Parallel mechanisms controlled by non-conventional control strategies

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Abstract: Many types of parallel mechanisms used as motion simulators, robotic manipulators or in test machines use the information generated by the displacement transducers coupled on its limbs to implement a closed-loop control. However, these transducers have higher costs and more influence over the system (due to the addition of mass and inertia) than inertial transducers. Thus, in this work a control strategy based on non-conventional methods applied in two types of parallel mechanisms are analyzed. This non-conventional method consists in not using the information of the controlled variable (the displacement of the linear actuator), but other information of the mechanism (linear accelerations and angular speeds of the moving platform) in order to implement the control strategy.

Keywords: parallel mechanisms, direct kinematics, control strategy

NOMENCLATURE

DOF = degree of freedom, dimensionless
u = horizontal coordinate (perpendicular to the plane vw) on frame B, mm
v = horizontal coordinate (perpendicular to the plane uw) on frame B, mm
w = vertical coordinate on frame B, mm
x = horizontal coordinate (perpendicular to the plane yz) on frame A, mm
y = horizontal coordinate (perpendicular to the plane xz) on frame A, mm
z = vertical coordinate on frame A, mm
s = Laplace variable, s^{-1}
K = controller gain
k = valve constant
b = coefficient of friction, kgf/s
p = pressure, Pa
q = volumetric flow rate, mm^3/s
m = mass, kg
g = acceleration of gravity, mm/s^2
t = time, s
n = matrix line
m = matrix column

Greek Symbols

φ = angle of pitch of the moving platform, rad
θ = angle of roll of the moving platform, rad
ψ = angle of yaw of the moving platform, rad

Subscripts

B relative to frame B
d relative to the desired trajectory
i relative to the *ith* limb
u relative to the coordinate u
1 relative to limb 1
2 relative to limb 2
3 relative to limb 3
4 relative to limb 4
5 relative to limb 5
6 relative to limb 6

Superscripts

A relative to frame A

INTRODUCTION

Compared to serial mechanisms, parallel mechanisms offer more stiffness, higher load-weight ratios and more uniform load distribution. Thereat this kind of mechanism is often used in motion simulators, robot manipulators, test machines and vibration control systems. Normally it consists of a moving platform connected to a fixed base (or more than one fixed base) by links driven by linear or rotary actuators. Rotary, universal or spherical joints (depending on the architecture of the mechanism) make these connections between the base and links and between the links and platform.

The inverse kinematics of a mechanism consists of obtaining the variables associated with the actuator displacements given the position and orientation of the moving platform. The inverse dynamics consists of obtaining the actuator forces given the forces and moments applied by the moving platform (Tsai, L. W., 1999). The direct kinematics and the direct dynamics are complementary. In parallel manipulators, the inverse kinematic is obtained straightly by a simple sum of vectors (Wang, Y., 2009). However, the direct kinematics is much more difficult to obtain. Depending on the complexity of the manipulator, the system of equations of its kinematics could have many solutions (more than 40 in the case of a 6 DOF parallel mechanism; Tsai, L. W., 1999). Many scientists are working towards the solution of the problems generated by this complexity. To solve the direct kinematics and dynamics problem, numerical solutions based on Newton-Raphson, Genetic Algorithms and other methods have been developed (Serrano, F. *et al*, 2007).

Objectives

This work aims on the analysis of a method to obtain the direct kinematics of two types of parallel mechanisms: a 3 DOF planar parallel mechanism and a Stewart Platform. In the first one, an analytical model is studied. The second model is quasi-analytical, since it uses the numerical inverse of the analytical jacobian matrix of the mechanism. This work also aims to find a control strategy based on the measure of other variables that describe the movement of the mechanism than the actuated variables. For example, in the Stewart Platform the measurement of the linear accelerations and the angular speeds of the moving platform are obtained, instead of the commonly used linear actuator displacements.

