SHORT CRACKS INFLUENCE ON FATIGUE LIMITS

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Abstract. *Structural components designed for very long fatigue lives should avoid fatigue crack initiation AND be tolerant to undetectable short cracks. But this requirement is still not used in fatigue design routines, which just intend to maintain the critical point loading below its fatigue limit. Nevertheless, most long-life designs work just fine, thus they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by SN procedures alone. This problem can only be solved by adding a proper fatigue crack propagation threshold requirement to the "infinite" life design criterion. This paper evaluates the tolerance to short 1D and 2D cracks, and proposes a design criterion for infinite fatigue life which explicitly considers it.*

Keywords*: short cracks, fatigue limits, crack tolerance*

1. INTRODUCTION

The notch sensitivity $0 \leq q \leq 1$ correlates the linear elastic (LE) stress concentration factor (SCF) $K_t = \sigma_{max}/\sigma_n$, to its corresponding fatigue SCF, $K_f = 1 + q \cdot (K_t - 1) = S_L/S_{Lntc}$, where S_L and S_{Lntc} are the fatigue limits measured on standard (smooth and polished) and on notched test specimens (TS), usually under fully alternated loads. But these limits can be defined for any $R = \sigma_{min}/\sigma_{max}$ ratio, $S_L(R)$ and $S_{Lntc}(R)$. Since q is associated with the generation of tiny non-propagating fatigue cracks when $S_L/K_t < \sigma_n < S_L/K_f$, it can be predicted from their fatigue behavior (Castro and Meggiolaro, 2009). It is the stress gradient around notch roots that controls the fatigue crack propagation (FCP) behavior of short cracks emanating from them. For any given material, q depends not only on the notch tip radius ρ, but also on its depth b, meaning that shallow and elongated notches of same radius ρ may have quite different sensitivities q. Note that "short crack" here means "mechanical" not "microstructural" short crack, since material isotropy is assumed in their modeling, a simplified hypothesis experimentally corroborated (Meggiolaro *et al.*, 2007, Wu *et al.*, 2010).

The FCP threshold of short cracks must be smaller than the long crack threshold $\Delta K_{th}(R)$, or else the stress range $\Delta\sigma$ required to propagate them would be higher than the fatigue limit $\Delta S_L(R)$. Indeed, if FCP is controlled by the stress intensity factor (SIF) range, $\Delta K \propto \Delta\sigma\sqrt{\pi a}$, and if short cracks with $a \to 0$ had the same $\Delta K_{th}(R)$ of long cracks, then their propagation by fatigue would require $\Delta\sigma \to \infty$ (Lawson *et al.*, 1999). The FCP threshold of short fatigue cracks under pulsating loads $\Delta K_{th}(a, R = 0)$ can be modeled using their ETS characteristic size a_0 (El Haddad-Topper-Smith 1979) estimated from $\Delta S_0 = \Delta S_L(R = 0)$ and $\Delta K_0 = \Delta K_{th}(R = 0)$. This clever trick reproduces the Kitagawa-Takahashi (1976) plot trend, using a modified SIF range $\Delta K'$ to describe the fatigue propagation of any crack, short or long,

$$\Delta K' = \Delta\sigma\sqrt{\pi(a + a_0)}\text{, where } a_0 = (1/\pi)(\Delta K_0/\Delta S_0)^2 \tag{1}$$

This a_0 trick reproduces the expected limits $\Delta K_{th}(a \to \infty) = \Delta K_0$ and $\Delta\sigma(a \to 0) = \Delta S_0$. Knowing that steels typically have $6 < \Delta K_0 < 12MPa\sqrt{m}$, ultimate tensile strength $400 < S_U < 2000MPa$, and fatigue limit $200 < S_L < 1000MPa$ (very clean high-strength steels tend to maintain the $S_L/S_U \cong 0.5$ trend of lower strength steels under $R = -1$); and estimating by Goodman the pulsating ($R = 0$) fatigue limit as $\Delta S_0 = 2S_U S_L/(S_U + S_L) \Rightarrow 260 < \Delta S_0 < 1300MPa$; it can then be expected that the maximum a_0 range for steels should be

$$(1/\pi)\left(\Delta K_{0_{min}}/\Delta S_{0_{max}}\right)^2 \cong 7 < a_0 < 700\mu m \cong (1/\pi)\left(\Delta K_{0_{max}}/\Delta S_{0_{min}}\right)^2 \tag{2}$$

