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EFFECT OF SHORT CRACKS ON THE FATIGUE LIMIT OF STRUCTURAL ALLOYS

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SYNOPSIS

Most structural components are designed against fatigue crack initiation, using εN or SN procedures which do not recognize any cracks, short or long. Hence, their "infinite life" predictions may become unreliable when such cracks are introduced by any means (say by an accident during manufacturing or operation) and not quickly detected and properly removed. Large cracks may be easily detected and dealt with, but small cracks do pass unnoticed even in careful inspections, if they are smaller than the detection threshold of the inspection method used to identify them. Thus, structural components designed for very long fatigue lives should be designed to avoid fatigue crack initiation AND to be tolerant to undetectable short cracks. However, this self-evident requirement is still not usually included in fatigue design routines, as most long-life designs just intend to maintain the stress range at the structural component critical point below its fatigue limit at the working R-ratio, $S_L(R)$, where $R = \sigma_{min}/\sigma_{max}$, guaranteeing that $\Delta\sigma < S_L(R)/\phi_F$, where ϕ_F is a suitable fatigue safety factor. Such calculations can, of course, become quite involved when designing e.g. against fatigue damage caused by random non-proportional multiaxial loads, but their philosophy remains the same. Nevertheless, most long-life designs work just fine, which means that they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by SN or εN procedures alone. This important problem can only be solved by adding a proper fatigue crack propagation threshold requirement to the "infinite" life design criterion. But the tolerance to non-propagating cracks must include appropriate short crack corrections to be reliable. This paper evaluates the tolerance to short 1D and 2D cracks, and proposes a design criterion for infinite fatigue life which explicitly considers them.

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1. INTRODUCTION

The notch sensitivity factor $0 \leq q \leq 1$ is widely used in structural design to quantify the difference between K_t, the linear elastic stress concentration factor (SCF), and K_f, its corresponding fatigue SCF, which quantifies the actual notch effect on the fatigue strength of structural components [1]. The SCF K_t is equal to σ_{max}/σ_n, where σ_{max} is the maximum (linear elastic) stress at the notch root caused by σ_n, and σ_n is the nominal stress that would act at that point if the notch did not affect the stress field around it. The fatigue SCF is usually defined by

$$K_f = 1 + q \cdot (K_t - 1) = S_L / S_{L_{ntc}} \tag{1}$$

where S_L and S_{Lntc} are the material fatigue limits (or else their fatigue strengths at a convenient very long life) measured on standard (smooth and polished) and on notched test specimens, respectively. But, as the fatigue process depends on two parameters, equation (1) can be generalized considering that K_f may depend e.g. on $R = \sigma_{min}/\sigma_{max}$, e.g. $K_f(R) = S_L(R)/S_{Lntc}(R)$.

It is well known that q can be associated with the relatively fast generation of tiny non-propagating fatigue cracks at notch roots, see Fig. 1. Indeed, according to Frost [2], early experimental evidence that small non-propagating fatigue cracks are found at notch roots when $S_L/K_t < \sigma_n < S_L/K_f$ goes back as far as 1949. Hence, it is certainly reasonable to expect that such tiny cracks can be used to quantitatively explain why $K_f \leq K_t$. Indeed, the notch sensitivity can be predicted from the fatigue behavior of short cracks emanating from notch tips, using relatively simple but sound mechanical principles, which do not require heuristic arguments, or arbitrary fitting parameters [3].

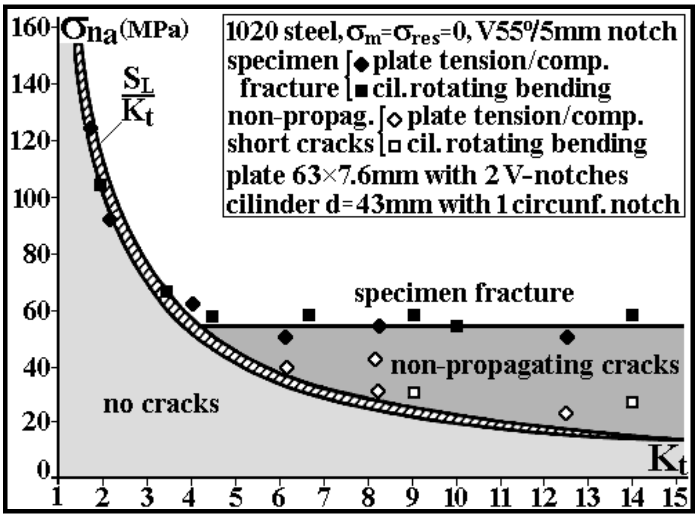

Fig. 1: Classical data showing that non-propagating fatigue cracks are generated at the notch roots if $S_L/K_t < \sigma_n < S_L/K_f$ [2].

The stress field gradients around notch roots controls the fatigue crack propagation (FCP) behavior of short cracks emanating from them. For any given material, q depends not only on the notch tip radius ρ, but also on its depth b, meaning that shallow and elongated notches of same radius ρ may have quite different sensitivities q. Note that "short crack" here means "mechanical" not "microstructural" short crack, since material isotropy is assumed in their modeling, a simplified hypothesis that has been experimentally corroborated [4].