Motivation

Displacement transducers are generally more expensive, have more inertia and are much heavier than an Inertial Measurement Unit (IMU). An IMU is a box containing three accelerometers and three gyroscopes. The accelerometers are placed such that their measurement axes are orthogonal to each other. Three gyroscopes are placed in a similar orthogonal pattern, measuring rotational velocity in reference to an arbitrarily chosen coordinate system (King, A. D., 1998). For example, a 6 DOF IMU could substitute 6 displacement transducers on a Stewart Platform. Making this substitution, the movement of the platform will be much less affected by the transducers, since displacement transducers' mass and inertia are normally of the same order of magnitude of the actuators, and the IMU could have 10 or 100 times less the mass and inertia than the actuator. Due to this, this work focuses on the development of a method to use the information of this IMU to control the position, orientation, linear and angular speed of the moving platform in two types of parallel mechanisms.

KINEMATICS OF THE PARALLEL MECHANISMS

The method used to obtain the direct kinematics on both cases is based on the analytical calculation of the inverse jacobian matrix. The inverse jacobian matrix relates the linear and angular speeds of the moving platform ($\dot{\mathbf{x}}$) with the linear velocity of the actuators ($\dot{\mathbf{q}}$), as shown in Eq. (1).

$$\dot{\mathbf{q}} = J^{-1}\dot{\mathbf{x}} \tag{1}$$

3 DOF planar mechanism

As shown in Fig. 1, this mechanism consists of three limbs (with variable lengths d_1, d_2 and d_3) connected to a fixed base ($\overline{A_1A_3}$) by three universal joints (A_1, A_2 and A_3) and connected to a moving platform ($\overline{B_1B_3}$) by other three universal joints (B_1, B_2 and B_3). The position and the orientation of the moving platform are given by x, z and θ.

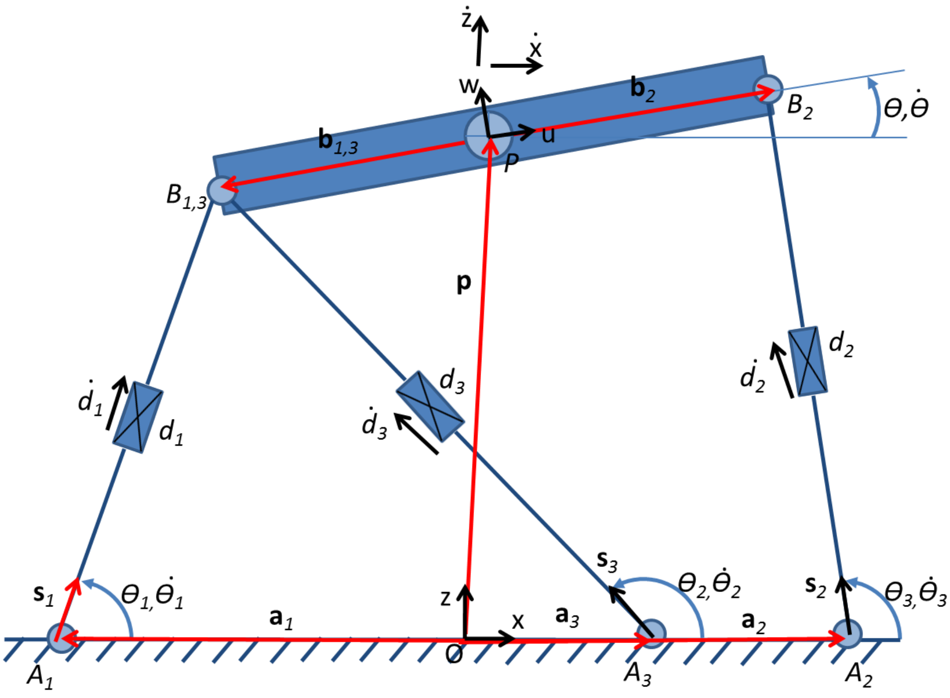

Figure 1 – Geometric scheme of a 3 DOF planar parallel mechanism.