This a_0 range may be overestimated, since ΔK_{0min} is not necessarily associated with ΔS_{0max}, neither is ΔK_{0max} always associated with ΔS_{0min}. But it justifies the "short crack" denomination used for cracks of a similar small size, and shows that cracks up to a few millimeters may still behave as short cracks in some steels, meaning they may have a smaller propagation threshold than that measured with long crack, which have $a >> a_0$. Since the strengths of typical aluminum alloys are $70 < S_U < 600MPa$, $30 < S_L < 230MPa$, $40 < \Delta S_0 < 330MPa$, and $1.2 < \Delta K_0 < 5MPa\sqrt{m}$, their maximum a_0 (over)estimated range, and thus their short crack influence scale, is wider than the steels range, $\sim 1\mu m < a_0 < \sim 5mm$.

As $\Delta K'$ has been deduced using Griffith's plate SIF, $\Delta K = \Delta\sigma\sqrt{\pi a}$, Yu *et al* (1988) used the non-dimensional geometry factor $g(a/w)$ from the SIF expression $\Delta K = \Delta\sigma\sqrt{\pi a}\cdot g(a/w)$ to deal with other geometries, re-defining

$$\Delta K' = g(a/w)\cdot\Delta\sigma\sqrt{\pi(a + a_0)}\text{, where } a_0 = (1/\pi)\left[\Delta K_0/\left(g(a/w)\cdot\Delta S_0\right)\right]^2 \tag{3}$$

Tolerable stress ranges $\Delta\sigma_0$ tend to the fatigue limit ΔS_0 when $a \to 0$ only if $\Delta\sigma$ is the range at the notch root, instead of the nominal one. But geometry factors $g(a/w)$ listed in SIF tables usually use $\Delta\sigma$ instead of $\Delta\sigma_n$ as their nominal stress. A clearer way to define a_0 is to explicitly recognize this practice, separating the geometry factor $g(a/w)$ into two parts: $g(a/w) = \eta \cdot \varphi(a)$, where $\varphi(a)$ depends on the stress gradient ahead of the notch tip, which departs from the notch SCF as the crack length $a \to 0$, whereas η encompasses all the remaining terms, such as the free surface correction:

$$\Delta K' = \eta \cdot \varphi(a) \cdot \Delta\sigma\sqrt{\pi(a+a_0)}\,, \quad \text{where } a_0 = (1/\pi)\left[\Delta K_0 / (\eta \cdot \Delta S_0)\right]^2 \tag{4}$$

The short crack problem can be clearly modeled by letting the SIF range retain its original equation, while the FCP threshold expression is modified to become a function of the crack length a, namely $\Delta K_0(a)$, resulting in

$$\Delta K_0(a) = \Delta K_0 \cdot \sqrt{a/(a+a_0)} \tag{5}$$

The ETS equation is one possible asymptotic match between the short and long crack behaviors. Following Bazant's (1977) reasoning, a more general equation can be used introducing an adjustable parameter γ to fit experimental data

$$\Delta K_0(a) = \Delta K_0 \cdot \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{6}$$

