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FATIGUE LIMIT SENSIBILITY TO SHORT CRACKS[1]

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Abstract
Most structural components are designed against fatigue crack initiation, by procedures which do not recognize cracks. Large cracks may be easily detected and dealt with, but small cracks may pass unnoticed even in careful inspections, if they are smaller than the detection threshold of the inspection method used to identify them. Thus, structural design for very long fatigue lives should avoid fatigue crack initiation AND be tolerant to undetectable short cracks. But this self-evident requirement is still not used in fatigue design routines, which just intend to maintain the loading at the structural component critical point below its fatigue limit. Nevertheless, most long-life designs work just fine, which means that they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by SN procedures alone. This important problem can only be solved by adding a proper short crack fatigue growth threshold requirement to the "infinite" life design criterion. This paper evaluates the tolerance to short 1D and 2D cracks, and proposes a design criterion for infinite fatigue life which explicitly considers it.
Keywords: Short cracks; Non-propagating cracks; Fatigue life prediction.

SENSIBILIDADE DO LIMITE À FADIGA ÀS TRINCAS CURTAS

Resumo
A maioria dos componentes estruturais é projetada contra a iniciação de trincas por fadiga, usando procedimentos que não reconhecem trincas. Trincas grandes podem ser facilmente detectadas e controladas, mas trincas curtas podem passar despercebidas mesmo em inspeções cuidadosas, se forem menores que o limiar de detecção do método usado para identificá-las. Logo, o projeto à fadiga para vidas muito longas deveria evitar o início de trincas **e** tolerar trincas curtas indetectáveis. Mas este requisito evidente ainda não é usado na maioria das rotinas de projeto, que visam apenas manter a tensão no ponto crítico abaixo do limite à fadiga. Entretanto, a maioria destes projetos funciona bem, logo tolera algumas trincas curtas não identificáveis, ou funcionalmente admissíveis. Mas a pergunta "quão tolerantes" não pode ser respondida apenas por procedimentos SN. Este problema importante só pode ser resolvido adicionando um limiar de propagação de trincas curtas apropriado ao critério de projeto à vida "infinita". Este artigo avalia a tolerância a trincas curtas 1D e 2D, e propõe um critério de vida infinita à fadiga que as considera explicitamente.
Palavras-chave: Trincas curtas; Trincas não-propagantes; Predição de vida à fadiga.

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1 INTRODUCTION

The notch sensitivity $0 \leq q \leq 1$ relates the linear elastic (LE) stress concentration factor (SCF) $K_t = \sigma_{max}/\sigma_n$, to $K_f = 1 + q \cdot (K_t - 1) = S_L(R)/S_{Lntc}(R)$, its corresponding fatigue SCF at $R = \sigma_{min}/\sigma_{max}$, which quantifies the actual notch effect on the fatigue strength of structural components.(1) σ_{max} and σ_{min} are the maximum and minimum LE stress at the notch root caused by σ_n; σ_n is the nominal stress that would act at that point if the notch did not affect the stress field around the notch; S_L and S_{Lntc} are the fatigue limits measured on standard (smooth and polished) and on notched test specimens (TS). It is well known that q can be associated with the relatively fast generation of tiny non-propagating fatigue cracks at notch roots when $S_L/K_t < \sigma_n < S_L/K_f$. The notch sensitivity can be predicted from the fatigue behavior of short cracks emanating from notch tips, using relatively simple but sound mechanical principles, which do not require heuristic arguments, or arbitrary fitting parameters.(2,3)

The stress field gradients around notch roots affect the fatigue crack propagation (FCP) behavior of short cracks emanating from them. For any given material, q depends not only on the notch tip radius ρ, but also on its depth b, meaning that shallow and elongated notches of same ρ may have quite different q. Note that “short crack” here means “mechanical” not “microstructural” short crack, since material isotropy is assumed in their modeling, a simplified hypothesis experimentally corroborated.

Short cracks must behave differently from long cracks, as their FCP threshold must be smaller than the long crack threshold $\Delta K_{th}(R)$, otherwise the stress range $\Delta\sigma$ required to propagate them would be higher than the material fatigue limit $\Delta S_L(R)$. Indeed, assuming that the FCP process is primarily controlled by the stress intensity factor (SIF) range, $\Delta K \propto \Delta\sigma\sqrt{(\pi a)}$, if short cracks with $a \to 0$ had the same $\Delta K_{th}(R)$ threshold of long cracks, their propagation by fatigue would require $\Delta\sigma \to \infty$, a physical non-sense.(4) The FCP threshold of short fatigue cracks under pulsating loads $\Delta K_{th}(a,\ R = 0)$ can be modeled using El Haddad-Topper-Smith (ETS) characteristic size a_0, which is estimated from $\Delta S_0 = \Delta S_L(R = 0)$ and $\Delta K_0 = \Delta K_{th}(R = 0)$.(5) This clever trick reproduces the Kitagawa-Takahashi(6) plot trend, using a modified SIF range $\Delta K'$ to describe the fatigue propagation of any crack, short or long,

$$\Delta K' = \Delta\sigma\sqrt{\pi(a + a_0)}\text{, where } a_0 = (1/\pi)\left(\Delta K_0/\Delta S_0\right)^2 \qquad (1)$$

Steels typically have $6 < \Delta K_0 < 12MPa\sqrt{m}$, tensile strength $400 < S_U < 2000MPa$, and fatigue limit $200 < S_L < 1000MPa$ (very clean high-strength steels tend to maintain the $S_L/S_U \cong 0.5$ trend of lower strength steels under fully alternated loads, $R = -1$). If by Goodman $\Delta S_0 = 2S_U S_L/(S_U + S_L) \Rightarrow 260 < \Delta S_0 < 1300MPa$, *where* $\Delta S_0 = 2S_L$ *is* the pulsating ($R = 0$) fatigue limit; then the estimated short crack characteristic size a_0 range for steels is $(1/\pi)\left(\Delta K_{0min}/\Delta S_{0max}\right)^2 \cong 7 < a_0 < 700\mu m \cong (1/\pi)\left(\Delta K_{0max}/\Delta S_{0min}\right)^2$.