The short cracks FCP threshold must be smaller than the long crack threshold $\Delta K_{th}(R)$, otherwise the stress range $\Delta\sigma$ required to propagate them would be higher than the material fatigue limit $\Delta S_L(R)$. Indeed, assuming that the FCP process is primarily controlled by the stress intensity factor (SIF) range, $\Delta K \propto \Delta\sigma\sqrt{(\pi a)}$, if short cracks with $a \to 0$ had the same threshold $\Delta K_{th}(R)$ of long cracks, then their propagation by fatigue would require $\Delta\sigma \to \infty$, a physical non-sense [5]. The FCP threshold of short fatigue cracks under pulsating loads $\Delta K_{th}(a, R = 0)$ can be modeled using El Haddad-Topper-Smith [6] or ETS characteristic size a_0, which can be estimated from $\Delta S_0 = \Delta S_L(R = 0)$ and $\Delta K_0 = \Delta K_{th}(R = 0)$. This clever trick reproduces the Kitagawa-Takahashi [7] plot trend, using a modified SIF range $\Delta K'$ to describe the fatigue propagation of any crack, short or long,

$$\Delta K' = \Delta\sigma\sqrt{\pi(a + a_0)}\text{, where } a_0 = (1/\pi)(\Delta K_0/\Delta S_0)^2 \qquad (2)$$

Using this a_0 trick, it is indeed possible to reproduce the expected limits $\Delta K_{th}(a \to \infty) = \Delta K_0$ and $\Delta\sigma(a \to 0) = \Delta S_0$, see Fig. 2. Knowing that steels typically have $6 < \Delta K_0 < 12MPa\sqrt{m}$, ultimate tensile strength $400 < S_U < 2000MPa$, and fatigue limit $200 < S_L < 1000MPa$ (since very clean high-strength steels tend to maintain the $S_L/S_U \cong 0.5$ trend of lower strength steels under fully alternated loads, with $R = -1$); and estimating by Goodman the pulsating ($R = 0$) fatigue limit as $\Delta S_0 = 2S_U S_L/(S_U + S_L) \Rightarrow 260 < \Delta S_0 < 1300MPa$; it can then be expected that the maximum range of the ETS short crack characteristic size a_0 for steels should be contained in the range

$$(1/\pi)(\Delta K_{0_{min}}/\Delta S_{0_{max}})^2 \cong 7 < a_0 < 700\mu m \cong (1/\pi)(\Delta K_{0_{max}}/\Delta S_{0_{min}})^2 \qquad (3)$$

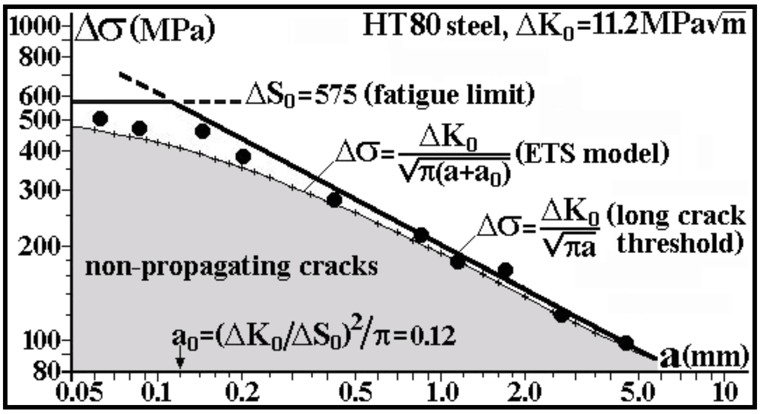

Fig. 2: Kitagawa-Takahashi plot describing the fatigue propagation of short and long cracks under $R = 0$ in a HT80 steel with $\Delta K_0 = 11.2MPa\sqrt{m}$ and $\Delta S_0 = 575MPa$.

Note in Fig. 2 that long cracks with $a >> a_0$ stop when $\Delta\sigma \le \Delta K_0/\sqrt{(\pi a)}$, that very short cracks with $a << a_0$ stop when $\Delta\sigma \le \Delta S_0$, and that the ETS curve predicts that any crack stop when $\Delta\sigma \le \Delta K_0/\sqrt{\pi}(a + a_0)$. Note also that this a_0 range may be overestimated, since the minimum threshold ΔK_{0min} is not necessarily associated with the maximum fatigue crack initiation limit ΔS_{0max}, neither is ΔK_{0max} always associated with ΔS_{0min}. But it nevertheless justifies the "short crack" denomination used for cracks of a similar small size, and highlights the short crack dependence on the FCP threshold and on the fatigue limit of the material. In other words, it can be expected that cracks up to a few millimeters may still behave as short cracks in some steels, meaning they may have a smaller propagation threshold than that measured with long crack, which have $a >> a_0$. The strength ranges of typical Al alloys are $70 < S_U < 600MPa$, $30 < S_L < 230MPa$, $40 < \Delta S_0 < 330MPa$, and $1.2 < \Delta K_0 < 5MPa\sqrt{m}$, thus their maximum a_0 (over)estimated range, and hence their short crack influence scale, is wider than the steels range, $\sim 1\mu m < a_0 < \sim 5mm$.

