FATIGUE NOTCH SENSITIVITY OF ELONGATED SLITS

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Abstract. *The fatigue notch sensitivity factor q can be associated with tiny non-propagating cracks at the notch root, thus it can be calculated if their propagation behavior is known. A generalized version of El Haddad-Topper-Smith's equation, used to reproduce the behavior of the Kitagawa-Takahashi plot, is adapted to evaluate the behavior of cracks emanating from circular holes and semi-elliptical notches. The q estimates obtained from this generalization, besides providing a sound physical basis for the notch sensitivity concept, show that the classical textbook q plots are only applicable to semicircular notches, since the elongated semi-elliptical slits can have very different q values. These predictions are supported by experimentally measuring the fatigue crack re-initiation lives after drilling a hole centred at the tip of deep pre-cracks, at a load-ratio R = 0.57, to avoid crack closure effects.*

Keywords*: Notch sensitivity, short cracks, fatigue life prediction, non-propagating cracks*

1. INTRODUCTION

The notch sensitivity factor q quantifies the difference between K_t, the notch linear elastic stress concentration factor (SCF), and its actual effect on the fatigue limit, $K_f = 1 + q(K_t - 1) = \Delta S_0/\Delta S_f$, where K_f is the so-called fatigue stress concentration factor, and ΔS_0 and ΔS_f are the fatigue limits of smooth and of notched SN specimens, respectively (Peterson, 1974). Small non-propagating fatigue cracks that are found at the notch roots when $\Delta S_0/K_t < \Delta\sigma_n < \Delta S_0/K_f$, where $\Delta\sigma_n$ is the nominal stress range applied in the notched piece can explain why $K_f \leq K_t$ (Frost et al, 1999). Thus, q values should be analytically predictable from the propagation behavior of short cracks emanating from the notches.

Several expressions have been proposed to model the influence of the size a of very small fatigue cracks on their propagation threshold value, $\Delta K_{th}(a)$, based on length parameters such as El Haddad-Topper-Smith's a_0 (1979) estimated from ΔS_0 and ΔK_0, the crack propagation threshold of long cracks, $\Delta K_{th}(a \to \infty)$, to reproduce the Kitagawa-Takahashi (1976) plot trend by a modified stress intensity factor (SIF) range, which for the Irwin plate is given by

$$\Delta K_I = \Delta\sigma\sqrt{\pi(a+a_0)}\ \text{, where } a_0 = (1/\pi)(\Delta K_0/\Delta S_0)^2 \tag{1}$$

See Lawson *et al* (1999) for a good review of near-threshold fatigue crack growth (FCG). Yu *et al* (1988) and Atzori *et al* (2003) used a geometry factor α to generalize the above equation to other geometries, resulting in

$$\Delta K_I = \alpha \cdot \Delta\sigma\sqrt{\pi(a+a_0)}\ ,\quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/(\alpha \cdot \Delta S_0)\right]^2 \tag{2}$$

However, for very small cracks with $a << a_0$ this expression implies that $\Delta\sigma$ tends to the fatigue limit ΔS_0, which is only true when $\Delta\sigma$ is the notch root stress range, instead of the nominal stress. But in most cases the geometry factor α found in SIF tables already includes the effects of the notch root SCF, defining $\Delta\sigma \equiv \Delta\sigma_n$ as the nominal stress. Hence, a clearer way to define the length parameter a_0 when the crack departs from a notch is by considering $\Delta\sigma$ as the nominal stress range and two factors: $\varphi(a)$, which tends to the notch root SCF as the crack length a tends to zero, and η, which only encompasses the remaining terms, such as the free surface correction:

$$\Delta K_I = \eta \cdot \varphi(a) \cdot \Delta\sigma\sqrt{\pi(a+a_0)}\ ,\quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/(\eta \cdot \Delta S_0)\right]^2 \tag{3}$$

Note that $\varphi(a)$ does not appear in the expression of a_0, because for very small cracks ($a \to 0$) the notch root stress range $\varphi(0)\cdot\Delta\sigma$ is equal to ΔS_0. Alternatively, the stress intensity range ΔK_I can retain its original equation, while the threshold expression is modified by a function of the crack length a, namely $\Delta K_{th}(a)$, resulting in

$$\Delta K_{th}(a)/\Delta K_0 = \sqrt{a/(a+a_0)} \tag{4}$$

In the following section, a generalization of El Haddad-Topper-Smith's equation is used to better fit the data on the crack size dependence of $\Delta K_{th}(a)$. This expression is then applied to cracks emanating from circular holes and semi-elliptical notches, resulting in improved estimates of the notch sensitivity q and the largest non-propagating crack size.