This a_0 range may be overestimated, since the minimum threshold ΔK_{0min} is not necessarily associated with the maximum fatigue crack initiation limit ΔS_{0max}, neither ΔK_{0max} is always associated with ΔS_{0min}. But it nevertheless justifies the “short crack” denomination used for cracks of a similar small size, and highlights the short crack dependence on the FCP threshold and on the fatigue limit of the material. In other words, it can be expected that cracks up to a few millimeters may still behave as short cracks in some steels, meaning they may have a smaller propagation threshold

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than that measured with long crack, which have $a >> a_0$. Since the strengths of typical aluminum alloys are $70 < S_U < 600MPa$, $30 < S_L < 230MPa$, $40 < \Delta S_0 < 330MPa$, and $1.2 < \Delta K_0 < 5MPa\sqrt{m}$, their maximum a_0 (over)estimated range, thus their short crack influence scale, is wider than the steels range, $1\mu m < a_0 < 5mm$.
As ETS $\Delta K'$ has been deduced using Griffith's plate SIF, $\Delta K = \Delta\sigma\sqrt{(\pi a)}$[7] used the non-dimensional geometry factor $g(a/w)$ of the general expression for SIF $\Delta K = \Delta\sigma\sqrt{(\pi a)}\cdot g(a/w)$ to deal with other geometries, re-defining

$$\Delta K' = g(a/w)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\,,\quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/(g(a/w)\cdot\Delta S_0)\right]^2 \tag{2}$$

But the tolerable stress range $\Delta\sigma$ under pulsating loads tends to the fatigue limit ΔS_0 when $a \to 0$ only if $\Delta\sigma$ is the notch root instead of the nominal stress range. However, $g(a/w)$ found in SIF tables usually include the notch SCF, thus they use $\Delta\sigma$ instead of $\Delta\sigma_n$ as the nominal stress. A clearer way to define a_0 when the short crack departs from a notch root is to explicitly recognize this practice, separating the geometry factor $g(a/w)$ into two parts: $g(a/w) = \eta\cdot\varphi(a)$, where $\varphi(a)$ describes the stress gradient ahead of the notch tip, which tends to the SCF as the crack length $a \to 0$, whereas η encompasses all the remaining terms, such as the free surface correction:

$$\Delta K' = \eta\cdot\varphi(a)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\,,\quad \text{where } a_0 = (1/\pi)\left[\Delta K_0/(\eta\cdot\Delta S_0)\right]^2 \tag{3}$$

Operationally, the short crack problem can be treated by letting the SIF range ΔK retain its original equation, while the FCP threshold expression (under pulsating loads) is modified to become a function of the crack length a, namely $\Delta K_0(a)$, resulting in

$$\Delta K_0(a) = \Delta K_0\cdot\sqrt{a/(a+a_0)} \tag{4}$$

The ETS equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant's[8] reasoning, a more general equation can be used introducing an adjustable parameter γ to fit experimental data

$$\Delta K_0(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{5}$$

Equations (1-4) result from (5) if $\gamma = 2.0$. The bi-linear limit, $\Delta\sigma(a \le a_0) = \Delta S_0$ for short cracks, and $\Delta K_0(a \ge a_0) = \Delta K_0$ for long ones, is obtained when $g(a/w) = \eta\cdot\varphi(a) = 1$ and $\gamma \to \infty$. Most short crack FCP data is fitted by $\Delta K_0(a)$ curves with $1.5 \le \gamma \le 8$, but $\gamma = 6$ better reproduces classical q-plots based on data measures by testing semi-circular notched fatigue TS [2-3]. Using (5) as the FCP threshold, then any crack departing from a notch under pulsating loads should propagate if

$$\Delta K = \eta\cdot\varphi(a/\rho)\cdot\Delta\sigma\sqrt{\pi a} > \Delta K_0(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{6}$$

where η = 1.12 is the free surface correction. As fatigue depends on two driving forces, $\Delta\sigma$ and σ_{max}, (6) can be extended to consider σ_{max} (indirectly modeled by the R-ratio) influence in short crack behavior. First, the short crack characteristic size

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should be defined using the FCP threshold for long cracks $\Delta K_R = \Delta K_{th}(a >> a_R, R)$, and the fatigue limit ΔS_R, both measured or properly estimated at the desired R-ratio.

$$a_R = (1/\pi)\left[\Delta K_R/(1.12 \cdot \Delta S_R)\right]^2 \tag{7}$$

Likewise, the corresponding short crack FCP threshold should be re-written as

$$\Delta K_R(a) = \Delta K_R \cdot \left[1 + (a_R/a)^{\gamma/2}\right]^{-1/\gamma} \tag{8}$$

All these details are important when such models are used to make predictions in real life situations, as they do influence the calculation results. In particular, neglecting the σ_{max} effect on fatigue can lead to severe non-conservative life estimations, a potentially dangerous practice unacceptable for design or analysis purposes.