As ETS $\Delta K'$ has been deduced using Griffith's plate SIF, $\Delta K = \Delta\sigma\sqrt{(\pi a)}$, Yu *et al* [8] used the non-dimensional geometry factor $g(a/w)$ from the SIF expression $\Delta K = \Delta\sigma\sqrt{(\pi a)}\cdot g(a/w)$ to deal with other geometries, re-defining

$$\Delta K' = g(a/w)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\,,\quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/\left(g(a/w)\cdot\Delta S_0\right)\right]^2 \tag{4}$$

Note that the tolerable stress range $\Delta\sigma$ under pulsating loads tends to the fatigue limit ΔS_0 when $a \to 0$ only if $\Delta\sigma$ is the stress range at the notch root, instead of the nominal range. But the geometry factors $g(a/w)$ listed in SIF tables usually include the notch SCF, thus use $\Delta\sigma$ instead of $\Delta\sigma_n$ as the nominal stress. Therefore, a clearer way to define a_0 when the short crack departs from a notch root is to explicitly recognize this practice, separating the geometry factor $g(a/w)$ into two parts: $g(a/w) = \eta\cdot\varphi(a)$, where $\varphi(a)$ depends on the stress gradient ahead of the notch tip, which departs from the notch SCF as the crack length $a \to 0$, whereas η encompasses all the remaining terms, such as the free surface correction:

$$\Delta K' = \eta\cdot\varphi(a)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\,,\quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/\left(\eta\cdot\Delta S_0\right)\right]^2 \tag{5}$$

However, for design and analysis applications, the short crack problem can be more clearly modeled by letting the SIF range ΔK retain its original equation, while the FCP threshold expression (under pulsating loads) is modified to become a function of the crack length a, namely $\Delta K_0(a)$, resulting in

$$\Delta K_0(a) = \Delta K_0\cdot\sqrt{a/(a+a_0)} \tag{6}$$

Moreover, the ETS equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant's reasoning [9], a more general equation can be used introducing an adjustable parameter γ to better fit experimental data

$$\Delta K_0(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{7}$$

Equations (2-6) result from (7) if $\gamma = 2.0$. The bi-linear limit, $\Delta\sigma(a \le a_0) = \Delta S_0$ for short cracks, and $\Delta K_0(a \ge a_0) = \Delta K_0$ for long ones, is obtained if $g(a/w) = \eta\cdot\varphi(a) = 1$ and $\gamma \to \infty$. Most short crack FCP data is fitted by $\Delta K_0(a)$ curves with $1.5 \le \gamma \le 8$, but $\gamma = 6$ better reproduces classical Peterson q-plots [1], which are based on fatigue data obtained by testing TS with semi-circular notches, as discussed in [3]. Using (7) as the FCP threshold, then any crack departing from a free smooth or notched surface under pulsating loads should propagate if

$$\Delta K = \eta\cdot\varphi(a/\rho)\cdot\Delta\sigma\sqrt{\pi a} > \Delta K_0(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{8}$$

where $\eta = 1.12$ is the free surface correction□.

As fatigue depends on two driving forces, the stress range $\Delta\sigma$ and its peak σ_{max}, (8) can be extended to consider σ_{max} (indirectly modeled by the R-ratio) influence in short crack behavior. First, the short crack characteristic size should be defined using the FCP threshold for long cracks $\Delta K_R = \Delta K_{th}(a >> a_R, R)$ and the fatigue limit ΔS_R, both measured or properly estimated at the desired R-ratio, then the corresponding short crack FCP threshold should be re-written to consider it:

$$a_R = (1/\pi)\left[\Delta K_R/(1.12 \cdot \Delta S_R)\right]^2 \tag{9}$$

$$\Delta K_R(a) = \Delta K_R \cdot \left[1 + (a_R/a)^{\gamma/2}\right]^{-1/\gamma} \tag{10}$$

All these details are important when such models are used to make predictions in real life situations, as they do influence the calculation results. In particular, neglecting the σ_{max} effect on fatigue can lead to severe non-conservative life estimations, a potentially dangerous practice unacceptable for design or analysis purposes.

2. FCP BEHAVIOR OF SHORT CRACKS WHICH DEPART FROM ELONGATED NOTCHES

Before jumping into more elaborated mechanics, it is well worth to justify using simpler mathematics why small cracks starting from sharp notch roots can propagate for a while before stopping and becoming non-propagating under fixed loading conditions. This fact may appear to be a paradox, since cracks are sharper than notches, thus it is not unreasonable to think that if a given fatigue load can start a crack from a notch, then it should be able to continue to propagate it. But life is more interesting than that.

Indeed, let's start estimating the SIF of a small crack a departing from the elliptical notch tip of an Inglis plate loaded in mode I, with semi-axes $b >> a$ and c, and root radius $\rho = c^2/b$. The $2b$ axis is centered at the x co-ordinate origin, σ_n is the nominal stress (perpendicular to a and b). In this case, $K_I(a) \cong \sigma_n \cdot \sqrt{(\pi a)} \cdot f_1(a, b, c) \cdot f_2(free\ surface)$, where $f_1(a, b, c) \cong \sigma_y(x)/\sigma_n$; $\sigma_y(x)$ is the maxima stress distribution at $(x = b + a, y = 0)$ ahead of the notch tip when there is no crack; and $f_2 = 1.12$. The function $f_1(x = b + a, y = 0)$ is given by Schijve [10]:

$$f_1 = \frac{\sigma_y(x, y=0)}{\sigma_n} = 1 + \frac{(b^2 - 2bc)(x - \sqrt{x^2 - b^2 + c^2})(x^2 - b^2 + c^2) + bc^2(b - c)x}{(b - c)^2(x^2 - b^2 + c^2)\sqrt{x^2 - b^2 + c^2}} \tag{11}$$