2. PROPAGATION OF SHORT CRACKS AND ITS INFLUENCE ON THE NOTCH SENSITIVITY

The El Haddad-Topper-Smith's equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant's (1997) reasoning, a more general equation can be used, involving a fitting parameter γ, which can be written as

$$\Delta K_{th}(a)/\Delta K_0 = \left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{5}$$

Equations (1-4) are obtained from (5) when $\gamma=2.0$, and the bi-linear estimate $\Delta\sigma(a \le a_0) = \Delta S_0$ for short cracks and $\Delta Kth(a \ge a_0) = \Delta K_0$ for long ones is obtained if $\gamma \to \infty$. The fitting parameter γ allows the $\Delta K_{th}(a)$ estimates to better correlate with experimental short crack propagation data from Tanaka et al (1981) and Livieri and Tovo (2004), see Fig. 1. Most of the data in this figure can be bounded by two curves obtained using $\gamma = 1.5$ and $\gamma = 8.0$ in Eq. (5).

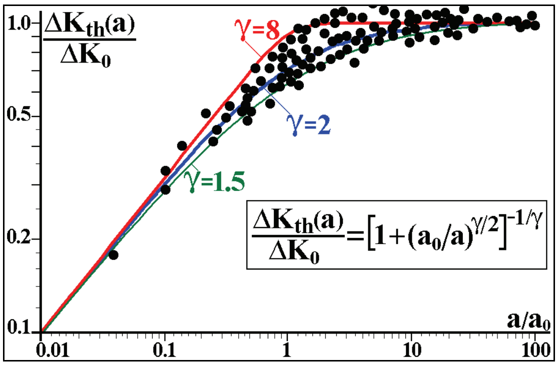

Figure 1: Ratio between short and long crack propagation thresholds as a function of a/a_0.

Equation (5) is now used to evaluate the FCG behavior of short cracks emanating from circular holes in Kirsh plates. The SIF of a single crack with length a emanating from a circular hole with radius ρ is expressed, within 1%, by

$$\Delta K_I = 1.12 \cdot \varphi(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} \tag{6}$$

where the factor $\varphi(a/\rho)$, related to the hole stress concentration, is given by (Tada et al, 1985)

$$\varphi\left(\frac{a}{\rho}\right) \equiv \varphi(x) = \left(1+\frac{0.2}{(1+x)}+\frac{0.3}{(1+x)^6}\right)\cdot\left(2-2.354\frac{x}{1+x}+1.2056\left(\frac{x}{1+x}\right)^2-0.2211\left(\frac{x}{1+x}\right)^3\right),\ x \equiv \frac{a}{\rho} \tag{7}$$

Note that when the crack size a tends to zero, Eq. (10) becomes

$$\lim_{a\to 0} \Delta K_I = 1.12 \cdot 3 \cdot \Delta\sigma\sqrt{\pi a} \tag{8}$$

as expected, since the above equation combines the solution for an edge crack in a semi-infinite plate with the SCF of a circular hole, $K_t = 3$ ($\therefore \varphi(0) = 3$). Note also that the other limit, when a tends to infinity, results in

$$\lim_{a\to\infty} \Delta K_I = \Delta\sigma\sqrt{\pi a/2} \tag{9}$$

which is the SIF for a crack with length a in an infinite plate, where the crack tip is so far from the hole it does not suffer its influence in the stress field (in fact, the equivalent crack length would be $a + 2\rho$, however as $a \to \infty$ the ρ value disappears from the equation). Thus, the SIF of a crack which departs from a (circular) Kirsh hole has $\varphi(x = 0) = 3$ and $\varphi(x \to \infty) = 1/1.12\sqrt{2} \cong 0.63$, thus the crack will propagate when

$$\Delta K_I = \eta \cdot \varphi(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} > \Delta K_{th} = \Delta K_0 \cdot \left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{10}$$

where $\eta = 1.12$ is the free surface correction. Knowing that $\Delta K_{th} \equiv \Delta K_0$ for a long crack, the crack length parameter a_0 from the above equation is

