A MODIFIED WANG-BROWN METHOD FOR MULTIAXIAL RAINFLOW COUNTING OF NON-PROPORTIONAL STRESS OR STRAIN HISTORIES

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Abstract. *Non-proportional (NP) multiaxial fatigue life predictions require the use of a multiaxial rainflow algorithm together with a method to calculate the effective stress or strain ranges associated with each counted cycle. The most successful multiaxial rainflow algorithm proposed so far is the Wang-Brown (WB) method, however it has a few idiosyncrasies that can lead to non-conservative damage predictions, incorrectly filtering out significant events within a multiaxial loading cycle. This work introduces a novel multiaxial rainflow counting algorithm, called Modified Wang-Brown (MWB), applicable to both non-periodic NP histories and periodic NP histories formed by complex blocks with multiple cycles each. There are two main improvements of the MWB over the original WB algorithm. First, the criterion to choose the point where the count is started is modified. Examples are shown to prove that the original criterion can overlook the most damaging event from the history, as opposed to the modified version. And second, the algorithm implementation is significantly simplified when formulated in the reduced five-dimensional Euclidean space defined by Papadopoulos. Under plane stress conditions, prevalent in most fatigue crack initiation problems, the algorithm is further simplified using a three-dimensional Euclidean space based on the deviatoric stresses or strains. The computational implementation of such algorithm is discussed in detail, including a flow chart with all its necessary steps, allowing a fast and efficient calculation of fatigue damage even for very long non-periodic NP histories.*

Keywords: *multiaxial fatigue; multiaxial rainflow; Wang-Brown method.*

1. INTRODUCTION

Most non-proportional (NP) fatigue design methods are only applicable to periodic histories or to infinite life calculations. Finite life calculations can be performed, but usually the available models implicitly assume that each block of the periodic loading path contains a single cycle. To generalize the existing methods to finite life predictions in periodic histories with multiple cycles at each block or period, or to non-periodic histories, a cycle counting algorithm must be introduced.

The rainflow algorithm (Matsuishi and Endo, 1968; ASTM E 1049) is reputably the best approach to identify the most damaging events embedded in a variable amplitude history. For linear elastic uniaxial histories, it is indifferent to perform the uniaxial rainflow count on the stresses or on the strains. In addition, the sequential rainflow count is always a better option over the traditional rainflow, since it preserves the original loading order. The sequential rainflow is obtained by simply reordering the resulting traditional rainflow count by the final counting point. Even for linear elastic problems, where the damage caused by each event should not depend on the other events or on the loading order, it is a good idea to choose the sequential rainflow to correctly predict in which event the accumulated damage reaches its critical value (usually 1.0, according to Miner's rule). It is also recommended for crack growth predictions, to correctly account for load interaction effects without changing the load order.

For elastoplastic uniaxial histories, it is fundamental to calculate the hysteresis loops before performing any rainflow count. After measuring/calculating both stress and strain histories, the sequential rainflow count must be applied to the strains, never to the stresses.

However, for multiaxial problems, the uniaxial rainflow of stresses or strains may lead to significant errors, even in the linear elastic case, as discussed in the next section.

2. RAINFLOW OF A MULTIAXIAL HISTORY

For general NP multiaxial histories, the traditional uniaxial rainflow count, applied to some stress or strain component that is assumed dominant to describe the history, does not provide good life predictions. This is so because the peaks and valleys of the stresses and the corresponding strains do not usually coincide, even in the same direction, not to mention in different directions.

Another important issue involves the common practice of filtering non-reversals from a measured history. When dealing with measured multiaxial loading histories, the sampling points that do not constitute a reversal in any of its stress or strain components are usually eliminated. Filtering points that do not constitute a reversal helps to decrease computational cost in multiaxial fatigue calculations, especially when dealing with over-sampled data (Bannantine and Socie, 1991).

But filtering out all points that do not constitute a reversal in one stress or strain component may cause significant damage prediction problems, discussed next. First, the reversal points obtained from a multiaxial rainflow algorithm may not occur at the reversal of one of the stress or strain components. E.g., the relative Mises strain, used in Wang-Brown's rainflow count (Wang and Brown, 1996), may reach a peak value at a point that is not a maximum or minimum of any strain component. But this most important point would have been filtered by any non-reversal filtering algorithm, compromising the results.

