress range $\Delta\sigma$ that can cause crack initiation *and* propagation without arrest. So, by definition, this specific $\Delta\sigma_0/\Delta\sigma$ is equal to the fatigue stress concentration factor K_f. To obtain K_f, it is then sufficient to guarantee that both functions *f* and *g* are tangent at a single point with $x = x_{max}$. This x_{max} value is associated with the largest non-propagating flaw that can arise from fatigue alone. So, given *n* and *k* from the material and notch, x_{max} and K_f can be solved from the system of equations:

$$\begin{cases} f\left(x_{max}\right) = g\left(x_{max}, K_f, k, n\right) \\ \dfrac{\partial}{\partial x} f\left(x_{max}\right) = \dfrac{\partial}{\partial x} g\left(x_{max}, K_f, k, n\right) \end{cases} \tag{14}$$

This system can be solved numerically for each combination of *k* and *n* values, and the notch sensitivity factor *q* is then obtained from

$$q(k,n) \equiv \frac{K_f(k,n) - 1}{K_t - 1} \tag{15}$$

Results

For several combinations of *k* and *n*, the smallest stress range necessary to both initiate and propagate a crack is calculated from Eq. (14), resulting in expressions for K_f and therefore *q*, see Fig. 3.

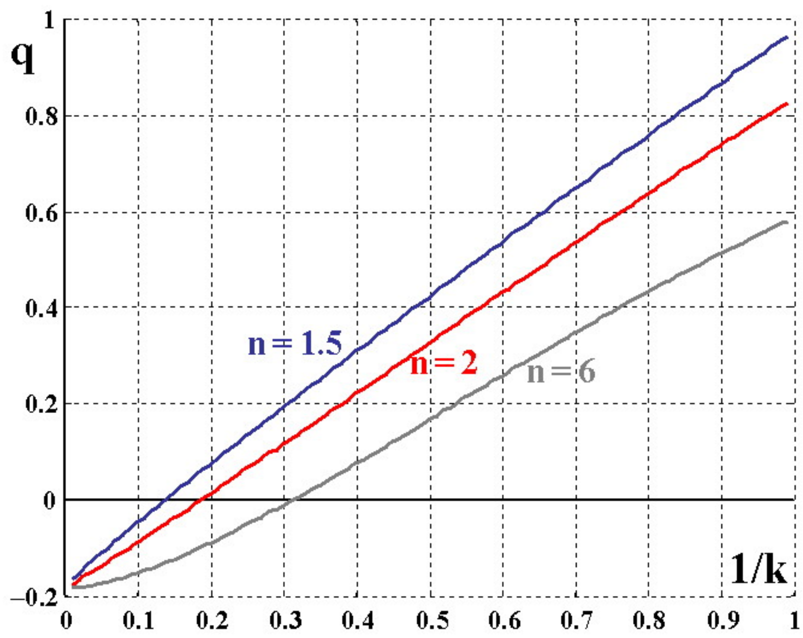

Figure 3. Notch sensitivity factors *q* as a function of the adimensional parameters *k* and *n*.

Note from the figure that q is approximately linear with $1/k$ for $q > 0$. This results in the proposed estimate:

$$q(k,n) \cong \frac{q_1(n)}{k} - q_0(n) = q_1(n)\frac{\Delta\sigma_0\sqrt{\rho}}{\Delta K_0} - q_0(n) \tag{16}$$

where $q_0(n)$ and $q_1(n)$ are functions of n, and $q_1(n)$ is typically between 0.85 and 1.15. Note that if the estimate above results in q larger than 1, then $q = 1$. This will happen at holes with a very large radius ρ_{upper} such that

$$\frac{\Delta\sigma_0\sqrt{\rho_{upper}}}{\Delta K_0} > \frac{1+q_0(n)}{q_1(n)} \quad \Rightarrow \quad \rho_{upper} > \left(\frac{1+q_0(n)}{q_1(n)} \cdot \frac{\Delta K_0}{\Delta\sigma_0}\right)^2 \tag{17}$$

Therefore, it is impossible to generate a non-propagating crack under constant amplitude loading in notches with a very large radius, regardless of the stress level. The stress gradient is so small in this case that any crack that initiates will cut through a long region still influenced by the stress concentration, preventing any possibility of crack arrest. Equation (14) will not have a solution for $x_{max} > 0$, because $\partial g/\partial x$ in this case will be more negative than $\partial f/\partial x$ at $x = 0$.