The slender the elliptical notch is, meaning the smaller their semi-axes c/b and tip radius to depth ρ/b ratios are, the higher is its SCF. But high K_t imply in steeper stress gradients ahead of the notch tip, $\partial\sigma_y(x, y = 0)/\partial x$. In fact, the linear elastic stress concentration induced by any elliptical hole drops from $K_t = 1 + 2b/c = 1 + 2\sqrt{(b/\rho)} = \sigma_y(1)/\sigma_n \geq 3$ at its tip border to about $1.82 < K_{1.2} = \sigma_y(1.2)/\sigma_n < 2.11$ (for $b \geq c$) at a point just $b/5$ ahead of it. This means that their SCF influence is associated with their depth b, not with their tip radii ρ. Such is the cause for the peculiar growth of short cracks which depart from elongated notch roots. Their SIF, which should tend to increase with their length $a = x - b$, may instead decrease after they grow for a short while because the SCF effect in $K_I \cong 1.12 \cdot \sigma_n \sqrt{(\pi a)} \cdot f_1$ may diminish sharply due the high stress drop close to the notch tip, overcompensating the crack growth effect.

This $K_I(a) \cong 1.12 \cdot \sigma_n \cdot \sqrt{(\pi a)} \cdot f_1$ estimate can be used to evaluate the size of non-propagating fatigue cracks tolerable at Inglis notch roots, using the short crack FCP behavior. A simple example can illustrate this [11]: assuming a large steel plate with $S_U = 600MPa$, $S_L = 200MPa$ and $\Delta K_0 = 9MPa\sqrt{m}$ works under $\Delta\sigma_n = 100MPa$ at $R = -1$, verify if it is possible to change a circular $d = 20mm$ central hole by an elliptical one with axis $2b = 20mm$ (perpendicular to σ_n) and $2c = 2mm$, without inducing the plate to fail by fatigue.

Neglecting the buckling problem (which is important in thin plates), the circular hole has a safety factor against fatigue crack initiation $\phi_F = S_L/K_f \cdot \sigma_n = 200/150 \cong 1.33$, as this large hole has $K_f \cong K_t = 3$. But the sharp elliptical hole would not be admissible by traditional SN design routines, since it has $\rho = c^2/b = 0.1mm$, thus a very high $K_t = 1 + 2b/c = 21$. Its notch sensitivity estimated from usual q plots [1] would be $q \cong 0.32 \Rightarrow K_f = 1 + q \cdot (K_t - 1) = 7.33$, thus it would induce $K_f \cdot \sigma_n = 376MPa > S_L$. However, as this K_f value is sensibly higher than typical values reported in the literature [1-2, 10-14], this problem can be re-studied considering the short crack FCP behavior.

Supposing $\Delta K_{th}(R < 0) \cong \Delta K_0$ as usual, $\Delta K_0(a) = \Delta K_0/[1+(a_0/a)]^{-0.5}$ (by ETS), $S'_L = 0.5S_U$ (the material fatigue limit, as FCP modeling does not need modifying factors required to estimate S_L), $\Delta S_0 = S_U/1.5$ (by Goodman) and $a_0 = (1/\pi)(1.5\Delta K_0/1.12 \cdot S_U)^2 \cong 0.13mm$, the SIF ranges $\Delta K_I(a)$ for the two holes are compared to the FCP threshold $\Delta K_0(a)$ in Fig. 3.

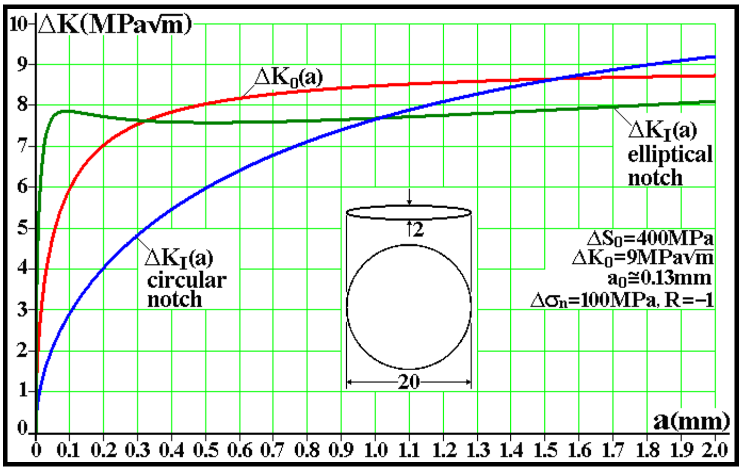

Fig. 3: According to (10), cracks should not initiate at the circular hole border, while the crack which initiates at the elliptical notch tip stops after reaching $a \cong 0.33mm$.

The SIF estimated for cracks departing from the circular notch remains below the $\Delta K_0(a)$ FCP threshold curve (which considers the short crack behavior) up to $a \cong 1.54mm$. Thus, if a small surface scratch locally augments the stress range and initiates a tiny crack at that hole border, it would not propagate under this fixed $\Delta\sigma_n = 100MPa$ and $R = -1$ load, confirming its "safe" prediction made by traditional SN procedures. Only if a crack with $a > 1.54mm$ is introduced at this circular hole border by any other means, it would propagate by fatigue under those otherwise safe loading conditions.