The second problem may occur because the entire path between two reversals is needed to evaluate the equivalent stress or strain associated with each count, e.g. using a convex hull method. Filtering out points along such path would almost certainly result in lower equivalent stress or strain estimates than expected.

Another issue with rainflow counting NP multiaxial histories is whether or not to use a critical plane approach. The Wang-Brown multiaxial rainflow algorithm is general enough to be directly applied to a multiaxial history involving all 6 strain components. But the counted cycles will probably occur in different planes, not reproducing the crack initiation mechanism. Instead, a critical plane approach must be followed: the multiaxial history must be projected onto a candidate plane, and only then should a multiaxial rainflow count be used.

In this critical plane approach, the stress and/or strain history is projected onto a candidate plane from the critical point. A uniaxial rainflow count is then applied to an appropriate strain or stress component, which depends on the chosen damage model: for the εN and SWT damage models, the normal strain perpendicular to the candidate plane is rainflow counted; in the Brown-Miller, Fatemi-Socie and Wang-Brown damage models, a shear strain component acting parallel to the candidate plane is rainflow counted; and in the Findley damage model, a shear stress component parallel to the candidate plane is counted.

While performing such rainflow count on the candidate plane, the other stress and strain components cannot be overlooked or discarded. For instance, if the SWT damage model is used, at every rainflow counted half cycle ε_I, the maximum value of the normal stress $\sigma_{\perp I}$ parallel to ε_I along the entire half cycle must be stored to compute $\sigma_{\perp Imax}$. Since for complex NP multiaxial load histories these maxima may happen at any point along the half cycle, not only at the peaks and valleys of a given component, non-reversals should never be filtered before performing the rainflow count.

Note also that, if only the strain (or stress) history is provided, one might need to calculate the entire stress-strain history from proportional multiaxial stress-strain relations or from incremental plasticity techniques, before performing the rainflow count. After performing the rainflow count at each candidate plane, the resulting damage is calculated. The critical plane is then the candidate plane that results in the highest fatigue damage.

However, it must be noted that Case B cracks (Socie, 1999) can have two shear strain (or stress) components acting parallel to each candidate plane. A uniaxial rainflow approach would either neglect the effect of one of such shear components, or consider that one of them is dominant over the other during the rainflow algorithm application. But this practice can be ***non***-conservative, since both shear components induce crack initiation. To deal with that, a true multiaxial rainflow algorithm must be used, accounting for all stress or strain components, such as Wang-Brown's algorithm, discussed next.

3. WANG-BROWN'S MULTIAXIAL RAINFLOW ALGORITHM

Wang and Brown (1996) proposed an interesting multiaxial generalization of the rainflow count that is applicable to any proportional or NP history of strains (or stresses, with simple modifications to the algorithm). Wang-Brown's multiaxial rainflow is based on the Mises strain ε_{Mises} as an indirect measure of fatigue damage.

The problem with using ε_{Mises} is the loss of the loading event sign, since Mises values are always positive. Therefore, in 90° out-of-phase histories it is even possible that ε_{Mises} remain constant, which would wrongfully result in an infinite life prediction. To solve this issue, the relative Mises strain ε_{RMises} is used, calculated from the difference between the strain components $(\varepsilon_{xj}, \varepsilon_{yj}, \varepsilon_{zj}, \gamma_{xyj}, \gamma_{xzj}, \gamma_{yzj})$ of each (j^{th}) point in the history and the strain components $(\varepsilon_{xi}, \varepsilon_{yi}, \varepsilon_{zi}, \gamma_{xyi}, \gamma_{xzi}, \gamma_{yzi})$ of the initial (i^{th}) point of the current count:

$$\varepsilon_{RMises} = \frac{\sqrt{(\Delta\varepsilon_x - \Delta\varepsilon_y)^2 + (\Delta\varepsilon_x - \Delta\varepsilon_z)^2 + (\Delta\varepsilon_y - \Delta\varepsilon_z)^2 + 1.5(\Delta\gamma_{xy}^2 + \Delta\gamma_{xz}^2 + \Delta\gamma_{yz}^2)}}{\sqrt{2}\cdot(1+\bar{\nu})} \quad (1)$$

where $\Delta\varepsilon_x \equiv \varepsilon_{xj} - \varepsilon_{xi}$, $\Delta\varepsilon_y \equiv \varepsilon_{yj} - \varepsilon_{yi}$, $\varepsilon_z \equiv \varepsilon_{zj} - \varepsilon_{zi}$, $\Delta\gamma_{xy} \equiv \gamma_{xyj} - \gamma_{xyi}$, $\Delta\gamma_{xz} \equiv \gamma_{xzj} - \gamma_{xzi}$, $\Delta\gamma_{yz} \equiv \gamma_{yzj} - \gamma_{yzi}$, and $j > i$.