2 BEHAVIOR OF SHORT CRACKS DEPARTING FROM SLENDER NOTCHES

It is easy to mechanically justify why a crack starting from a sharp notch root can propagate for a while before stopping and becoming non-propagating (under fixed loading conditions.) A reasonable estimate for the SIF of a small crack a departing from the elliptical notch tip in an Inglis plate, with semi-axes $b >> a$ and c, and root radius $\rho = c^2/b$, is $K_I(a) \cong \sigma_n \cdot \sqrt{(\pi a)} \cdot f_1(a, b, c) \cdot f_2(free\ surface)$, where the $2b$ axis is centered at the x co-ordinate origin, σ_n is the nominal stress (perpendicular to a and b); $f_1(a, b, c) \cong \sigma_y(x)/\sigma_n$; $\sigma_y(x)$ is the stress at $(x = b + a, y = 0)$ ahead of the notch tip when there is no crack; and $f_2 = 1.12$. $\sigma_y(x = b + a, y = 0)$ is given by:[9]

$$f_1 = \frac{\sigma_y(x, y=0)}{\sigma_n} = 1 + \frac{(b^2 - 2bc)(x - \sqrt{x^2 - b^2 + c^2})(x^2 - b^2 + c^2) + bc^2(b-c)x}{(b-c)^2(x^2 - b^2 + c^2)\sqrt{x^2 - b^2 + c^2}} \tag{9}$$

The slender the elliptical notch is, meaning the smaller their semi-axes c/b and tip radius to depth ρ/b ratios are, the higher is its SCF. But high K_t imply in steeper stress gradients $\partial\sigma_y(x, y = 0)/\partial x$ around notch tips, since LE stress concentration induced by any elliptical hole drops from $K_t = 1 + 2b/c = 1 + 2\sqrt{(b/\rho)} = \sigma_y(1)/\sigma_n \geq 3$ at its tip border to $1.82 < K_{1.2} = \sigma_y(1.2)/\sigma_n < 2.11$ (for $b \geq c$) at a point just $b/5$ ahead of it, meaning their Saint Venant's controlling distance is associated with their depth b, not with their tip radii ρ.[1] This is the cause for the peculiar growth of short cracks which depart from elongated notch roots. Their SIF, which should tend to increase with their length $a = x - b$, may instead decrease after they grow for a short while because the SCF effect in $K_I \cong 1.12 \cdot \sigma_n \sqrt{(\pi a)} \cdot f_1$ may decrease sharply due the high stress drop close to the notch tip, overcompensating the crack growth effect. This $K_I(a)$ estimate can be used to evaluate non-propagating fatigue cracks tolerable at notch roots, using the short crack FCP behavior. E.g., if a large steel plate with $S_U = 600MPa$, $S_L = 200MPa$ and $\Delta K_0 = 9MPa\sqrt{m}$ works under $\Delta\sigma_n = 100MPa$ at $R = -1$, verify if it is possible to change a circular $d = 20mm$ central hole by an elliptical one with $2b = 20mm$ (perpendicular to σ_n) and $2c = 2mm$, without inducing the plate to fail by fatigue.
Neglecting the buckling problem, important in thin plates, the circular hole has safety factor against fatigue crack initiation $\phi_F = S_L/K_f \cdot \sigma_n = 200/150 \cong 1.33$, as this large hole has $K_f \cong K_t = 3$. But the sharp elliptical hole would not be admissible by traditional SN design routines, since it has $\rho = c^2/b = 0.1mm$, thus a very high $K_t = 1 + 2b/c = 21$. Its

notch sensitivity estimated from the usual Peterson q plot[10] would be $q \cong 0.32 \Rightarrow K_f = 1 + q \cdot (K_t - 1) = 7.33$, thus it would induce $K_f \cdot \sigma_n = 376MPa > S_L$. However, as this K_f value is considerably higher than typical values reported in the literature,[1] it is worth to re-study this problem considering the short crack FCP behavior. Supposing $\Delta K_{th}(R < 0) \cong \Delta K_0$ as usual, $\Delta K_0(a) = \Delta K_0/[1+(a_0/a)]^{-0.5}$ (by ETS), $S_L' = 0.5S_U$ (the material fatigue limit, as FCP modeling does not need modifying factors required to estimate S_L), $\Delta S_0 = S_U/1.5$ (by Goodman) and $a_0 = (1/\pi)(1.5\Delta K_0/1.12 \cdot S_U)^2 \cong 0.13mm$, the SIF ranges $\Delta K_I(a)$ for the two holes are compared to the FCP threshold $\Delta K_0(a)$ in Figure 1. The SIF for cracks departing from the circular notch remains below the $\Delta K_0(a)$ FCP threshold curve (which considers the short crack behavior) up to $a \cong 1.54mm$. Thus, if a small surface scratch locally augments the stress range and initiates a tiny crack at that hole border, it would not propagate under this fixed $\Delta\sigma_n = 100MPa$ and $R = -1$ load, confirming its "safe" prediction made by traditional *SN* procedures. Only if a crack with $a > 1.54mm$ is introduced at this hole border by any other means, it would propagate by fatigue under those otherwise safe loading conditions.

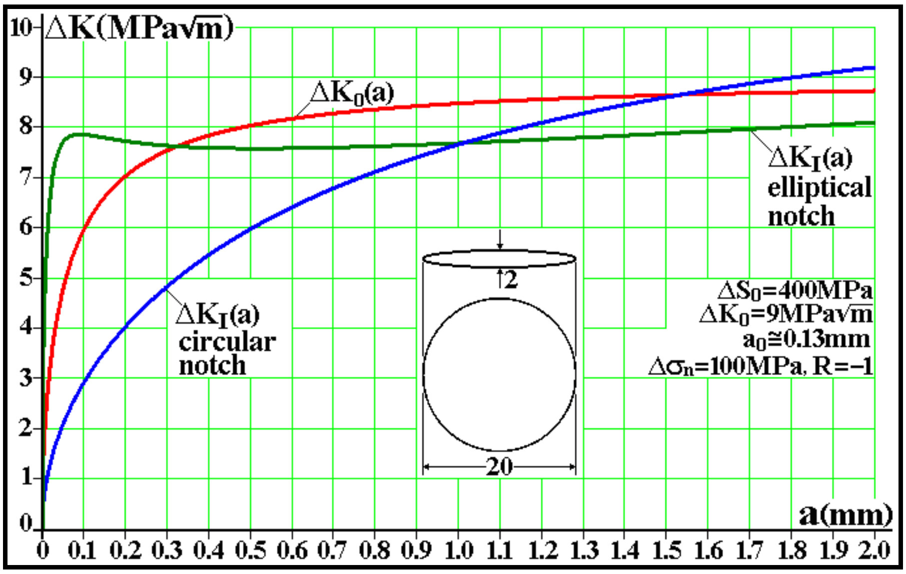

Figure 1: Using equation (9), it is estimated that cracks should not initiate at the circular hole border, which tolerates cracks $a < 1.54mm$, while the crack which initiates at the elliptical notch tip stops after reaching $a \cong 0.33mm$.