Equations (1-5) result from (6) if $\gamma = 2.0$. The limits $\Delta\sigma(a \le a_0) = \Delta S_0$ for short cracks and $\Delta K_0(a \ge a_0) = \Delta K_0$ for long ones are obtained when $g(a/w) = \eta \cdot \varphi(a) = 1$ and $\gamma \to \infty$. Most short crack FCP data is fitted by $\Delta K_0(a)$ curves with $1.5 \le \gamma \le 8$, but $\gamma = 6$ better reproduces classical Peterson q-plots based on fatigue data obtained by testing TS with semi-circular notches (Castro and Meggiolaro, 2009). Using (6) as the FCP threshold, then any crack departing from a free smooth or notched surface under pulsating loads should propagate if

$$\Delta K = \eta \cdot \varphi(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} > \Delta K_0(a) = \Delta K_0 \cdot \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{7}$$

where $\eta = 1.12$ is the free surface correction. As fatigue depends on two driving forces, the stress range $\Delta\sigma$ and its peak σ_{max}, (7) can be extended to consider σ_{max} (indirectly modeled by the R-ratio) influence in short crack behavior. First, the short crack characteristic size should be defined using the FCP threshold for long cracks $\Delta K_R = \Delta K_{th}(a >> a_R, R)$, and the fatigue limit ΔS_R, both measured or properly estimated at the desired R-ratio, then

$$a_R = (1/\pi)\left[\Delta K_R / (1.12 \cdot \Delta S_R)\right]^2 \tag{8}$$

Likewise, the corresponding short crack FCP threshold should be re-written as

$$\Delta K_R(a) = \Delta K_R \cdot \left[1 + (a_R/a)^{\gamma/2}\right]^{-1/\gamma} \tag{9}$$

All these details are important when such models are used to make predictions in real life situations, as they do influence the calculation results. In particular, neglecting the σ_{max} effect on fatigue can lead to severe non-conservative life estimations, a potentially dangerous practice unacceptable for design or analysis purposes.

2. BEHAVIOR OF SHORT CRACKS WHICH DEPART FROM ELONGATED NOTCHES

An estimate for the SIF of a small crack a departing from an Inglis plate elliptical notch tip, with semi-axes $b >> a$ and c, and root radius $\rho = c^2/b$, is $K_I(a) \cong \sigma_n \cdot \sqrt{(\pi a)} \cdot f_1(a, b, c) \cdot f_2(\text{free surface})$. The $2b$ axis is centered at the x co-ordinate origin, σ_n is the nominal stress (perpendicular to a and b); $f_1(a, b, c) \cong \sigma_y(x)/\sigma_n$; $\sigma_y(x)$ is the stress at $(x = b + a, y = 0)$ ahead of the notch tip when there is no crack; and $f_2 = 1.12$. $\sigma_y(x = b + a, y = 0)$ is given by (Schijve, 2001):

$$f_1 = \frac{\sigma_y(x, y=0)}{\sigma_n} = 1 + \frac{(b^2 - 2bc)(x - \sqrt{x^2 - b^2 + c^2})(x^2 - b^2 + c^2) + bc^2(b-c)x}{(b-c)^2(x^2 - b^2 + c^2)\sqrt{x^2 - b^2 + c^2}} \tag{10}$$

The slender the elliptical notch is, the higher is its SCF. But high K_t imply in steeper stress gradients $\partial\sigma_y(x, y = 0)/\partial x$ around notch tips, since elliptical holes LE SCF for drop from $K_t = 1 + 2b/c = 1 + 2\sqrt{(b/\rho)} = \sigma_y(1)/\sigma_n \ge 3$ at their tip to about $1.82 < K_{1.2} = \sigma_y(1.2)/\sigma_n < 2.11$ (for $b \ge c$) at a point just $b/5$ ahead of it, meaning their Saint Venant's controlling distance is associated with their depth b, not with their tip radii ρ . This is the cause for the peculiar growth of short cracks which depart from elongated notch roots. Their SIF, which should tend to increase with their length $a = x - b$, may instead decrease after they grow for a short while because the SCF effect in $K_I \cong 1.12 \cdot \sigma_n \sqrt{(\pi a)} \cdot f_1$ may diminish sharply due the high stress drop close to the notch tip, overcompensating the crack growth effect. This $K_I(a)$ estimate can be used to evaluate non-propagating fatigue cracks tolerable at notch roots, using the short crack FCP behavior. For analysis purposes, the SIF range of a single crack with length *a* emanating from a semi-elliptical notch with semi-axes *b* (co-linear to *a*) and *c* at the edge of a very large plate loaded in mode I can be written as