The relative strains need to be re-calculated for every initial counting point, a computationally intensive task for very long histories. Note however that the relative strain ε_{RMises} is only used to locate the initial and final counting points of each half cycle, after which it is possible to apply at these points any multiaxial damage model (even models that do not include a Mises strain parameter).

As in the uniaxial case, Wang-Brown's multiaxial rainflow is based on 3 simple rules:

1. The first count must start at the point with the largest value of ε_{Mises} from the entire history.

2. Each count must be initiated sequentially at each peak or valley of a strain component, and the relative Mises strain ε_{RMises} of the subsequent history must be computed with respect to the initial point.
3. The final point of each count is obtained when reaching:
   a) the largest value of ε_{RMises} with respect to the initial point of the history, or
   b) any path used in a previous count.

Note that the maxima and minima of each stress or strain component may not happen at the beginning or at the end of the counted half cycle, as discussed before. It may happen at any point along the cycle. Therefore, any stress or strain range must be computed considering the maximum and minimum values along the entire path between two reversions, not only the initial and final values from the half cycle.

4. MODIFIED WANG-BROWN (MWB) ALGORITHM

The original Wang-Brown algorithm is not difficult to be implemented in histories of uniaxial tension/bending combined with torsion, which can be represented only by one normal σ_x and one shear τ_{xy} stress components (or one normal ε_x and one shear γ_{xy} strain components). In this case, the subspace of normal and shear components is planar (it is represented by a diagram in only 2 dimensions), and the only difficulty in applying the algorithm happens when solving for the equations of the ellipses associated with the points with same relative Mises stress or strain.

However, in a generic multiaxial history, the dimension of the diagram may be increased, requiring the calculation of intersections between straight lines and ellipsoid or hyper-ellipsoid surfaces, increasing the computational complexity.

The Modified Wang-Brown method solves this problem by working in the reduced 5-dimensional stress $E_{5\sigma}$ or strain $E_{5\varepsilon}$ subspaces (Papadopoulos, 1997), or in a lower dimension subspace from them. In this way, a general multiaxial strain or stress history is represented by a set of points $P_i = (e_1, e_2, e_3, e_4, e_5)$ or $P_i = (S_1, S_2, S_3, S_4, S_5)$, respectively, where

$$S_1 \equiv \sigma_x - \frac{\sigma_y}{2} - \frac{\sigma_z}{2} = \frac{3}{2} S_x, \quad S_2 \equiv \frac{\sigma_y - \sigma_z}{2}\sqrt{3} = \frac{S_y - S_z}{2}\sqrt{3}$$
$$S_3 \equiv \tau_{xy}\sqrt{3}, \quad S_4 \equiv \tau_{xz}\sqrt{3}, \quad S_5 \equiv \tau_{yz}\sqrt{3} \tag{2}$$

$$e_1 \equiv \frac{3}{2} \cdot \frac{e_x}{1+\bar{\nu}} = \frac{2\varepsilon_x - \varepsilon_y - \varepsilon_z}{2\cdot(1+\bar{\nu})}, \quad e_2 \equiv \frac{e_y - e_z}{2\cdot(1+\bar{\nu})}\sqrt{3} = \frac{\varepsilon_y - \varepsilon_z}{2\cdot(1+\bar{\nu})}\sqrt{3},$$
$$e_3 \equiv \frac{\gamma_{xy}\sqrt{3}}{2\cdot(1+\bar{\nu})}, \quad e_4 \equiv \frac{\gamma_{xz}\sqrt{3}}{2\cdot(1+\bar{\nu})}, \quad e_5 \equiv \frac{\gamma_{yz}\sqrt{3}}{2\cdot(1+\bar{\nu})} \tag{3}$$

Wang-Brown's multiaxial rainflow algorithm is rather simplified when working in such spaces, because the distance between two points is already the relative Mises strain (or stress) between them. The 3 rules of the rainflow count have now simple geometric interpretations, resulting in:

1. The count must be initiated at the point with highest norm, i.e., with the longest Euclidean distance to the origin of the diagram. This first initial counting point is called P_1, and the subsequent ones are called P_2..., P_n, in the same sequence of the original history.
2. Each count must be sequentially initiated at each point P_i of the diagram.
3. The final point of each counting is obtained when reaching:
   a) the point P_j most distant from the initial point P_i (with $j > i$) in the reduced subspace, or
   b) any path used in a previous count.