$$a_0 = (1/\pi)\left[\Delta K_0/(1.12 \cdot \Delta S_0)\right]^2 \tag{11}$$

Note that, as discussed before, the factor $\varphi(a/\rho)$ does not appear in the definition of a_0. The crack propagation criterion can be based on dimensionless functions $\varphi(a/\rho)$ and $g(a/\rho, \Delta S_0/\Delta\sigma, \Delta K_0/\Delta S_0\sqrt{\rho}, \gamma)$ and written as

$$\varphi(a/\rho) > \frac{\left(\Delta K_0/\Delta S_0\sqrt{\rho}\right)\cdot\left(\Delta S_0/\Delta\sigma\right)}{\left[\left(\eta\sqrt{\pi a/\rho}\right)^{\gamma}+\left(\Delta K_0/\Delta S_0\sqrt{\rho}\right)^{\gamma}\right]^{1/\gamma}} \equiv g\left(a/\rho, \Delta S_0/\Delta\sigma, \Delta K_0/\Delta S_0\sqrt{\rho}, \gamma\right) \tag{12}$$

If $x \equiv a/\rho$ and $\kappa \equiv \Delta K_0/\Delta S_0\sqrt{\rho}$, then the crack grows whenever $\varphi(x) > g(x, \Delta S_0/\Delta\sigma, \kappa, \gamma)$. Figure 2 plots φ and g, assuming a material/notch combination with $\kappa = 1.5$ and $\gamma = 6$, as a function of the normalized crack length x. For a high applied $\Delta\sigma$, the ratio $\Delta S_0/\Delta\sigma$ becomes small, and the function g is always below φ, meaning that a crack of any length will propagate. The lower curve in Fig. 2 shows the function g obtained from a ratio $\Delta S_0/\Delta\sigma = 1.4$, never crossing φ. On the other hand, for a $\Delta\sigma$ small enough such that $\Delta S_0/\Delta\sigma \geq K_t = 3$, g is always above φ and no crack will initiate nor propagate, as shown by the top curve in the figure. But three other cases must be noted.

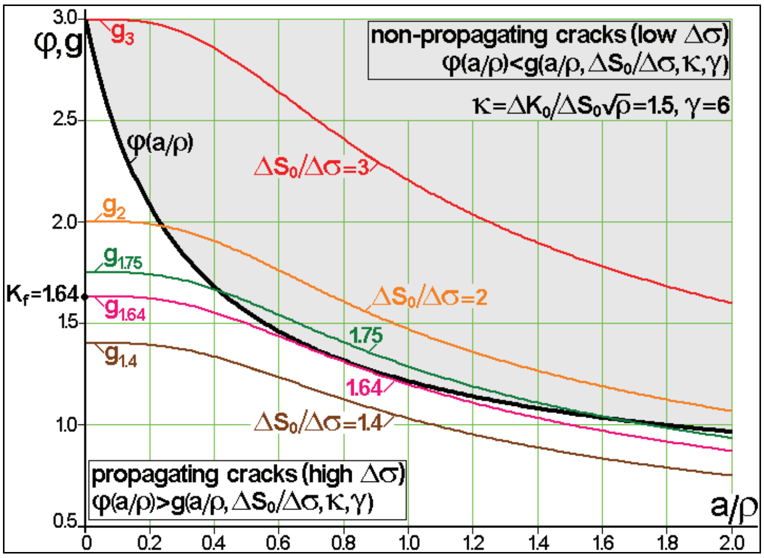

Figure 2: The fatigue stress concentration factor K_f is obtained by finding the load functions $g(a/\rho, \Delta S_0/\Delta\sigma, k, \gamma)$ which is tangent to the resistance curve $\varphi(a/\rho)$.

The first one, illustrated by the g curve with $\Delta S_0/\Delta\sigma = 2$ in Fig. 2, has only one intersection point with φ. This means that such stress levels cause a crack to initiate at the notch, however it will only propagate until a size $a = x\cdot\rho$ obtained from the x value at the intersection point. Therefore, non-propagating cracks will appear at the notch root.

The second case, illustrated by the g curve with $\Delta S_0/\Delta\sigma = 1.75$ in Fig. 2, has two intersection points with φ. Thus, non-propagating cracks will also appear, with maximum sizes obtained from the first intersection point (on the left). Interestingly, cracks longer than the value defined by the second intersection will re-start propagating until fracture. However, crack growth between the two intersections should be caused by a different mechanism, e.g. corrosion or creep.