However, under these same loading conditions, the $\Delta K_I(a)$ curve estimated for the elliptical hole starts above $\Delta K_0(a)$, thus a crack should initiate at its border, as expected from its high K_t. But as this tiny crack propagates through the high stress gradient ahead of the notch root, it sees rapidly diminishing stresses around its tip during its early growth, which overcompensate the increasing crack size effect on $\Delta K_I(a)$. This means that the estimated SIF becomes smaller than $\Delta K_0(a)$ at $a \cong 0.33mm$, when the crack stops and becomes non-propagating (if $\Delta\sigma_n$ and R remain fixed), see Fig. 3. As fatigue failures include crack initiation and growth up to fracture, both notches could be considered safe for this service loading. But the non-propagating crack at the elliptical notch tip, a clear evidence of fatigue damage, renders it much less robust than the circular one, as discussed in Castro and Meggiolaro [11].

For more elaborated analysis purposes, the SIF range of a single crack with length a emanating from a semi-elliptical notch with semi-axes b (co-linear to a) and c, located at the edge of a very large plate loaded in mode I, can be written as

$$\Delta K_I = \eta \cdot \Delta\sigma\sqrt{\pi a} \cdot G\left(a/b, c/b\right) \tag{12}$$

where $\eta = 1.12$. The geometric function $G(a/b, c/b)$ can be expressed as a function of the dimensionless parameter $s = a/(b + a)$ and of the notch SCF, given by

$$K_t = \left[1 + 2\left(b/c\right)\right] \cdot \left\{1 + \left[0.12/\left(1 + c/b\right)^{2.5}\right]\right\} \tag{13}$$

To obtain expressions for G, extensive finite element calculations were performed for several cracked semi-elliptical notches. The numerical results, which agreed well with standard solutions [15], were fitted within 3% using empirical equations [3] by

$$G\left(a/b, c/b\right) \equiv f\left(K_t, s\right) = K_t\sqrt{\left[1 - exp\left(-sK_t^2\right)\right]/sK_t^2}\ ,\ c \le b \text{ and } s = a/(b + a) \tag{14}$$

$$G'\left(a/b, c/b\right) \equiv f'\left(K_t, s\right) = K_t\left[1 - exp\left(-K_t^2\right)\right]^{-s/2}\sqrt{\left[1 - exp\left(-sK_t^2\right)\right]/sK_t^2}\ ,\ c \ge b \tag{15}$$

Equation (12) includes the semi-elliptical notch effect in K_I through G or G'. Indeed, as $s \to 0$ when $a \to 0$, the maximum stress at its tip $\sigma_{max} \to G(0, c/b)\cdot\sigma_n = K_t\cdot\sigma_n$. Thus, the η-factor, but not the $G(a/b,c/b)$ part of K_I, should be considered in the short surface crack characteristic size a_0, as done in equation (4). The semi-ellipsis SCF includes a term $[1 + 0.12/(1 + c/b)^{2.5}]$ which can be interpreted as a free surface correction (FSC), since as $c/b \to 0$ and the semi-elliptical notch tends to a crack, its $K_t \to 1.12 \cdot 2\sqrt{(b/\rho)}$. Such 1.12 factor is the notch FSC, not the crack FSC η. Indeed, when $c/b \to 0$, this 1.12 factor disappears from the G expression, which gives $G(a/b, 0) = 1/\sqrt{s}$, and therefore $\Delta K_I = \eta\cdot G\cdot\Delta\sigma\cdot[\pi\cdot a]^{0.5} = \eta\cdot\Delta\sigma\cdot[\pi\cdot(a + b)]^{0.5}$, as expected, since the resulting crack for $c \to 0$ would have length $a + b$.

Traditional Peterson's q-estimates, obtained by fitting questionable semi-empirical equations to few experimental data points, assume q depends only on the notch root ρ and on the alloy ultimate strength S_U. Thus, similar alloys with same S_U but different ΔK_0 should have identical notch sensitivities. The same should occur with shallow and deep elongated notches of identical tip radii. However, whereas well established empirical relations relate the fatigue limit ΔS_0 to the tensile strength S_U of many materials, there are no such relations between their FCP threshold ΔK_0 and S_U. Moreover, it is also important to point out that the q estimation for elongated notches by the traditional procedures can generate unrealistic K_f values, as exemplified above. In conclusion, such traditional estimates should not be taken for granted.

The proposed model, on the other hand, is based on the FCP mechanics of short cracks which depart from elliptical notch roots, recognizing that their q values are associated with their tolerance to non-propagating cracks. It shows that their notch sensitivities, besides depending on ρ, ΔS_0, ΔK_0 and γ, are also strongly dependent on their shape, given by their c/b ratio. Their corresponding Peterson's curve is well approximated by the semi-circular $c/b = 1$ notch, but this curve is ***not*** applicable for much different c/b ratios. Therefore, the proposed predictions indicate that these traditional notch sensitivity estimates should ***not*** be used for elongated notches. Due to space limits, this analysis is limited to this compact discussion here, but it is completely developed in [3], and experimentally verified in [4]. See Castro and Meggiolaro [11] for further details.

3. TOLERABLE SHORT CRACK SIZES

The methodology presented here can be used to generate an unambiguous acceptance criterion for small cracks, a potentially much useful tool for practical applications. Most structural components are designed against fatigue crack initiation, using εN or SN procedures which do not recognize cracks. Hence, their "infinite life" predictions may become unreliable when such cracks are introduced by any means, say by manufacturing or assembling problems, and not quickly detected and properly removed. Large cracks may be easily detected and dealt with, but small cracks may pass unnoticed even in careful inspections. In fact, if they are smaller than the guaranteed detection threshold of the inspection method used to identify them, they simply cannot be detected. Thus, structural components designed for very long fatigue lives should be designed to be tolerant to such short cracks.