The first rule in Wang-Brown's algorithm was conceived to try to guarantee that the largest ε_{RMises} (or relative Mises stress σ_{RMises}) of the history is identified, one of the main objectives of a rainflow count. However, this rule can fail to reach this objective if the point P_1 with largest norm is not one of two points of the diagram farthest apart from each other.

This is easy to check in the example from Fig. 1, which shows an e_1-e_3 strain diagram with a triangular path. The point $(e_1, e_3) = (0.8\%, 0\%)$ is clearly the one with largest norm, equal to 0.8%, however its Wang-Brown count results in two half cycles with $\varepsilon_{RMises} = 1.0\%$. Instead, if the count is started at the point $(e_1, e_3) = (0\%, 0.6\%)$, both half cycles result in $\varepsilon_{RMises} = 1.1\%$. It is not difficult to prove that the largest relative Mises strain (or stress) of the history can be ***underestimated*** by up to $1-\sqrt{2}/2 = 29.3\%$ using the original Wang-Brown algorithm. Even if a convex hull method or the MOI method are applied to the resulting half cycles, to account for the shape of the entire path, and not only the value of ε_{RMises}, the original Wang-Brown algorithm still underestimates the resulting equivalent ranges. The conclusion is that the starting point of the first count must be better chosen.

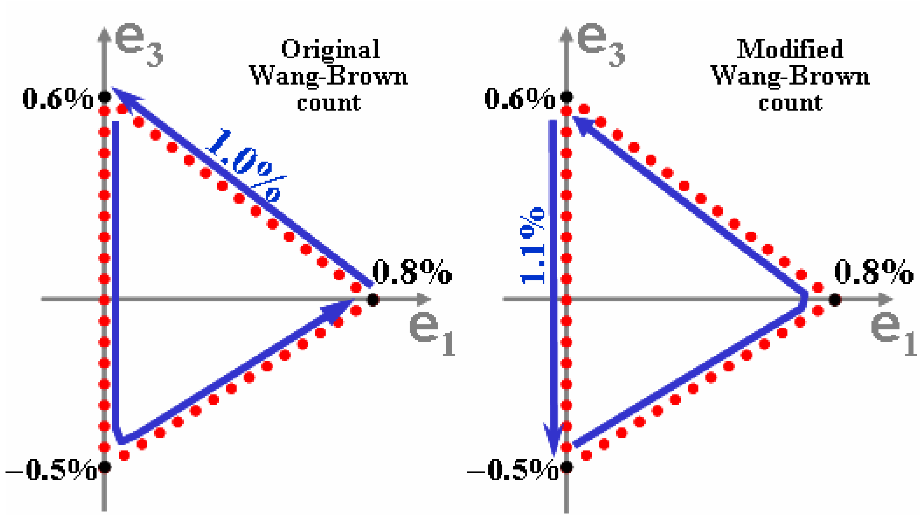

Fig. 1: Rainflow counts using the original Wang-Brown algorithm (left) and the modified version.

So, the first rule of the multiaxial rainflow count is now modified, to search for the pair of points in the diagram with largest relative distance, and between them the point P_1 farthest from the origin. But the Modified Wang-Brown (MWB) algorithm differs from the original method not only due to such first rule. Other rules are modified and introduced as well. The MWB method can be summarized by a set of 8 rules:

1. Find among the $n \cdot (n-1)/2$ pairs of points from an n-point path the one(s) that form the longest chord in the 5D strain (or stress) subspace, and choose among them the one with greatest distance from the origin; label this point P_1, and the subsequent P_2, ..., P_n following their original order;
2. Each count should be sequentially initiated at P_1, P_2, ..., P_i, ..., P_n;
3. The final point in each count is obtained when reaching:

   a) the point P_j farthest away (in an Euclidian sense) from the initial point P_i ($j > i$), or

   b) any finite segment (not just a point or a finite number of points) from a previous count;
4. Once found the initial and final points P_i and P_j, the count is defined by the traveled path portions closest to the straight segment P_i-P_j in an Euclidean sense (to avoid long "detours" from the straight line P_i-P_j that defines such half cycle);
5. Every time a full cycle is counted, i.e. two half cycles with identical extreme points are counted, use e.g. a convex hull method to calculate the equivalent strain (or stress) range or amplitude and mean or maximum from the full cycle to obtain the associated fatigue damage using some multiaxial model;
6. After rainflow counting the entire load history, repeat step 5 to calculate the damage contribution of the half cycles that did not close into a full cycle;
7. Use some damage accumulation rule, e.g., Miner's rule, to find the total multiaxial damage;
8. If using a critical plane approach, repeat steps 1-7 for every candidate plane, to find the critical plane that maximizes the accumulated multiaxial damage; note that only Case B cracks or tension-torsion histories will need a multiaxial rainflow count, because the single shear component in Case A cracks can be counted using a uniaxial rainflow algorithm.

5. COMPUTATIONAL IMPLEMENTATION OF THE MWB ALGORITHM

The practical implementation of the proposed MWB multiaxial rainflow count is described next, including a detailed description of its computational algorithm. During the execution of the algorithm, when a segment P_i-P_{i+1} is counted, totally or partially, an interpolation variable α_i ($1 \leq i \leq n$) is associated to it, such that $0 \leq \alpha_i \leq 1$. If the entire segment P_i-P_{i+1} has been counted, then $\alpha_i = 0$, otherwise α_i is computed from the intersection P_i' between P_i-P_{i+1} and the most recent count, see Fig. 2, using

$$\alpha_i = \frac{|P_i' - P_i|}{|P_{i+1} - P_i|} \tag{4}$$

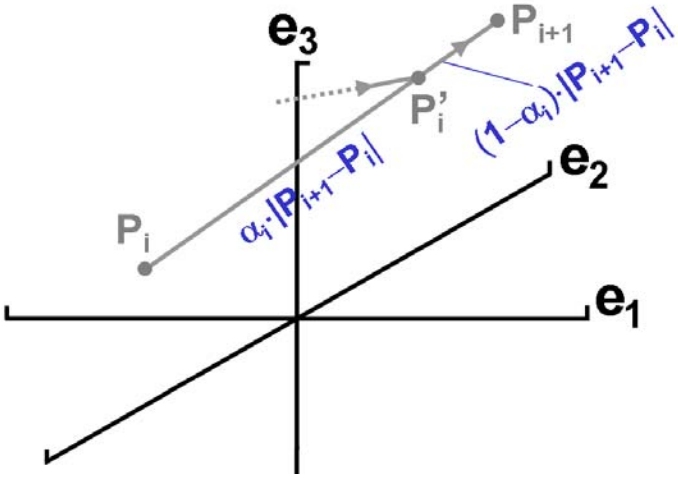

Fig. 2: Definition of the variable α_i that delimits the segment P_i'-P_{i+1} already accounted for and the segment P_i-P_i' that will still be counted by the algorithm.

In this way, a segment associated with $0 < \alpha_i < 1$ will have its segment $P_i'–P_{i+1}$ already counted, whereas the portion $P_i–P_i'$ is still available for future counts, where $P_i' = P_i + \alpha_i \cdot (P_{i+1} - P_i)$.

In the computational algorithm, all α_i are initialized with some value outside the interval $[0, 1]$ (e.g. $\alpha_i = -1$, for $i = 1, 2..., n$), to indicate that, initially, no path has been counted. Note that it is possible to have $\alpha_i = 1.0$ if a previous count crossed the segment exactly at P_{i+1}, which would create a stopping point for future counts, but without using up any portion of the segment $P_i–P_{i+1}$. But rule 3 above states that a point or a finite number of points previously counted cannot define the end of a count, therefore any $\alpha_i = 1.0$ must be reset to $\alpha_i = -1$ in the algorithm not to create a stopping point at P_{i+1} in this case. Note that in the algorithm the history paths are all assumed as formed by straight segments. If however the paths are curved, then they must be approximated by sufficiently refined polygonal paths.