Finally, in the third case, functions φ and g are tangent, thus meet at a single point (the curve with $\Delta S_0/\Delta\sigma = 1.64$ in Fig. 2). This $\Delta S_0/\Delta\sigma$ value is therefore associated with the smallest stress range $\Delta\sigma$ that can cause crack initiation ***and*** propagation without arrest. Hence, by definition, this specific $\Delta S_0/\Delta\sigma$ ratio is equal to the fatigue SCF K_f. In other words, to obtain K_f it is necessary to make the functions φ and g tangent at the point with $x = x_{max}$, which is the largest non-propagating flaw that can arise from fatigue alone. So, given γ and κ from the material and the notch, x_{max} and K_f can be found from the system of equations:

$$\begin{cases} \varphi(x_{max}) = g\left(x_{max}, K_f, \kappa, \gamma\right) \\ \partial\varphi(x_{max})/\partial x = \partial g\left(x_{max}, K_f, \kappa, \gamma\right)/\partial x \end{cases} \tag{13}$$

This system can be solved numerically for each combination of κ and γ values, and the notch sensitivity factor q is then obtained from

$$q(\kappa,\gamma) \equiv \left(K_f(\kappa,\gamma) - 1\right)/\left(K_t - 1\right) \tag{14}$$

This approach has three main advantages. First, it considers the material-dependent data fit parameter γ, which has a significant influence on the calculations. Second, it is an exact procedure, not an approximation such as the ones based on the limit case inequalities. And third, it can be easily extended to semi-elliptical notches, which can be evaluated in the same way. The SIF range of a single crack with length a emanating from a semi-elliptical notch with semi-axes b and c (where b is in the same direction as a) at the edge of a very large plate can be written as

$$\Delta K_I = \eta \cdot F(a/b, c/b) \cdot \Delta\sigma\sqrt{\pi a} \tag{15}$$

where $\eta = 1.12$ is the free surface correction, and $F(a/b, c/b)$ is a geometry factor associated with the notch stress concentration, which can be expressed as a function of the dimensionless parameter $s = a/(b + a)$ and of the notch SCF

$$K_t = \left[1 + 2(b/c)\right] \cdot \left[1 + 0.12 \big/ (1 + c/b)^{2.5}\right] \tag{16}$$

To obtain expressions for F, Finite Element (FE) calculations were performed using the Quebra2D program (Meggiolaro et al, 2007) considering several cracked semi-elliptical notch configurations. The numerical results, which agreed well with standard solutions (Tada et al, 1985), were fitted within 3% using empirical equations, resulting in

$$F(a/b, c/b) \equiv f(K_t, s) = K_t \cdot \sqrt{\left[1 - exp\left(-K_t^2 \cdot s\right)\right] \big/ \left(K_t^2 \cdot s\right)} \quad \text{for } c \le b \tag{17}$$

$$F(a/b, c/b) \equiv f'(K_t, s) = K_t \cdot \sqrt{\left[1 - exp\left(-K_t^2 \cdot s\right)\right] \big/ \left(K_t^2 \cdot s\right)} \cdot \left[1 - exp\left(-K_t^2\right)\right]^{-s/2} \quad \text{for } c \ge b \tag{18}$$

Figure 3 compares equation (17) with the FE calculations. Similar results are found for equation (18).

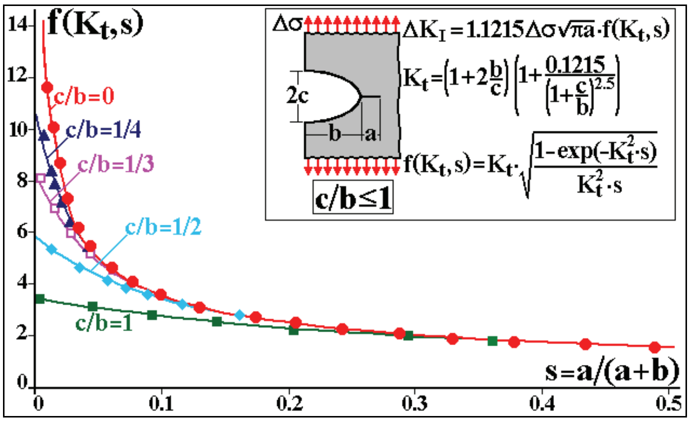

Figure 3: Finite Element calculations and proposed fit for the geometry factor of semi-elliptical notches with $c \le b$.