However, this self-evident requirement is still not usually included in fatigue design routines, as most long-life designs just intend to maintain the stress range at critical points below their fatigue limits, guaranteeing that $\Delta\sigma < S_R/\phi_F$, where ϕ_F is a suitable safety factor. Nevertheless, most long-life designs work well, which means that they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by SN or εN procedures alone. Such problem can be avoided by adding equations (8-10) to the "infinite" life design criterion to tolerate a (small) crack of size a. Therefore, in its simplest version, this improved criterion should then be written as

$$\Delta\sigma < \Delta K_R \Big/ \left\{\sqrt{\pi a}\cdot g(a/w)\cdot\left[1+(a_R/a)^{\gamma/2}\right]^{1/\gamma}\right\}, \text{ where } a_R = (1/\pi)\cdot[\Delta K_R/\eta\Delta S_R]^2 \quad (16)$$

As fatigue limits ΔS_R consider microstructural defects inherent to the material, equation (16) complements them by considering the component tolerance to short cracks. A simple case study can clarify how useful this concept can be, as discussed next.

Due to a rare manufacturing problem, a batch of an important component was marketed with small surface cracks, causing some unexpected annoying field failures. The task was to estimate how much such cracks affect the stresses those steel components could tolerate under uniaxial fatigue loads, knowing that their rectangular cross section has $2mm$ by $3.4mm$; that their measured fatigue limit under $R = -1$ is $S_L = 246MPa$; and that they have $S_U = 990MPa$. Note that as $S_L \cong S_U/4$, it probably includes surface roughness and/or similar effects which should not affect the cracks. But in the absence of more precise information, the only safe option is to use the measured S_L value to estimate S_R and a_R. Therefore, by Goodman

$$S_R = \left[S_L S_U (1-R) \right] / \left[S_U (1-R) + S_L (1+R) \right] \quad (17)$$

The mode I stress range $\Delta\sigma$ tolerable by this component when it has a uniaxial surface crack of depth a is then given by

$$\Delta\sigma < \frac{\Delta K_R / \varphi_F}{\sqrt{\pi a}\,[\,0.752 + 2.02\frac{a}{w} + 0.37(1 - \sin\frac{\pi a}{2w})^3\,]\sec\frac{\pi a}{2w}\sqrt{\frac{2w}{\pi a}\tan\frac{\pi a}{2w}} \cdot \left[1 + \left(\frac{a_R}{a}\right)^{\gamma/2}\right]^{1/\gamma}} \quad (18)$$

Equation (18) includes a suitable $g(a/w)$ geometry factor for this component (assuming it is a long strip,) obtained from [15]. Fig. 4-6 plot the maximum tolerable stress ranges (assuming $\phi_F = 1$) for several R-ratios, using $w = 3.4mm$ as its width. However, since the FCP threshold ΔK_R was not available, it had to be estimated, a risky but unavoidable procedure, as engineering decisions must be taken even when proper data is missing.

The typical threshold range for steels is $6 < \Delta K_0 < 12MPa\sqrt{m}$. Moreover, it is usual to assume $\Delta K_R \cong \Delta K_0$ for $R < 0$ loads (except if the load history contains severe underloads). Lower limit estimates for $R > 0$ are $\Delta K_{th}(0 < R \leq 0.17) = 6MPa\sqrt{m}$, and $\Delta K_{th}(R > 0.17) = 7 \cdot (1 - 0.85R)$ [11]. Using $\eta = 1.12$ and $\Delta K_0 = 6MPa\sqrt{m}$, the short crack characteristic value is estimated as $a_0 = 59\mu m$. Fig. 4 shows that if this component works for example under $\Delta\sigma = 286MPa$ and $R = -0.12$, it can tolerate cracks up to $a \cong 105\mu m$, whereas if it works under $\Delta\sigma = 176MPa$ and $R = 0.44$, it can sustain cracks up to $a \cong 150\mu m$. Figure 5 present the same curve, but using semi-log coordinates to enhance this component small tolerance to short cracks.