Under these same loading conditions, the $\Delta K_I(a)$ curve for the elliptical hole starts above $\Delta K_0(a)$, thus a crack should initiate at its border, as expected from its high K_t. But as this tiny crack propagates through the high stress gradient ahead of the notch root, it sees rapidly diminishing stresses around its tip during its early growth, which overcompensate the increasing crack size effect on $\Delta K_I(a)$. This crack SIF becomes smaller than $\Delta K_0(a)$ at $a \cong 0.33mm$, when it stops and becomes non-propagating (if $\Delta\sigma_n$ and R remain fixed), see Fig. 1. As fatigue failures include crack initiation and growth up to fracture, both notches could be considered safe for this service loading. But the non-propagating crack at the elliptical notch tip, a clear evidence of fatigue damage, renders it much less robust than the circular one, as discussed in Castro e Meggiolaro.[1]

For analysis purposes, the SIF range of a single crack with length *a* emanating from a semi-elliptical notch with semi-axes *b* and *c* (where *b* is in the same direction as *a*) at the edge of a very large plate loaded in mode I can be written as

$$\Delta K_I = \eta \cdot F(a/b, c/b) \cdot \Delta\sigma\sqrt{\pi a} \tag{10}$$

where η = 1.12, and $F(a/b, c/b)$ can be expressed as a function of the dimensionless parameter $s = a/(b + a)$ and of the notch SCF, given by

$$K_t = [1 + 2(b/c)] \cdot \left\{1 + \left[0.12/(1 + c/b)^{2.5}\right]\right\} \tag{11}$$

To obtain expressions for *F*, extensive finite element calculations were performed for several cracked semi-elliptical notches. The numerical results, which agreed well with standard solutions,[11] were fitted within 3% using empirical equations [2-3]

$$F(a/b, c/b) \equiv f(K_t, s) = K_t\sqrt{\left[1 - \exp\left(-sK_t^2\right)\right]/sK_t^2}\ ,\ c \le b \text{ and } s = a/(b + a) \tag{12}$$

$$F'(a/b, c/b) \equiv f'(K_t, s) = K_t\left[1 - \exp\left(-K_t^2\right)\right]^{-s/2}\sqrt{\left[1 - \exp\left(-sK_t^2\right)\right]/sK_t^2}\ ,\ c \ge b \tag{13}$$

The SIF expressions include the semi-elliptical notch effect through *F* or *F'*. Indeed, as $s \to 0$ when $a \to 0$, the maximum stress at its tip $\sigma_{max} \to F(0, c/b) \cdot \sigma_n = K_t \cdot \sigma_n$. Thus, the η-factor, but not the $F(a/b,c/b)$ part of K_I, should be considered in the short surface crack characteristic size a_0, as done in equation (3). Note also that the semi-elliptical K_t includes a term $[1 + 0.12/(1 + c/b)^{2.5}]$ which could be interpreted as the notch free surface correction (FSC). Thus, as $c/b \to 0$ and the semi-elliptical notch tends to a crack, its $K_t \to 1.12 \cdot 2\sqrt{(b/\rho)}$. Such *1.12* factor is the notch FSC, not the crack FSC η. Indeed, when $c/b \to 0$, this 1.12 factor disappears from the *F* expression, which gives $F(a/b, 0) = 1/\sqrt{s}$, and thus $\Delta K_I = \eta \cdot F \cdot \Delta\sigma \cdot [\pi \cdot a]^{0.5} = \eta \cdot \Delta\sigma \cdot [\pi \cdot (a + b)]^{0.5}$, as expected, since the resulting crack for $c \to 0$ would have length $a + b$.
Traditional *q* estimates, based on the fitting of questionable semi-empirical equations to few experimental data points, assume it depends only on the notch root ρ and on the material ultimate strength S_U. Thus, similar materials with the same S_U but different ΔK_0 should have identical notch sensitivities. The same should occur with shallow and deep or elongated notches of identical tip radii. However, whereas well established empirical relations relate the fatigue limit ΔS_0 to the tensile strength S_U of many materials, there are no such relations between their FCP threshold ΔK_0 and S_U. Moreover, it is also important to point out that the *q* estimation for elongated notches by the traditional procedures can generate unrealistic K_f values, as exemplified above. In conclusion, such traditional estimates should not be taken for granted.
The proposed model, on the other hand, is based on the FCP mechanics of short cracks which depart from elliptical notch roots, recognizing that their *q* values are associated with their tolerance to non-propagating cracks. It shows that their notch sensitivities, besides depending on ρ, ΔS_0, ΔK_0 and γ, are also strongly dependent on their shape, given by their *c/b* ratio.[2,3] Their corresponding Peterson's curve is well approximated by the semi-circular $c/b = 1$ notch, but this curve is ***not*** applicable for much different c/b ratios. Therefore, the proposed predictions indicate that these tra-

ditional notch sensitivity estimates should ***not*** be used for elongated notches, a forecast experimentally verified, as discussed in the following section.