3. NOTCH SENSITIVITY FOR SEMI-ELLIPTICAL NOTCHES

Traditional notch sensitivity estimates suppose that the q depends only on the notch root ρ and on the material ultimate strength S_U. But the sensitivity of semi-elliptical notches, besides depending on ρ, ΔS_0, ΔK_0 and γ, is also strongly dependent on the c/b ratio, see Fig. 4. As there are reasonable relationships between ΔS_0 and the ultimate tensile strength S_U, but not between ΔK_0 and S_U, two (e.g.) steels with same S_U but different ΔK_0 can have different notch sensitivities q. The curves in Fig. 4 are calculated for typical Al alloys with mean $S_U = 225MPa$, fatigue limit $S_L = 90MPa$ $\Rightarrow \Delta S_0 = 2S_LS_R/(S_L + S_R) = 129MPa$, propagation threshold $\Delta K_0 = 2.9MPa\sqrt{m}$, $\gamma = 6$, and short crack length parameters $a_0 = 0.26$mm Their corresponding Peterson's curve is well approximated by the semi-circular $c/b = 1$ notch, which is not applicable for high c/b ratios, indicating that these old estimates should ***not*** be used for elongated notches.

Fatigue tests carried out to find the number of cycles required to re-initiate the crack after drilling a stop-hole of radius ρ centred at the tip of cracks on modified SE(T) specimens of thickness $B = 8$ mm and width $W = 80$ mm, generating an elongated slit with b = 27.5mm, confirms these q predictions. The tested material was an Al alloy 6082 T6, with yielding strength $S_Y = 280MPa$, ultimate tensile strength $S_U = 327MPa$ and Young's modulus $E = 68GPa$. The fatigue tests have been made at 30Hz under constant load range ΔP at $R = P_{min}/P_{max} = 0.57$, to avoid crack closure interference on the FCG behavior. The fatigue crack re-initiation lives at the notch root can be modeled by εN procedures, using (i) the Coffin-Manson parameters σ'_f, b, ε'_f and c, $\sigma'_f = 485$ MPa, $b = -0.0695$, $\varepsilon'_f = 0.733$, $c = -0.827$, and the Ramberg-Osgood coefficient and exponent of the cyclic stress-strain curve, $H' = 443$ MPa, $h' = 0.064$; (ii) the nominal stress; and (iii) the stress concentration factor K_t of the notches generated by repairing the cracks by a stop-hole at their tips, which can be estimated by Inglis, giving for hole radius $\rho = 1 \Rightarrow K_t \cong 1 + 2\sqrt{(a/\rho)} = 11.49$, see Wu et al (2009) for details.

The elongated slit can be modeled by first calculating the stresses and strains maxima and ranges at its root according to a proper stress/strain concentration rule, and using them to calculate the crack re-initiation lives by a proper $\Delta\varepsilon \times N$ rule, considering the influence of the mean load. Neglecting this effect could lead to severely non-conservative predictions, as the *R*-ratio used in the tests was high (and indeed the Coffin-Manson predictions are highly non-conservative, thus absolutely useless in this case). Figure 5 shows that the lives predicted by Morrow El, Morrow EP and SWT are similar in this case (but it must be emphasized that such a similarity cannot be assumed beforehand, since in many other cases these rules can predict very different fatigue lives!), but too conservative in comparison to the measured data.

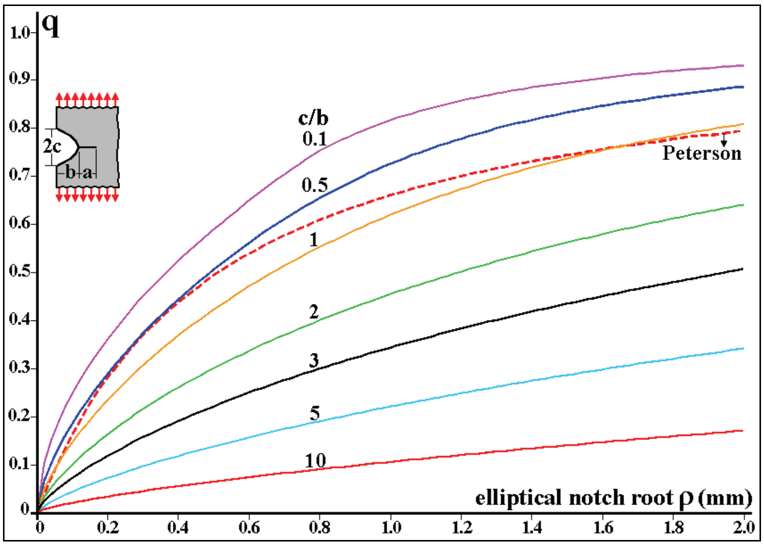

Figure 4: Notch sensitivity q as a function of the semi-elliptical notch root radius $\rho = c^2/b$ for aluminium alloys having a_0 = 0.26mm ($S_u \cong$ 225MPa).