$$\Delta K_I = \eta \cdot F\left(a/b, c/b\right) \cdot \Delta\sigma\sqrt{\pi a} \tag{11}$$

where η = 1.12, and $F(a/b, c/b)$ can be expressed as a function of the dimensionless parameter $s = a/(b + a)$ and of the notch SCF, given by

$$K_t = \left[1 + 2\left(b/c\right)\right] \cdot \left\{1 + \left[0.12 \Big/ \left(1 + c/b\right)^{2.5}\right]\right\} \tag{12}$$

To obtain expressions for *F*, extensive finite element calculations were performed for several cracked semi-elliptical notches. The numerical results, which agreed well with standard solutions (Tada *et al.*, 1985), were fitted within 3% using empirical equations (Meggiolaro *et al.*, 2007, Wu *et al.*, 2010)

$$F\left(a/b, c/b\right) \equiv f\left(K_t, s\right) = K_t\sqrt{\left[1 - exp\left(-sK_t^2\right)\right] \Big/ sK_t^2}\ ,\ c \le b \text{ and } s = a/(b + a) \tag{13}$$

$$F'\left(a/b, c/b\right) \equiv f'\left(K_t, s\right) = K_t\left[1 - exp\left(-K_t^2\right)\right]^{-s/2}\sqrt{\left[1 - exp\left(-sK_t^2\right)\right] \Big/ sK_t^2}\ ,\ c \ge b \tag{14}$$

The SIF expressions include the semi-elliptical notch effect through *F* or *F'*. Indeed, as $s \to 0$ when $a \to 0$, the maximum stress at its tip $\sigma_{max} \to F(0, c/b) \cdot \sigma_n = K_t \cdot \sigma_n$. Thus, the η-factor, but not the $F(a/b,c/b)$ part of K_I, should be considered in the short surface crack characteristic size a_0, as done in equation (3). Note also that the semi-elliptical K_t includes a term $[1 + 0.12/(1 + c/b)^{2.5}]$ which can be interpreted as a free surface correction (FSC), since as $c/b \to 0$ and the semi-elliptical notch tends to a crack, its $K_t \to 1.12 \cdot 2\sqrt{(b/\rho)}$. Such *1.12* factor is the notch FSC, not the crack FSC η. Indeed, when $c/b \to 0$, this 1.12 factor disappears from the *F* expression, which gives $F(a/b, 0) = 1/\sqrt{s}$, and therefore $\Delta K_I = \eta \cdot F \cdot \Delta\sigma \cdot [\pi \cdot a]^{0.5} = \eta \cdot \Delta\sigma \cdot [\pi \cdot (a + b)]^{0.5}$, as expected, since the resulting crack for $c \to 0$ would have length $a + b$.

Traditional *q* estimates (Peterson, 1974) assume it depends only on the notch root ρ and on the material strength S_U. Thus, materials with same S_U but different ΔK_0 should have identical notch sensitivities. But whereas good empirical relations relate the fatigue limit ΔS_0 to the tensile strength S_U of many materials, there are no such relations between their FCP threshold ΔK_0 and S_U. Moreover, traditional *q* estimation for elongated notches by the procedures can generate unrealistic high K_f values. In conclusion, such traditional estimates should not be taken for granted.

The proposed model, on the other hand, is based on the FCP mechanics of short cracks which depart from elliptical notch roots, recognizing that their *q* values are associated with their tolerance to non-propagating cracks. It shows that their notch sensitivities, besides depending on ρ, ΔS_0, ΔK_0 and γ, are also strongly dependent on their shape, given by their *c/b* ratio. Their corresponding Peterson's curve is well approximated by the semi-circular $c/b = 1$ notch, but this curve is ***not*** applicable for much different c/b ratios. Therefore, the proposed predictions indicate that these traditional notch sensitivity estimates should ***not*** be used for elongated notches (Castro and Meggiolaro, 2009, Meggiolaro *et al.*, 2007), an issue experimentally verified by Wu *et al.* (2010), as discussed in the following section.