3 VERIFICATION OF ELONGATED NOTCH SENSITIVITY PREDICTIONS

Fatigue tests were carried out on modified SE(T) specimens of thickness $t = 8mm$ and width $W = 80mm$, to find the number of cycles required to re-initiate the crack after drilling a stop-hole of radius ρ centered at its tip, generating an elongated slit with $b = 27.5mm$, as detailed in Wu et al.[3] The original objective of those tests was to study life improvements obtained by the stop-hole repair technique, but these tests can also be used to support the validity of the proposed model. The TS were made from an Al alloy 6082 T6, with $S_Y = 280MPa$, $S_U = 327MPa$, and Young's modulus $E = 68GPa$. The particularly careful tests were made at 30Hz under fixed load range at $R = 0.57$, to avoid any crack closure influence on their FCP behavior. The TS were first pre-cracked until reaching the required crack size. Then they were removed to introduce the stop-holes in a milling machine, using a slight under-size drill precisely centered at their crack tips. Finally, the holes were enlarged to reach their size using a reamer. The stop-hole sizes were large enough to remove the previous plastic zones.

The fatigue crack re-initiation lives at the tip of the resulting elongated notch can be modeled by εN procedures using (i) the alloy parameters $\sigma'_f = 485$MPa, $b = -0.0695$, $\varepsilon'_f = 0.733$ and $c = -0.827$, and Ramberg-Osgood's coefficient and exponent of the cyclic stress-strain curve, $H = 443$MPa and $h = 0.064$;[12] (ii) the nominal stress range and R-ratio; and finally (iii) the stress concentration factor of the notches generated after repairing the cracks by a stop-hole at their tips, which can be calculated by FE ($K_t = 11.8$, 8.1, and 7.6 for the 3 stop-hole radii, $\rho = 1$, 2.5, and 3 mm.)

The repaired crack can be modeled by calculating the stress and strain maxima and ranges at the stop-hole root border by Neuber's rule, and by using them to calculate the crack re-initiation lives by an appropriate $\Delta\varepsilon \times N$ rule, considering the influence of the mean loads. Neglecting this effect could lead to severely non-conservative predictions, as the R-ratio used in the tests was high (Coffin-Manson predictions are highly non-conservative, thus useless in this case). Figure 2 shows that the lives predicted by the elastic version of Morrow's equation (which is an extension of the classical Goodman line) and by the Smith-Watson-Topper (SWT) equation are similar in this case, and predict well the measured data. Further details are available in Wu et al.[3]

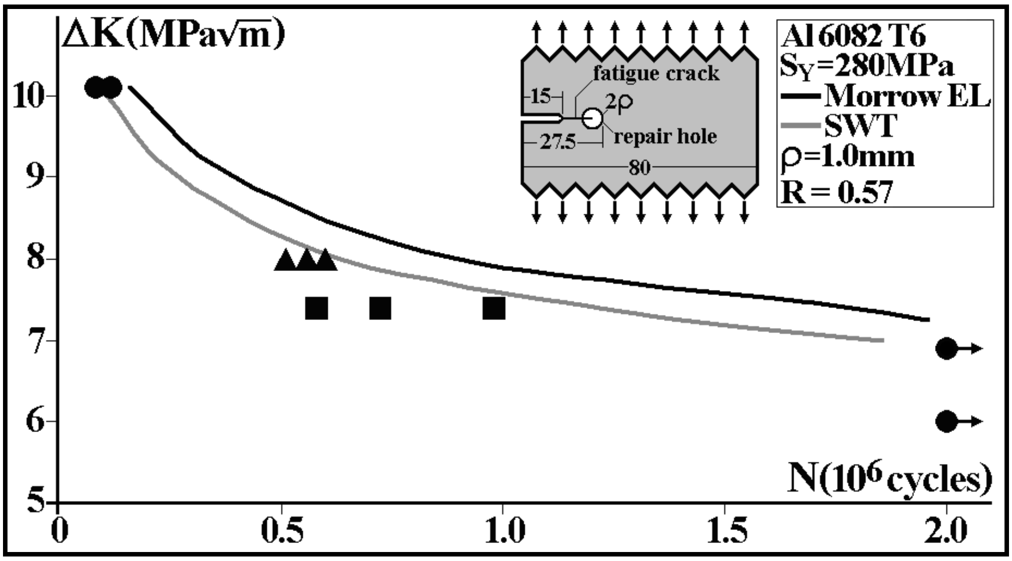

Figura 2: Predicted and measured crack re-initiation lives after introducing stop-holes with radii ρ = *1.0mm* at the tip of the crack, using the properly calculated K_f of the resulting elongated slit (instead of its K_t) and appropriate εN procedures.

4 A CRITERION TO ACCEPT SHORT CRACKS

Based on the encouraging life estimations for these fatigue crack re-initiation data, the reverse path can be followed, assuming the methodology presented here can be used to generate an unambiguous acceptance criterion for small cracks, a potentially much useful tool for practical applications. Most structural components are designed against fatigue crack initiation, using εN or *SN* procedures which do not recognize cracks. Hence, their "infinite life" predictions may become unreliable when such cracks are introduced by any means, and not quickly detected and properly removed. Large cracks may be easily detected and dealt with, but small cracks may pass unnoticed even in careful inspections, if they are smaller than the detection threshold of the inspection method used to identify them. Thus, structural components designed for very long fatigue lives should be designed to be tolerant to short cracks.

However, this self-evident requirement is still not usually included in fatigue design routines, as most long-life designs just intend to maintain the stress range at critical points below their fatigue limits, guaranteeing that $\Delta\sigma < S_R/\phi_F$, where ϕ_F is a suitable safety factor. Nevertheless, most long-life designs work well, which means that they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by *SN* or εN procedures alone. Such problem can be avoided by adding (6) and (7) to the "infinite" life design criterion which, to tolerate a crack of size *a* in its simplest version, should be written as

$$\Delta\sigma < \Delta K_R \Big/ \left\{ \sqrt{\pi a} \cdot g(a/w) \cdot \left[1 + (a_R/a)^{\gamma/2} \right]^{1/\gamma} \right\}, \; a_R = (1/\pi) \cdot [\Delta K_R / \eta \Delta S_R]^2 \qquad (14)$$

As fatigue limits ΔS_R include the influence of microstructural defects inherent to the material, (14) complements it considering the component tolerance to cracks. A simple case study can clarify how useful this concept can be, as discussed next.