Since the transformations that converted the stress and strain components into their deviatoric forms, as well as the transformations that projected them onto the reduced subspaces $E_{5\sigma}$ and $E_{5\varepsilon}$, are all linear (even for the elastoplastic case), the stresses and strains at a point P_i' in the straight segment P_i-P_{i+1} can be linearly calculated from α_i and the coordinates of points P_i and P_{i+1}. E.g., the projected deviatoric strain e_1 at point P_i' is simply $\varepsilon_{1,i} + \alpha_i \cdot (\varepsilon_{1,(i+1)} - \varepsilon_{1,i})$. This linearity simplifies very much the calculations in the proposed multiaxial rainflow algorithm, eliminating the need to calculate intersections between segments and ellipses, ellipsoids or hyper-ellipsoids, as it was necessary in the original Wang-Brown algorithm.

As mentioned before, the multiaxial rainflow count starts at each point P_i of the history, $i = 1, 2..., n$. The algorithm to perform the count from an initial point P_i is described next.

If the path P_i-P_{i+1} is already associated with a variable α_i different than -1, calculated during a previous count, then the count stops, and the stopping point will be $P_i' = P_i + \alpha_i \cdot (P_{i+1} - P_i)$. Otherwise, if the path P_i-P_{i+1} is not associated with any α_i different than -1, then this entire segment is counted and α_i is set to zero.

Next, the algorithm searches for the first point P_{j+1} ($j > i$) that has a greater or equal distance to P_{i+1} with respect to P_i. If it does not exist, then P_{i+1} will be the final point of the count. Otherwise, the intersection with the segment P_j-P_{j+1} is calculated at the point P_j' with same distance to point P_i as P_{i+1}, see Fig. 3. The value of α_j associated with point P_j' is obtained from Stewart's Theorem (Coxeter and Greitzer, 1967) applied to triangle $P_i–P_j–P_{j+1}$

$$b^2 \cdot [(1-\alpha_j)\cdot a] + c^2 \cdot [\alpha_j \cdot a] - p^2 \cdot a = [\alpha_j \cdot a] \cdot [(1-\alpha_j)\cdot a] \cdot a \tag{5}$$

where, b, c and p are defined in Fig. 3, resulting in

$$a^2 \cdot \alpha_j^2 + (c^2 - b^2 - a^2)\cdot \alpha_j + (b^2 - p^2) = 0 \tag{6}$$

$$\alpha_j = \frac{(a^2 + b^2 - c^2) \pm \sqrt{(a^2 + b^2 - c^2)^2 - 4a^2(b^2 - p^2)}}{2a^2} \tag{7}$$

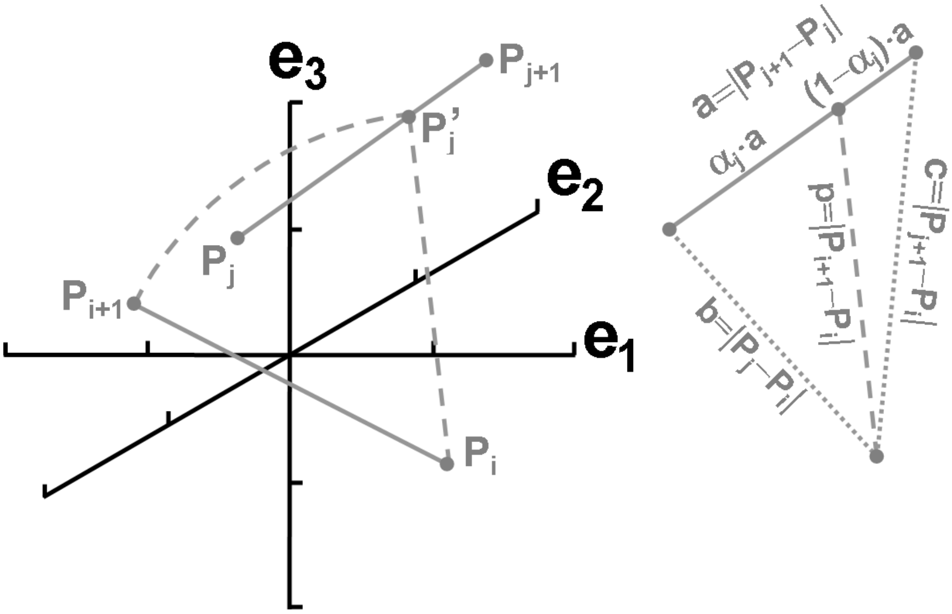

Fig. 3: Calculation of the intersection point P_j' between the current multiaxial rainflow count and the segment P_j-P_{j+1} in a subspace of $E_{5\varepsilon}$ (or $E_{5\sigma}$). Note that, usually, the triangles P_i-P_j-P_{j+1} and P_i-P_{i+1}-P_j' do not share the same plane.