3. EXPERIMENTAL VERIFICATION OF ELONGATED NOTCH SENSITIVITY PREDICTIONS

Fatigue tests were carried out on modified SE(T) specimens of thickness $t = 8mm$ and width $w = 80mm$, to find the number of cycles required to re-initiate the crack after drilling a stop-hole of radius ρ centered at its tip, generating an elongated slit with $b = 27.5mm$. These tests can also be used to check the model proposed to describe short crack FCP behavior. They are very briefly described here, but details are available in Wu *et al.* (2010). The TS were made from an Al alloy 6082 T6, with $S_Y = 280MPa$, $S_U = 327MPa$, and Young's modulus $E = 68GPa$. The particularly careful tests were made at 30Hz under fixed load range at $R = 0.57$, to avoid any crack closure influence on their FCP behavior. The TS were first pre-cracked until reaching the required crack size. Then they were removed to introduce the stop-holes in a milling machine, using a slight under-size drill precisely centered at their crack tips. Finally, the holes were enlarged to reach their final diameter using a reamer. The stop-hole sizes were large enough to remove the previous plastic zones.

After the stop-hole repair, the fatigue crack re-initiation lives at the tip of the resulting elongated notch can be modeled by εN procedures using (i) the alloy parameters $\sigma'_f = 485$MPa, $b = -0.0695$, $\varepsilon'_f = 0.733$ and $c = -0.827$, and Ramberg-Osgood's coefficient and exponent of the cyclic stress-strain curve, $H = 443$MPa and $h = 0.064$ (Borrego et al, 2003); (ii) the nominal stress range and R-ratio; and finally (iii) the stress concentration factor of the notches generated after repairing the cracks by a stop-hole at their tips, which can be calculated by FE ($K_t = 11.8$, 8.1, and 7.6 for the 3 stop-hole radii, $\rho = 1$, 2.5, and 3 mm.)

The stress and strain maxima and ranges at the stop-hole tip can be calculated by Neuber's rule, and used to estimate crack re-initiation lives by a $\Delta\varepsilon{\times}N$ rule, considering the mean loads influence. Neglecting it could lead to severely non-conservative predictions, due to the high R-ratio used. Lives predicted for the two larger holes reproduced reasonably well the tests results, but for the smaller $\rho = 1mm$ hole they turn out to be too conservative. Few mechanical reasons can explain this. One would be significant compressive residual stresses at the $\rho = 1mm$ stop-hole tips. But all stop-holes were identically drilled and reamed, to remove their previous crack tip plastic zones. Hence, it is difficult to justify why high compressive residual stresses would be present only at the $\rho = 1mm$ stop-hole roots. The same occur with their surface finish. However, the smaller stop-holes generate elongated notches with a larger K_t, thus with a much steeper stress gradient near their roots. This effect can significantly affect the growth of short cracks and, consequently, the stop-hole fatigue notch sensitivity, possibly providing a sound mechanical explanation for the measured behavior.

Indeed, when using K_f instead of K_t with the traditional εN procedures, calculating the elongated notch sensitivity q by the method proposed here, all the estimated fatigue crack re-initiation lives reproduce quite well the measured results. The Al 6082 T6 fatigue limit and fatigue crack propagation threshold under pulsating loads required to calculate K_f are estimated as $\Delta K_0 = 4.8\ MPa\sqrt{m}$ and $\Delta S_0 = 110MPa$, following traditional structural design practices. The γ exponent was chosen as $\gamma = 6$, as recommended by Castro and Meggiolaro (2009). Figure 1 presents the lives predicted by Morrow's equation and by Smith-Watson-Topper (SWT), which reproduce well the measured data for the $\rho = 1mm$ hole. Note that the term "prediction" can in fact be used here, since the curves result from re-initiation life estimations calculated using material properties, without considering any of the measured data points.