On the other hand, it is possible to obtain a value of q smaller than zero, down to $q = -0.2$ for a circular hole, see Fig. 3. This can indeed happen for holes with a very small radius ρ_{lower} such that

$$\frac{\Delta\sigma_0\sqrt{\rho_{lower}}}{\Delta K_0} < \frac{q_0(n)}{q_1(n)} \quad \Rightarrow \quad \rho_{lower} < \left(\frac{q_0(n)}{q_1(n)} \cdot \frac{\Delta K_0}{\Delta\sigma_0}\right)^2 \tag{18}$$

The physical meaning of a negative q is that it is easier to initiate and propagate a fatigue crack at a notchless border of the plate than at a very small hole inside the plate. The ΔK_I of a crack at the small hole will soon tend to Eq. (8) due to the large stress gradient, without the 1.1215 factor, while the stress intensity solution for an edge crack will be larger since it includes the 1.1215 factor. In addition, for most materials, the size of this critical radius ρ_{lower} is just a few micrometers. This leads to the conclusion that internal defects with equivalent radius smaller than such ρ_{lower} of a few micrometers are harmless, since its K_f will be smaller than 1, and the main propagating crack will initiate at the surface.

Note that several estimates, such as Peterson's, assume that the notch sensitivity is only a function of ρ and the ultimate strength S_u. Equation (16), however, suggests that q depends basically on ρ, $\Delta\sigma_0$ and ΔK_0, in addition to n. Even though there are reasonable estimates relating $\Delta\sigma_0$ and S_u, there is no clear relationship between ΔK_0 and S_u. This means, e.g., that two steels with same S_u but very different ΔK_0 would have different behaviours that Peterson's equation would not be able to reproduce. Therefore, notch sensitivity experiments should always include a measure of the ΔK_0 of the material.

Finally, data on 450 steels and aluminum alloys with fully measured S_u, $\Delta\sigma_0$ and ΔK_0 are collected from the ViDa software database [12]. The average values of $\Delta\sigma_0$ and ΔK_0 are evaluated for steels with S_u near the ranges 400, 800, 1200, 1600 and 2000MPa, and aluminum alloys near 225MPa. Equation (16) is then plotted as a function of the notch radius ρ, using the above averages and assuming $n = 6$, see Fig. 4. Note that Peterson's equations, which were originally fitted to notch sensitivity experiments, can be reasonably predicted and reproduced using the proposed analytical approach.

Conclusions

A generalization of El Haddad-Topper-Smith's parameter was presented to model the crack size dependence of the threshold stress intensity range for short cracks. The proposed expressions were used to calculate the behaviour of non-propagating cracks. New estimates for the notch sensitivity factor were obtained and compared with Peterson's results. It was found that the q estimates obtained from this generalization correlate well with crack initiation data.

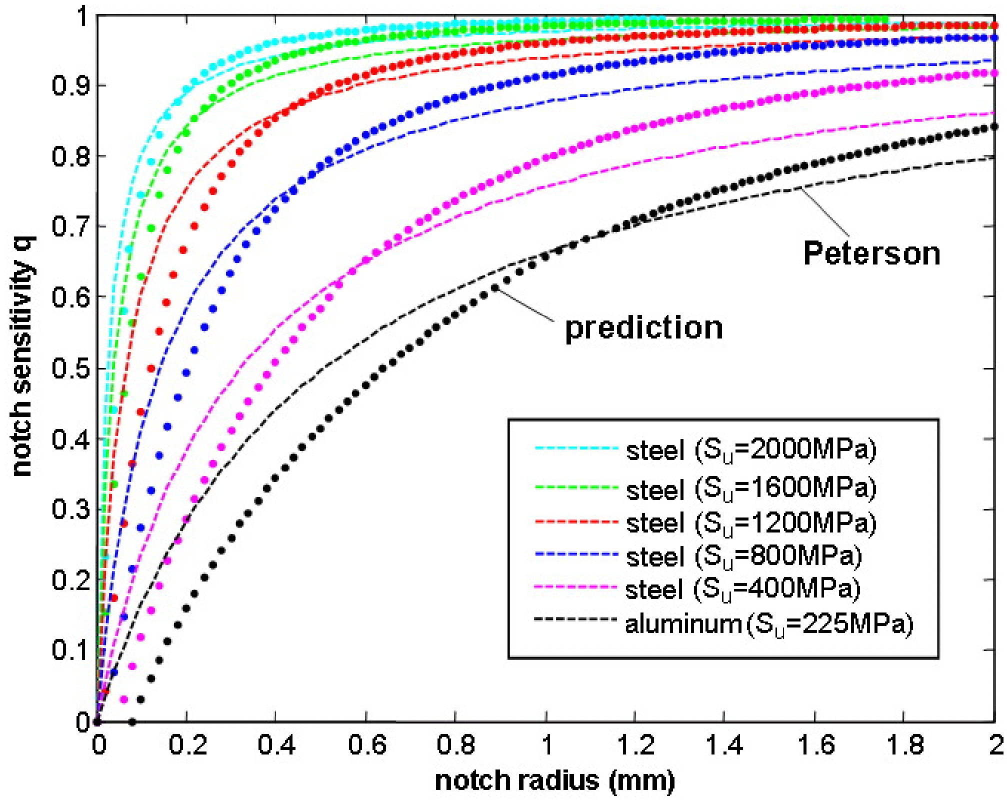

Figure 4. Predicted and experimentally fitted notch sensitivity factors as a function of notch radius for several materials.

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