Due to an unusual manufacturing problem, a batch of an important component was sold with small surface cracks, causing some unexpected annoying failures. The task was to estimate how such cracks affect the stresses those steel components could

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tolerate under uniaxial fatigue loads, knowing that their rectangular cross section has *2mm* by *3.4mm*; their measured fatigue limit under $R = -1$ is $S_L = 246MPa$; and their $S_U = 990MPa$. As $S_L > S_U/2$, it may include surface roughness effects which should not affect the cracks. But, in the absence of reliable information, the only safe option is to use the measured S_L value to estimate S_R and a_R. Therefore, by Goodman

$$S_R = \left[S_L S_U (1-R) \right] / \left[S_U (1-R) + S_L (1+R) \right] \tag{15}$$

The mode I stress range $\Delta\sigma$ tolerable by this component when it has a uniaxial surface crack of depth a can thus be expressed by

$$\Delta\sigma < \frac{\Delta K_R / \varphi_F}{\sqrt{\pi a}\,[\,0.752 + 2.02\frac{a}{w} + 0.37(1-\sin\frac{\pi a}{2w})^3\,]\sec\frac{\pi a}{2w}\sqrt{\frac{2w}{\pi a}\tan\frac{\pi a}{2w}}\cdot\left[1+\left(\frac{a_R}{a}\right)^{\gamma/2}\right]^{1/\gamma}} \tag{16}$$

where w = *3.4mm* was the component width, and its $g(a/w)$ geometry factor was obtained from Tada, Paris e Irwin.[11] Figures 3-5 plot the maximum tolerable stress ranges (for $\phi_F = 1$) for several R-ratios. As the FCP threshold of this component was not available, it had to be estimated. The typical threshold range for steels is $6 < \Delta K_0 < 12MPa\sqrt{m}$. It is usual to assume $\Delta K_R \cong \Delta K_0$ for $R < 0$ loads (except if the load history contains severe underloads). Lower limit estimations for positive R are $\Delta K_{th}(0 < R \leq 0.17) = 6MPa\sqrt{m}$, and $\Delta K_{th}(R > 0.17) = 7\cdot(1 - 0.85R)$.[1] Using $\eta = 1.12$ and $\Delta K_0 = 6MPa\sqrt{m}$, the short crack characteristic value is estimated as $a_0 = 59\mu m$. Figure 3 shows that if this component works under $\Delta\sigma = 286MPa$ and $R = -0.12$, it tolerates cracks up to $a \cong 105\mu m$, and under $\Delta\sigma = 176MPa$ and $R = 0.44$, cracks up to $a \cong 150\mu m$, e.g. Figure 4 uses semi-log coordinates to enhance this component small tolerance to short cracks.

Therefore, this simple (but sensible) model indicates that this component is not too tolerant to 1D surface cracks. However, as this conclusion is based on estimated properties, it is worth to study its sensibility to the assumed values. Figure 5 shows the prediction range associated with the typical interval expected for the estimated properties, enhancing how important it is to measure them. Note that (14) assumes that the short crack is unidimensional and grows without changing its original plane. Note also that this model only describes the behavior of macroscopically short cracks, as it uses macroscopic material properties. Thus it can only be applied to short cracks which are large in relation to the characteristic size of the intrinsic material anisotropy (e.g. its grain size). Smaller cracks grow inside an anisotropic and usually inhomogeneous scale, thus their FCP is also affected by microstructural barriers, such as second phase particles or grain boundaries. However, as grains cannot be mapped in most practical applications, such problems, in spite of their academic interest, are not really a major problem from the fatigue design point of view.

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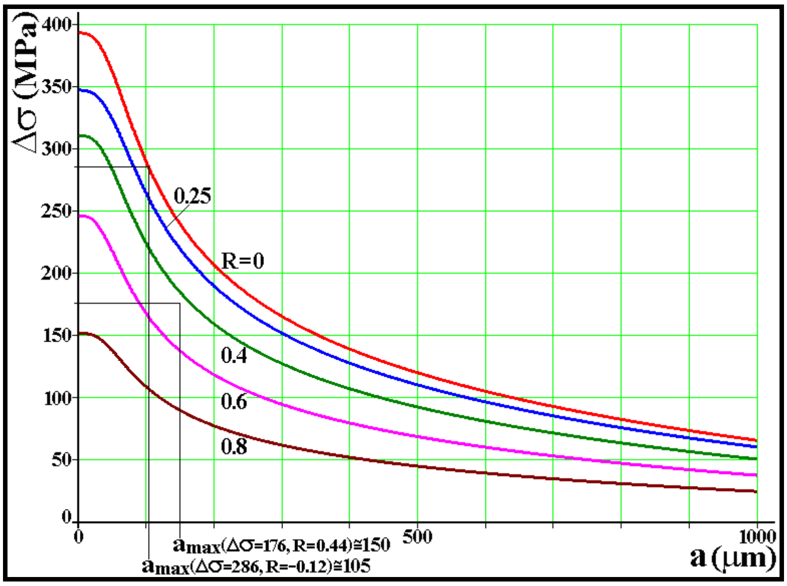

Figure 3: Effect of a surface crack of size *a* in the largest stress range $\Delta\sigma_R(a)$ tolerable by a strip of width $w = 3.4mm$ loaded in mode I, for various *R*-ratios (it is assumed that it has $\Delta K_0 = 6MPa\sqrt{m}$ and $\gamma = 6$, thus $a_0 = 59$ and $a_{0.8} = 55\mu m$).