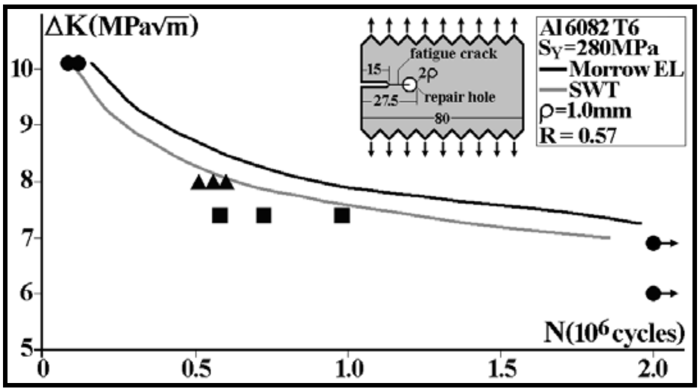

Figure 1: Predicted and measured crack re-initiation lives after introducing stop-holes with radii $\rho = 1.0mm$ at the tip of the previous crack, using the properly calculated K_f of the resulting elongated slit (instead of its K_t) and εN procedures.

4. A CRITERION TO DEFINE FUNCTIONALLY ADMISIBLE SHORT CRACKS

This same methodology can be used to generate an acceptance criterion for small cracks. Since most long-life designs work well, structural components are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by SN or εN procedures alone. Such problem can be avoided by adding equations (7-9) to the "infinite" life design criterion to tolerate a crack of size a. In its simplest version, this criterion should then be written as

$$\Delta\sigma < \Delta K_R \Big/ \left\{ \sqrt{\pi a} \cdot g(a/w) \cdot \left[1 + (a_R/a)^{\gamma/2} \right]^{1/\gamma} \right\}, \text{ where } a_R = (1/\pi) \cdot [\Delta K_R / \eta \Delta S_R]^2 \tag{15}$$

As fatigue limits ΔS_R considers microstructural defects inherent to the material, (15) complements it considering the component tolerance to cracks. A simple case study can clarify how useful this concept can be, as discussed next.

It was needed to estimate how short cracks affected tolerable stresses under uniaxial fatigue loads of a component with $2mm$ by $3.4mm$ rectangular cross section; measured fatigue limit $S_L(R = -1) = 246MPa$; and $S_U = 990MPa$. Note that as $S_L \cong S_U/4$, it should include surface roughness effects which should not affect the cracks. But, in the absence of reliable information, the only safe option is to use the measured S_L value to estimate S_R and a_R. Therefore, by Goodman

$$S_R = \left[S_L S_U (1-R) \right] \Big/ \left[S_U(1-R) + S_L(1+R) \right] \tag{16}$$

The mode I stress range $\Delta\sigma$ tolerable by this component when it has a uniaxial surface crack of depth a is then

$$\Delta\sigma < \frac{\Delta K_R / \varphi_F}{\sqrt{\pi a}\ [\, 0.752 + 2.02\frac{a}{w} + 0.37(1 - \sin\frac{\pi a}{2w})^3\,]\ \sec\frac{\pi a}{2w}\sqrt{\frac{2w}{\pi a}\tan\frac{\pi a}{2w}} \cdot \left[1 + \left(\frac{a_R}{a}\right)^{\gamma/2} \right]^{1/\gamma}} \tag{17}$$