The valid solution will be the lowest $\alpha_j = \alpha_j'$ (between the 2 solutions) that satisfies $0 \leq \alpha_j' \leq 1$. If the segment P_j-P_{j+1} was already associated to some $\alpha_j = \alpha_j^*$ (i.e., a portion from it had already been counted), then there are 2 hypotheses: (i) if $\alpha_j' < \alpha_j^*$, then the portion between α_j' and α_j^* is counted, the stopping point becomes the one associated with α_j^* (since it crossed a segment from a previous count), and $\alpha_j = \alpha_j'$ is stored, replacing α_j^*; or (ii) if $\alpha_j' \geq \alpha_j^*$, then the intersection point P_j' would be invalid since it would take place on a previously counted segment, therefore the stopping point must be set as P_{i+1}.

On the other hand, if no portion of the segment P_j–P_{j+1} had been accounted for (i.e., α_j was equal to -1), then the calculated α_j' is stored in α_j, counting then the segment P_j'–P_{j+1}. As mentioned before, if α_j results in 1.0 then it must be reset to $\alpha_j = -1$ not to create an unnecessary stopping point for future counts. This count continues from P_{j+1} in a similar way as the one coming from P_{i+1}. The algorithm then searches for the first point P_{m+1} ($m > j$) with a greater or equal distance than P_{j+1} with respect to P_i. If it does not exist, then P_{j+1} will be the final point of this count. Otherwise, the intersection with the segment P_m–P_{m+1} is calculated at a point P_m', with the same distance as P_{j+1} with respect to P_i. The value of $\alpha_m = \alpha_m'$ associated with P_m' is obtained applying Stewart's Theorem to the triangle P_i–P_m–P_{m+1}. The expression of α_m is analogous to the one obtained before for α_j, being enough to exchange P_j for P_m, P_{j+1} for P_{m+1}, and P_{i+1} for P_{j+1}.

The above procedure continues in a similar way. If the segment P_m–P_{m+1} was already associated to some $\alpha_m = \alpha_m^*$ different than -1, then there are 2 hypotheses: (i) if $\alpha_m' < \alpha_m^*$, then the segment bounded by the points associated with α_m' and α_m^* is counted, the stopping point is associated with α_m^*, and $\alpha_m = \alpha_m'$ is stored, replacing α_m^*; or (ii) if $\alpha_m' \geq \alpha_m^*$, then the intersection point P_m' would be invalid and the stopping point is P_{j+1}. If no portion of the path P_m–P_{m+1} had been counted, then the value $\alpha_m = \alpha_m'$ is stored, the segment P_m'–P_{m+1} is counted, and the count continues from P_{m+1}. The algorithm then searches for the first point P_{r+1} ($r > m$) with a longer distance than P_{m+1} with respect to P_i, and so on.

The algorithm continues until the stopping point for the count started at P_i is found. The linearity of the adopted transformations allows all resulting stresses and strains at any intersection point to be obtained from a simple interpolation involving the α_i coefficients.

The entire process is performed for all starting points P_i ($i = 1, 2..., n$). Note that a count can stop at point P_1 if the history is periodic, in which case the segment P_n–P_1 exists and it cannot be left out. At the end of the algorithm, all segments will be associated with values $\alpha_i = 0$, indicating that all of them were entirely counted. Figure 4 shows the flow chart of the entire MWB algorithm.