However, this model has another limitation which may be more important for practical applications: it assumes that the short crack can be completely characterized by its depth *a*. But most short cracks are surface or corner cracks, which tend to grow by fatigue at least in two directions, maintaining their original plane when they are loaded under pure mode I conditions. In these cases, they can be modeled as bidi-mensional (2D) cracks which grow both in depth and width. In reality, both long and short cracks (these meaning cracks not much larger than a_R) only behave as 1D cracks after having cut all the component width to become a through crack, with a more or less straight front which propagates in an approximately uniform way. Thus, equation (16) must be adapted to consider this fact.

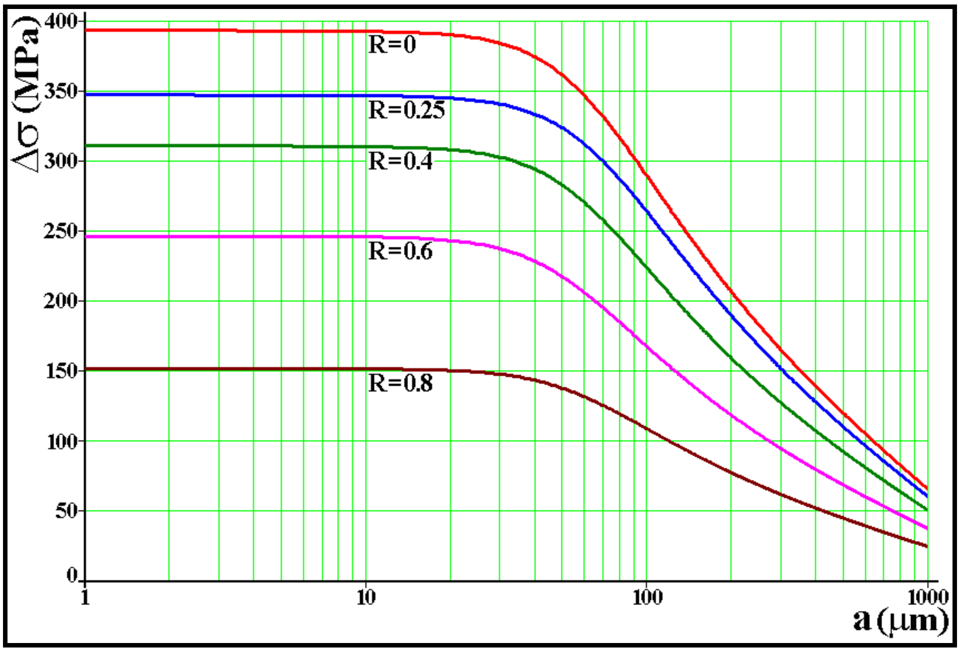

Figure 4: Similar to Figure 3, but with semi-log scale to enhance the short crack tolerance. Small cracks with $a < 30\mu m$ have practically no effect in its fatigue resistance.

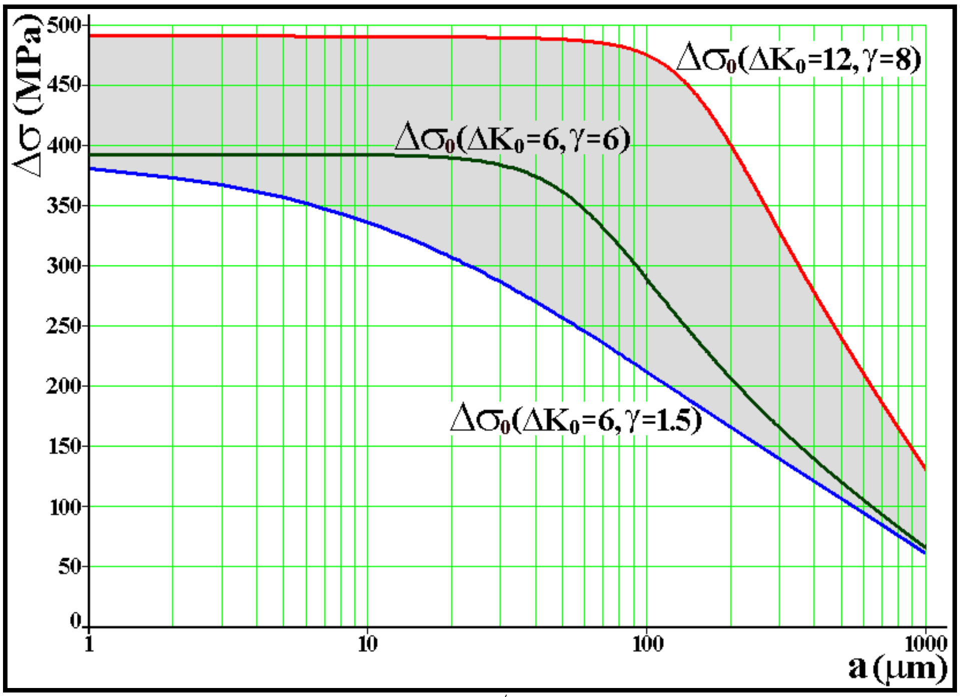

Figure 5: Typical steel threshold $6 < \Delta K_0 < 12 MPa\sqrt{m}$ and γ exponent $1.5 < \gamma < 8$ ranges influence in the largest mode I stress ranges $\Delta\sigma_0$ tolerated by the w □ $3.4mm$ strip, as a function of the 1D superficial crack size a.