where $g(a/w)$ was obtained from Tada et al (1985). Figure 2 plots the maximum tolerable stress (assuming $\phi_F = 1$) for several R-ratios. As the FCP threshold was not available, it had to be estimated. The typical threshold range for steels is $6 < \Delta K_0 < 12MPa\sqrt{m}$. It is usual to assume $\Delta K_R \cong \Delta K_0$ for $R < 0$ loads. Lower limit estimations for positive R are $\Delta K_{th}(0 < R \leq 0.17) = 6MPa\sqrt{m}$, and $\Delta K_{th}(R > 0.17) = 7\cdot(1 - 0.85R)$ (Castro and Meggiolaro, 2009). Using $\eta = 1.12$ and $\Delta K_0 = 6MPa\sqrt{m}$, $a_0 = 59\mu m$. Figure 2 shows that if this piece works e.g. under $\Delta\sigma = 286MPa$ and $R = -0.12$, it tolerates cracks up to $a \cong 105\mu m$, and if it works under $\Delta\sigma = 176MPa$ and $R = 0.44$, it can sustain cracks up to $a \cong 150\mu m$.

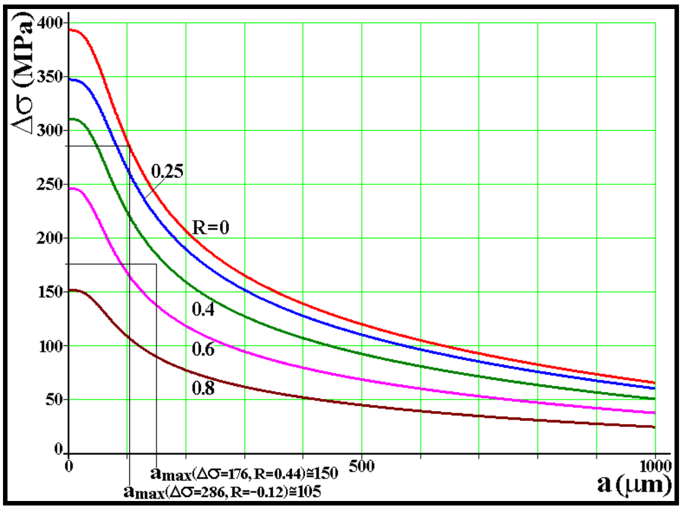

Figure 2: Surface crack of size a effect in the largest stress range $\Delta\sigma_R(a)$ tolerable by a strip of width $w = 3.4mm$ loaded in mode I, for various R-ratios (supposing $\Delta K_0 = 6MPa\sqrt{m}$ and $\gamma = 6$, thus $a_0 = 59$ and $a_{0.8} = 55\mu m$).

Therefore, this model indicates that this piece is not too tolerant to 1D surface cracks. But as this conclusion is based on estimated properties, Figure 3 studies its sensibility to the assumed values. Equation (15) assumes that the short crack is 1D and grows without changing its original plane, and this model describes the behavior of macroscopically short cracks, as it uses macroscopic material properties. Thus it can only be applied to short cracks which are large in relation to the characteristic size of the intrinsic material anisotropy (e.g. its grain size). Smaller cracks grow inside an anisotropic and usually inhomogeneous scale, thus their FCP is also affected by microstructural barriers, such as second phase particles or grain boundaries. However, as grains cannot be mapped in most practical applications, such problems, in spite of their academic interest, are not really a major problem from the fatigue design point of view.

However, this model has another limitation: it assumes that the short crack can be completely characterized by its depth a. But most short cracks are surface or corner cracks, which tend to grow by fatigue at least in two directions, maintaining their original plane when they are loaded under pure mode I conditions. In these cases, they can be modeled as bidimensional (2D) cracks which grow both in depth and width. In reality, both long and short cracks (these meaning cracks not much larger than a_R) only behave as 1D cracks after having cut all the component width to become a through crack, with a more or less straight front which propagates in an approximately uniform way. Thus, equation (17) must be adapted to consider the influence of 2D short cracks in the fatigue limit.