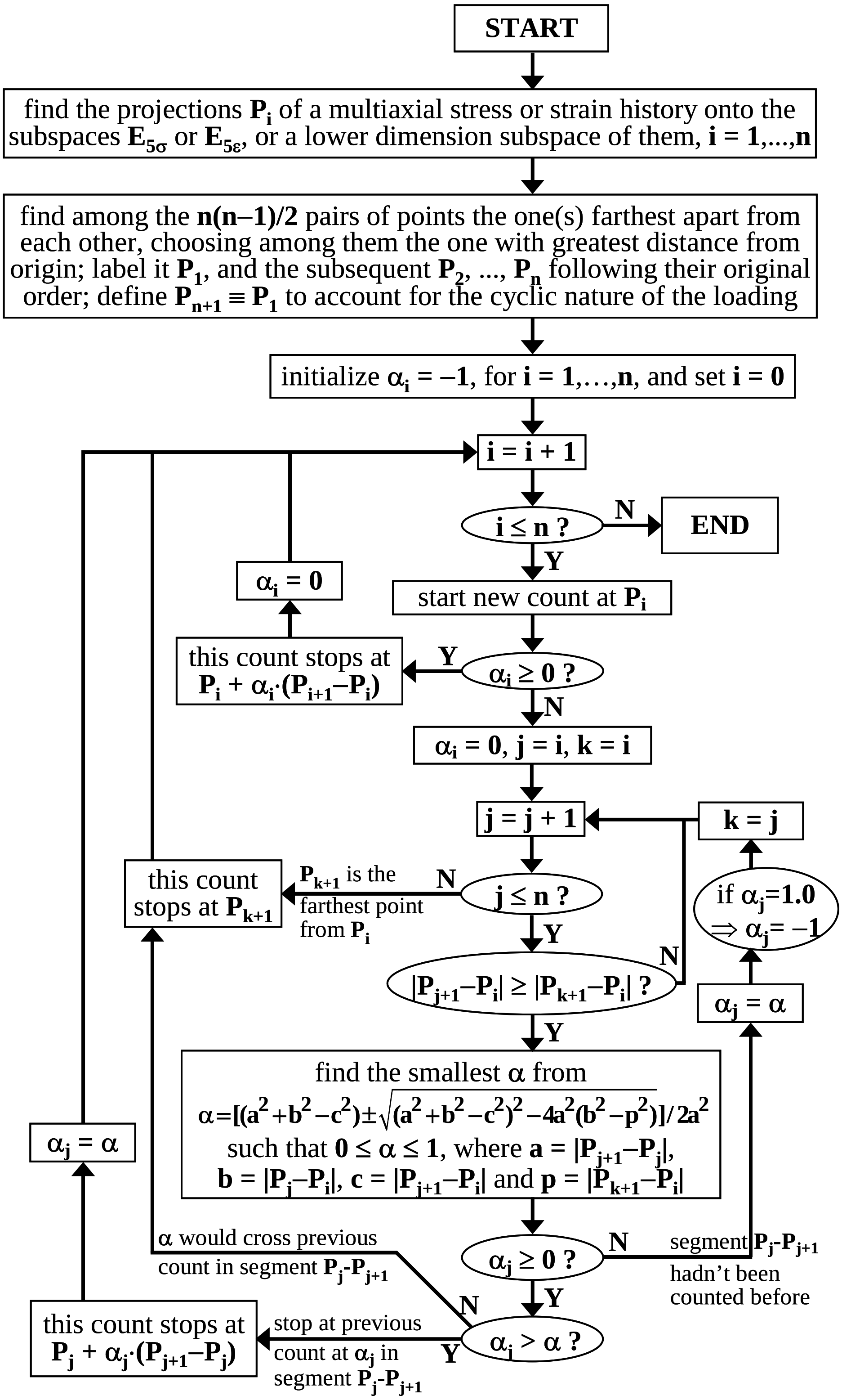

Fig. 4: Flow chart of the proposed Modified Wang-Brown (MWB) algorithm.

6. CONCLUSIONS

Wang and Brown proposed a multiaxial rainflow count based on the relative Mises strain ε_{RMises} as an indirect measure of the damage during a half cycle. But the original method requires the calculation of ε_{RMises} at every rainflow count for all subsequent starting points. In this work, a Modified Wang Brown (MWB) rainflow counting method was proposed, based on the representation of the stress or strain history in a reduced 5D subspace of the 6 deviatoric strain (or stress) components. The MWB uses improved rules to guarantee that the event with highest ε_{RMises} is always counted.

Coupled with some convex hull method, the MWB can better account for the path shape influence on the associated fatigue damage. The method has simple geometric interpretations that considerably simplify its implementation, e.g. the distance between 2 points in the considered deviatoric strain subspace is the ε_{RMises} between them. The computational implementation of the algorithm was discussed in detail, including a flow chart with all necessary steps.

7. ACKNOWLEDGEMENTS

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