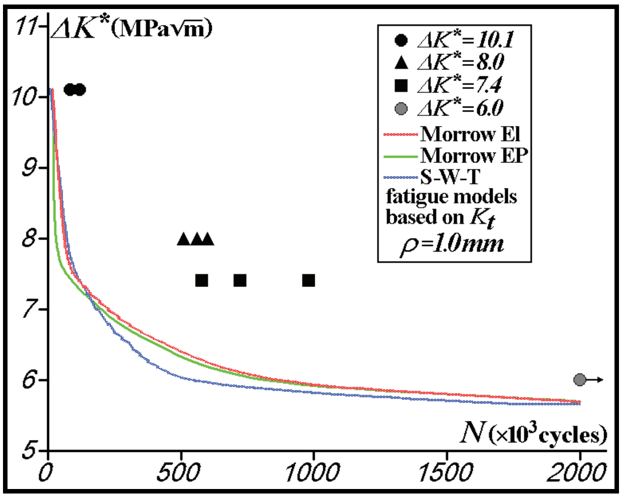

Figure 5: Predicted and measured crack re-initiation lives at the stop-holes roots of radius $\rho = 1.0mm$, using the K_t of the resulting elongated slit, modelled by the semi-elliptical notch which has the same length and radius.

However, when using K_f instead of K_t on the εN rules, calculating the elongated notch sensitivity q by the procedures discussed above, the predictions reproduce quite well the measured results, see Figure 6. The Al 6082 T6 fatigue limit and fatigue crack propagation threshold under pulsating loads required to calculate K_f are estimated as $\Delta K_0 = 4.8\ MPa\sqrt{m}$ and $\Delta S_0 = 110MPa$, following traditional structural design practices, and the Bazant's exponent was chosen, as recommended by Meggiolaro et al (2007), as $\gamma = 6$.

4. CONCLUSIONS

A generalized El Haddad-Topper-Smith's parameter was used to model the crack size dependence of the threshold stress intensity range for short cracks and the behavior of non-propagating fatigue cracks. This dependence has been used to estimate the notch sensitivity factor q cracks that depart form circular holes and from semi-elliptical notches. The predicted notch sensitivities reproduce well the classical Peterson's q estimates for circular holes or approximately semi-circular notches, but it is found that the notch sensitivity of elongated slits has a very strong dependence on the notch aspect ratio, defined by the ratio c/b of the semi-elliptical notch that encloses the slit and has the same tip radius.

these predictions are confirmed by experimental measurements of the re-initiation life of repaired fatigue cracks, using their calculated K_f and proper εN procedures.

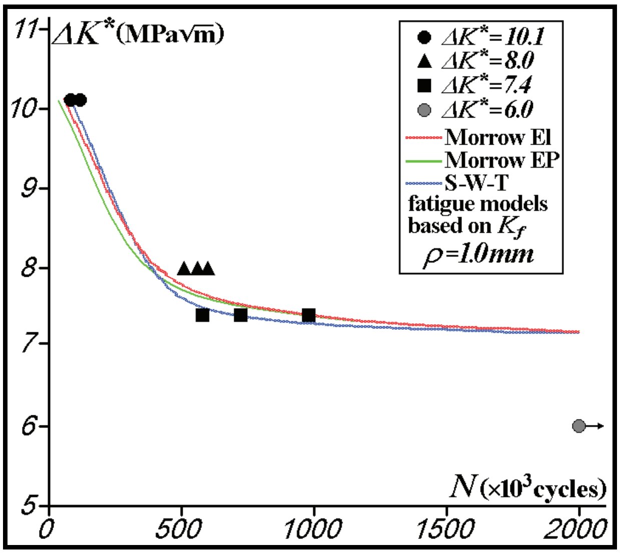

Figure 6: Predicted and measured crack re-initiation lives at the stop-holes roots of radius $\rho = 1.0mm$, using the K_f (instead of K_t) of the repaired crack.

5. ACKNOWLEDGEMENTS

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