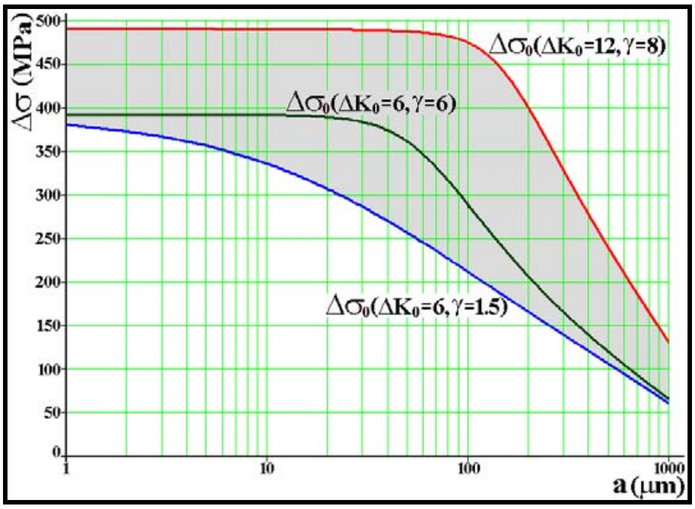

Figure 3: Typical steel threshold $6 < \Delta K_0 < 12MPa\sqrt{m}$ and γ exponent $1.5 < \gamma < 8$ ranges influence in the largest mode I stress ranges $\Delta\sigma_0$ tolerated by the $w = 3.4mm$ strip, as a function of the 1D superficial crack size a.

This can be done by assuming that: (i) the cracks are loaded in pure mode I, under quasi-constant $\Delta\sigma$ and R conditions, with no major overloads; (ii) material properties measured (or estimated) testing 1D specimens may be used to simulate the FCP behavior of 2D cracks; and (iii) 2D surface or corner cracks can be well modeled as having an approximately elliptical front, thus their SIF can be described by the classical Newman-Raju equations (1984). In this case, it can be expected that the component tolerance to cracks be given by:

$$\Delta\sigma < \begin{cases} \Delta K_R \Big/ \left\{ \sqrt{\pi a} \cdot \Phi_a(a,c,w,t) \cdot \left[1 + (a_R/a)^{\gamma/2} \right]^{1/\gamma} \right\} \\ \Delta K_R \Big/ \left\{ \sqrt{\pi c} \cdot \Phi_c(a,c,w,t) \cdot \left[1 + (a_R/c)^{\gamma/2} \right]^{1/\gamma} \right\} \end{cases} \tag{18}$$

For semi-elliptical or quart-elliptical surface cracks in a plate of thickness t, the SIF in the semi-axis directions, or in the depth a and width c directions, $K_{I,a} = \sigma\sqrt{(\pi a)}\cdot\Phi_a$ and $K_{I,c} = \sigma\sqrt{(\pi c)}\cdot\Phi_c$, are given by complicated functions, which enhance the operational advantage of treating the FCP threshold as a function of the crack size, $\Delta K_{th}(a)$, as claimed above. For structural calculations and design purposes, it is indeed relatively simple to use either equation (15) or (18) to evaluate the influence of surface cracks on the component fatigue strength. Moreover, it is not too difficult to adapt the 2D equations to include notch effects. Φa and Φc expressions are reproduced in Castro and Meggiolaro (2009).

5. CONCLUSIONS

A generalized El Haddad-Topper-Smith's parameter was used to model the threshold stress intensity range for short cracks dependence on the crack size, as well as the behavior of non-propagating fatigue cracks. This dependence was used to estimate the notch sensitivity factor q of semi-elliptical notches, from studying the propagation behavior of short non-propagating cracks that may initiate from their tips. The predicted notch sensitivities reproduced well the classical Peterson's q estimates for circular holes or approximately semi-circular notches, but it was found that the notch sensitivity of elongated slits has a very strong dependence on the notch aspect ratio, defined by the ratio c/b of the semi-elliptical notch that approximates the slit shape having the same tip radius. These predictions were confirmed by experimental measurements of the re-initiation life of long fatigue cracks repaired by introducing a stop-hole at their tips, using their calculated K_f and appropriate εN procedures. Based on this promising performance, a criterion to evaluate the influence of small or large surface cracks in the fatigue resistance was proposed.

6. ACKNOWLEDGEMENTS

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