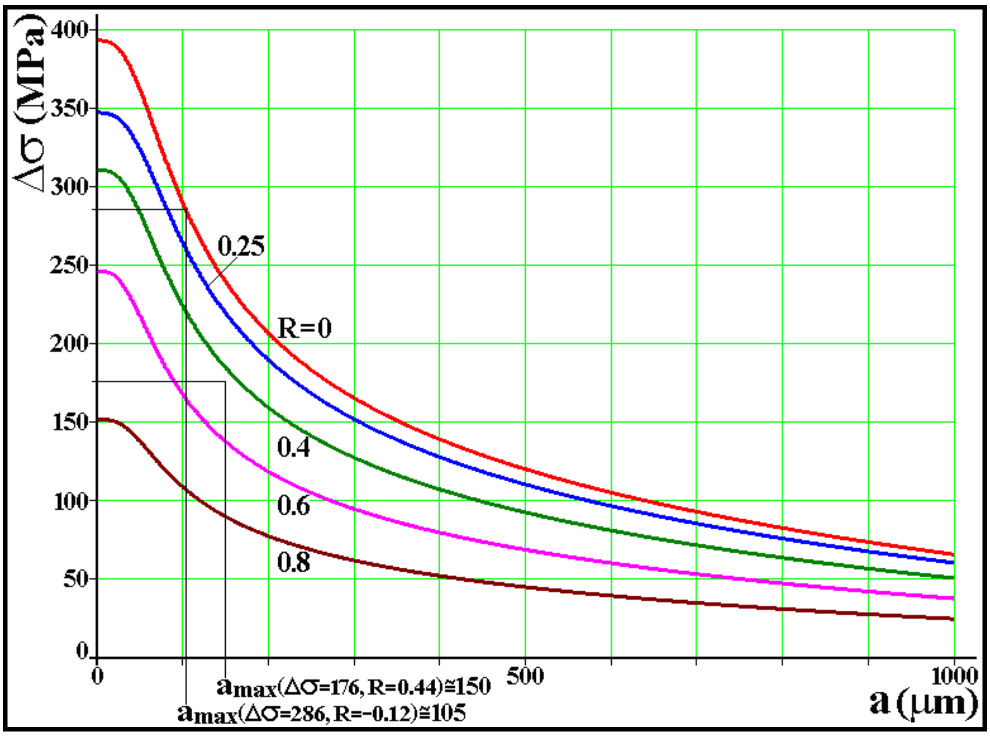

Fig. 4: Surface crack of size a effect in the largest stress range $\Delta\sigma_R(a)$ tolerable by a strip of width $w = 3.4mm$ loaded in mode I, for various R-ratios (supposing $\Delta K_0 = 6MPa\sqrt{m}$ and $\gamma = 6$, thus $a_0 = 59$ and $a_{0.8} = a(R = 0.8) = 55\mu m$).

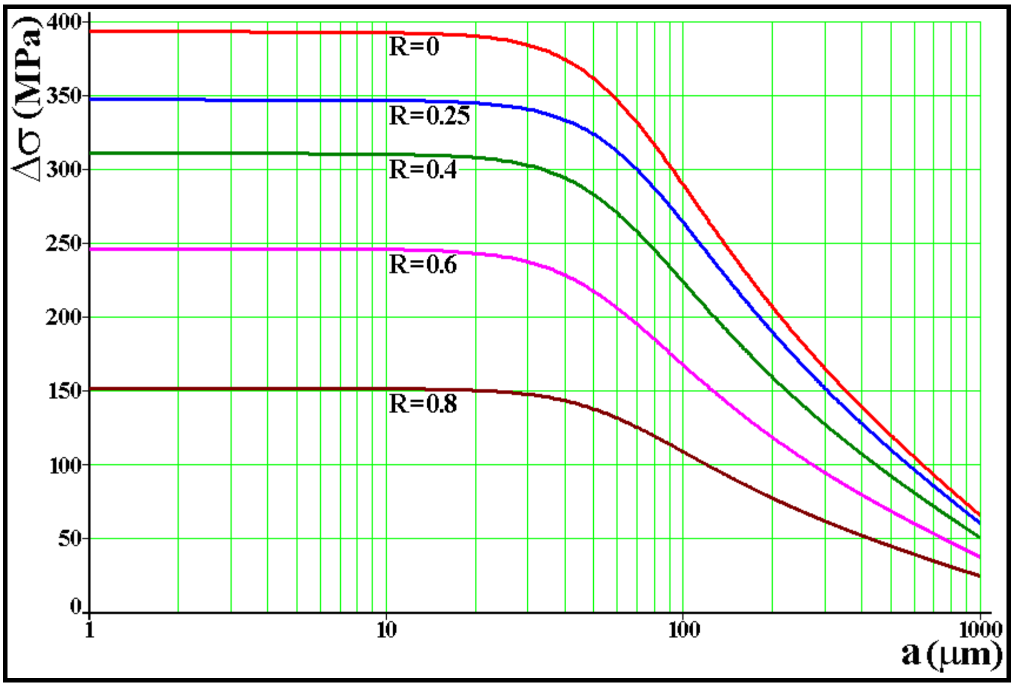

Fig. 5: Similar to Fig. 4, but with semi-log scale to enhance the short crack tolerance. Small cracks with $a < 30\mu m$ have practically no effect in its fatigue resistance.

Therefore, this simple (but sensible) model indicates that this component is not too tolerant to 1D surface cracks. However, as this conclusion is based on estimated properties, it is worth to study its sensibility to the assumed values. Fig. 6 shows the prediction range associated with the typical interval expected for the estimated properties, enhancing how important it is to measure them.