Therefore, assuming that: (i) the cracks are loaded in pure mode I, under quasi-constant $\Delta\sigma$ and R conditions, with no major overloads; (ii) material properties measured (or estimated) testing 1D specimens may be used to simulate the FCP behavior of 2D cracks; and (iii) 2D surface or corner cracks can be well modeled as having an approximately elliptical front, thus their SIF can be described by the classical Newman-Raju equations.[13] In this case, it can be expected that the component tolerance to cracks be given by:

$$\Delta\sigma < \begin{cases} \Delta K_R \Big/ \left\{ \sqrt{\pi a} \cdot \Phi_a(a,c,w,t) \cdot \left[1 + (a_R/a)^{\gamma/2} \right]^{1/\gamma} \right\} \\ \Delta K_R \Big/ \left\{ \sqrt{\pi c} \cdot \Phi_c(a,c,w,t) \cdot \left[1 + (a_R/c)^{\gamma/2} \right]^{1/\gamma} \right\} \end{cases} \tag{17}$$

For semi-elliptical surface cracks in a plate of thickness t, the SIF in the depth a and width c directions, $K_{I,a} = \sigma\sqrt{(\pi a)}\cdot\Phi_a$ and $K_{I,c} = \sigma\sqrt{(\pi c)}\cdot\Phi_c$, are given by:

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$$
\begin{cases}
K_{I,a} = \sigma\sqrt{\pi a}\cdot\Phi_a = \sigma\sqrt{\pi a}\cdot F\cdot M/\sqrt{Q} \\
K_{I,c} = \sigma\sqrt{\pi a}\cdot\Phi_c = \sigma\sqrt{\pi c}\cdot F\cdot(M/\sqrt{Q})\cdot a/c\cdot G \\
F(c/w, a/t) = \sqrt{\sec[(\pi c/2w)\sqrt{a/t}]}\cdot\left[1-0.025[(c/w)\sqrt{a/t}]^2+0.06[(c/w)\sqrt{a/t}]^4\right] \\
M = \begin{cases} 1.13-0.09\dfrac{a}{c}+\left[\dfrac{0.89}{0.2+a/c}-0.54\right]\dfrac{a^2}{t^2}+\left[0.5-\dfrac{1}{0.65+a/c}+14(1-\dfrac{a}{c})^{24}\right]\dfrac{a^4}{t^4}, & a\le c \\ c/a+0.04(c/a)^2+(c/a)^{4.5}(a/t)^2\left[0.2-0.11(a/t)^2\right], & a>c \end{cases} \\
Q = \begin{cases} 1+1.464(a/c)^{1.65}, & a\le c \\ 1+1.464(c/a)^{1.65}, & a>c \end{cases} \\
G = \begin{cases} 1.1+0.35(a/t)^2, & a\le c \\ 1.1+0.35(a/t)^2(c/a), & a>c \end{cases}
\end{cases} \quad (18)
$$

For quarter-elliptical corner cracks, the Φ_a and Φ_c geometry functions are:

$$
\begin{cases}
\Phi_a = \sec\left(\dfrac{\pi c}{2w}\sqrt{\dfrac{a}{t}}\right)\left[0.752+2.02\dfrac{c}{w}\sqrt{\dfrac{a}{t}}+0.37\left(1-\sin\left(\dfrac{\pi c}{2w}\sqrt{\dfrac{a}{t}}\right)\right)^3\right]\sqrt{\dfrac{2w}{\pi c}\sqrt{\dfrac{t}{a}}\tan\left(\dfrac{\pi c}{2w}\sqrt{\dfrac{a}{t}}\right)}\times \\
\times\left(1.08-0.03\dfrac{a}{c}+\left(-0.44+\dfrac{1.06}{0.3+a/c}\right)\left(\dfrac{a}{t}\right)^2+\left[-0.5+0.25\dfrac{a}{c}+14.8\left(1-\dfrac{a}{c}\right)^{15}\right]\left(\dfrac{a}{t}\right)^4\right)\times \\
\times\dfrac{1.08+0.15\left(\dfrac{a}{t}\right)^2}{\sqrt{1+1.464(a/c)^{1.65}}} \\
\\
\Phi_c = \sec\left(\dfrac{\pi c}{2w}\sqrt{\dfrac{a}{t}}\right)\left[0.752+2.02\dfrac{c}{w}\sqrt{\dfrac{a}{t}}+0.37\left(1-\sin\left(\dfrac{\pi c}{2w}\sqrt{\dfrac{a}{t}}\right)\right)^3\right]\sqrt{\dfrac{2w}{\pi c}\sqrt{\dfrac{t}{a}}\tan\left(\dfrac{\pi c}{2w}\sqrt{\dfrac{a}{t}}\right)}\times \\
\times\left(1.08-0.03\dfrac{a}{c}+\left(-0.44+\dfrac{1.06}{0.3+a/c}\right)\left(\dfrac{a}{t}\right)^2+\left[-0.5+0.25\dfrac{a}{c}+14.8\left(1-\dfrac{a}{c}\right)^{15}\right]\left(\dfrac{a}{t}\right)^4\right)\times \\
\times\dfrac{1.08+0.4\left(\dfrac{a}{t}\right)^2}{\sqrt{1+1.464(a/c)^{1.65}}}
\end{cases} \quad (19)
$$

These complicated SIF functions enhance the operational advantage of treating the FCP threshold as a function of the crack size, $\Delta K_{th}(a)$, as claimed above. For structural calculations and mechanical design purposes, it is indeed relatively simple to use either equation (16) or (17) to evaluate the influence of surface cracks on the component fatigue strength. Moreover, it is not too difficult to adapt the 2D equations to include notch effects.

5 CONCLUSIONS

A generalized El Haddad-Topper-Smith's parameter was used to model the threshold stress intensity range for short cracks dependence on the crack size, as well as the behavior of non-propagating fatigue cracks. This dependence was used to estimate the notch sensitivity factor *q* of semi-elliptical notches, from studying the propagation behavior of short non-propagating cracks that may initiate from their tips. The predicted notch sensitivities reproduced well the classical Peterson's *q* estimates for circular holes or approximately semi-circular notches, but it was found that the notch sensitivity of elongated slits has a very strong dependence on the notch aspect ratio, defined by the ratio *c/b* of the semi-elliptical notch that approximates the slit shape having the same tip radius. These predictions were confirmed by experimental measurements of the re-initiation life of long fatigue cracks repaired by introducing a stop-hole at their tips, using their calculated K_f and appropriate εN procedures. Based on this promising performance, a criterion to evaluate the influence of small or large surface cracks in the fatigue resistance was proposed.

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