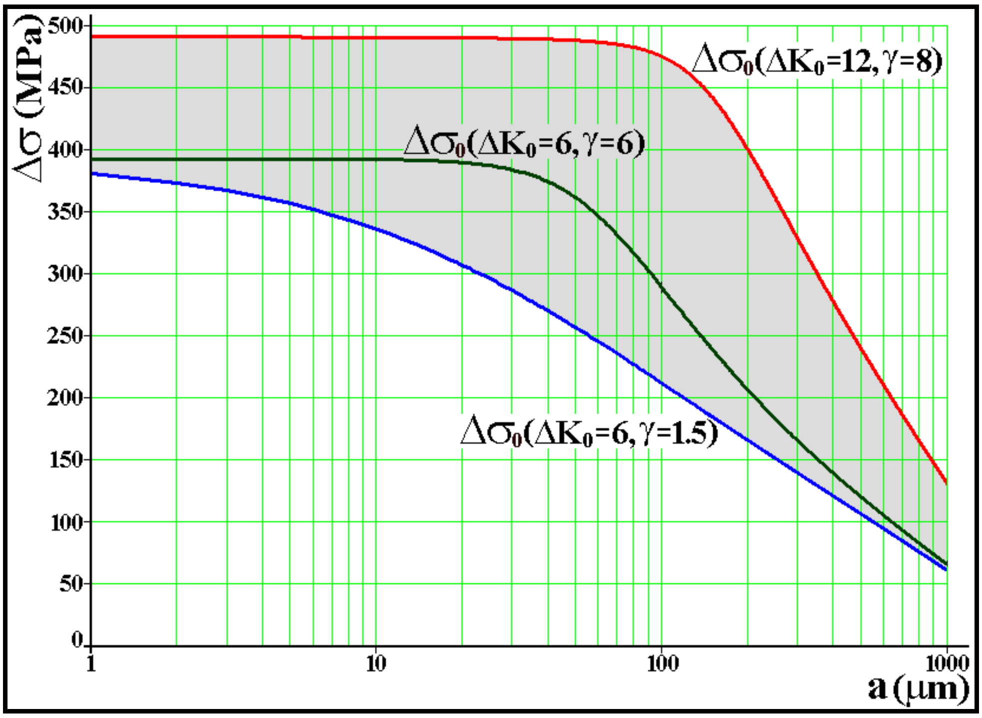

Fig. 6: Typical steel threshold $6 < \Delta K_0 < 12MPa\sqrt{m}$ and γ exponent $1.5 < \gamma < 8$ ranges influence in the largest mode I stress ranges $\Delta\sigma_0$ tolerated by the $w = 3.4mm$ strip, as a function of the 1D superficial crack size a.

Note that equation (16) assumes that the short crack is unidimensional and grows without changing its original plane. Note also that this model only describes the behavior of macroscopically short cracks, as it uses macroscopic material properties. Thus it can only be applied to short cracks which are large in relation to the characteristic size of the intrinsic material anisotropy (e.g. its grain size). Smaller cracks grow inside an anisotropic and usually inhomogeneous scale, thus their FCP is also affected by microstructural barriers, such as second phase particles or grain boundaries. However, as grains cannot be mapped in most practical applications, such problems, in spite of their academic interest, are not really a major problem from the fatigue design point of view.

But this model first limitation which may be much more important for practical applications, as it assumes that the short crack can be completely characterized by its depth *a*. However, most short cracks are surface or corner cracks, which to grow by fatigue in two directions, maintaining their original plane when they are loaded under pure mode I conditions. In these cases, they can be modeled as bidimensional (2D) cracks which grow both in depth and width. In reality, both long and short cracks (these meaning cracks not much larger than a_R) only behave as 1D cracks after having cut all the component width to become a through crack, with a more or less straight front which propagates in an approximately uniform way. Thus, equation (16) must be adapted to consider the influence of 2D short cracks in the fatigue limit. This can be done by assuming that:

(i) the cracks are loaded in pure mode I, under quasi-constant $\Delta\sigma$ and R conditions, with no major overloads;

(ii) material properties measured (or estimated) testing 1D specimens may be used to simulate the FCP behavior of 2D cracks; and

(iii) 2D surface or corner cracks can be well modeled as having an approximately elliptical front, thus their SIF can be described by the classical Newman-Raju [16]. In this case, it can be expected that the component tolerance to cracks be given by:

$$\Delta\sigma < \begin{cases} \Delta K_R \Big/ \left\{ \sqrt{\pi a}\cdot\Phi_a(a,c,w,t)\cdot\left[1+(a_R/a)^{\gamma/2}\right]^{1/\gamma} \right\} \\ \Delta K_R \Big/ \left\{ \sqrt{\pi c}\cdot\Phi_c(a,c,w,t)\cdot\left[1+(a_R/c)^{\gamma/2}\right]^{1/\gamma} \right\} \end{cases} \tag{19}$$

For semi-elliptical or quart-elliptical surface cracks in a plate of thickness *t* loaded in mode I, the SIF in the semi-axis directions, or in the depth *a* and width *c* directions, $K_{I,a} = \sigma\sqrt{(\pi a)}\cdot\Phi_a$ and $K_{I,c} = \sigma\sqrt{(\pi c)}\cdot\Phi_c$, are given by quite complicated functions, which enhance the operational advantage of treating the FCP threshold as a function of the crack size, $\Delta K_{th}(a)$, as claimed above. For structural calculations and design purposes, it is indeed relatively simple to use either equation (16) or (19) to evaluate the influence of surface cracks on the component fatigue strength. Moreover, it is not too difficult to adapt the 2D equations to include notch effects. Φa and Φc expressions are reproduced in Castro and Meggiolaro [11].

4. CONCLUSIONS

A generalized El Haddad-Topper-Smith's parameter was used to model the threshold stress intensity range for short cracks dependence on the crack size, as well as the behavior of non-propagating fatigue cracks. This dependence was used to estimate the notch sensitivity factor *q* of semi-elliptical notches, from studying the propagation behavior of short non-propagating cracks that may initiate from their tips. The predicted notch sensitivities reproduced well the classical Peterson's q estimates for circular holes or approximately semi-circular notches, but it was found that the notch sensitivity of elongated slits has a very strong dependence on the notch aspect ratio, defined by the ratio c/b of the semi-elliptical notch that approximates the slit shape having the same tip radius. These predictions were confirmed by experimental measurements of the re-initiation life of long fatigue cracks repaired by introducing a stop-hole at their tips, using their calculated K_f and appropriate εN procedures. Based on this promising performance, a criterion to evaluate the influence of small or large surface cracks in the fatigue resistance was proposed.

5. ACKNOWLEDGEMENTS

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