Inverse Kinematics

The inverse geometry could be obtained by the vector sum shown in Eq. (2), where ${}^{A}R_{B}$ is the transformation matrix between the fixed frame A(x, z) and the moving frame B(u, w).

$$\overline{A_i B_i} = \mathbf{p} + {}^{A}R_{B}\,{}^{B}\mathbf{b}_i - \mathbf{a}_i \tag{2}$$

Applying the differential in relation with time of Eq. (2) we obtain Eq. (3), where $\mathbf{s}_i$ is the unit vector on the direction of the segment $\overline{A_i B_i}$.

$$\mathbf{s}_i \cdot \begin{bmatrix} \dot{x}_d \\ \dot{z}_d \end{bmatrix} + (\mathbf{b}_i \times \mathbf{s}_i) \cdot \dot{\theta} = \dot{d}_i \tag{3}$$

Separating the variables related to the limbs from the variables related to the moving platform, we can write the inverse jacobian of the mechanism, see Eq. (4). θ_1, θ_2 and θ_3 are the angles between the limbs and the fixed base, while b_{1u}, b_{2u} and b_{3u} are the horizontal coordinates of the points B_1, B_2 and B_3 with respect to the frame B(u,w).

$$J^{-1} = \begin{bmatrix} \mathbf{s}_1^T & (\mathbf{b}_1 \times \mathbf{s}_1)^T \\ \mathbf{s}_2^T & (\mathbf{b}_2 \times \mathbf{s}_2)^T \\ \mathbf{s}_3^T & (\mathbf{b}_3 \times \mathbf{s}_3)^T \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \operatorname{sen}\theta_1 & b_{1u}\operatorname{sen}(\theta_1 - \theta) \\ \cos\theta_2 & \operatorname{sen}\theta_2 & b_{2u}\operatorname{sen}(\theta_2 - \theta) \\ \cos\theta_3 & \operatorname{sen}\theta_3 & b_{3u}\operatorname{sen}(\theta_3 - \theta) \end{bmatrix} \tag{4}$$

To solve the control problem based on the acceleration of the moving platform, one has to obtain the differential of the inverse jacobian in order to have the relation between the linear and angular velocities and accelerations of the moving platform and the accelerations of the limbs of the mechanism, as shown in Eq. (5). The matrix of the derivatives of the jacobian matrix is given by Eq. (6).

$$\ddot{\mathbf{q}} = \dot{J^{-1}}\dot{\mathbf{x}} + J^{-1}\ddot{\mathbf{x}} \tag{5}$$

$$\dot{J^{-1}} = \begin{bmatrix} -\operatorname{sen}\theta_1\,\dot{\theta}_1 & \cos\theta_1\dot{\theta}_1 & b_{1u}\cos(\theta_1 - \theta).\left(\dot{\theta}_1 - \dot{\theta}\right) \\ -\operatorname{sen}\theta_2\,\dot{\theta}_2 & \cos\theta_2\,\dot{\theta}_2 & b_{2u}\cos(\theta_2 - \theta).\left(\dot{\theta}_2 - \dot{\theta}\right) \\ -\operatorname{sen}\theta_3\,\dot{\theta}_3 & \cos\theta_3\,\dot{\theta}_3 & b_{3u}\cos(\theta_3 - \theta).\left(\dot{\theta}_3 - \dot{\theta}\right) \end{bmatrix} \tag{6}$$

Direct Kinematics

The direct jacobian (or simply jacobian) of this mechanism could be obtained analytically by following the procedure shown in Eq. (7), where $Adj(J^{-1})$ is the adjoint matrix, which is the transpose of the cofactors matrix of the inverse jacobian.

$$J = (J^{-1})^{-1} = \frac{1}{|J^{-1}|} \cdot [C(J^{-1})]^T = \frac{1}{|J^{-1}|} \cdot Adj(J^{-1}) \tag{7}$$

Again, to solve the problem of the control based on the acceleration of the moving platform, one has to obtain the differential of the jacobian in order to have the relation between the velocities and accelerations of the limbs and the linear and angular accelerations of the moving platform s, shown in Eq. (8).

$$\ddot{\mathbf{x}} = \dot{J}\dot{\mathbf{q}} + J\ddot{\mathbf{q}} \tag{8}$$

The solution of the matrix with the derivatives of the direct jacobian is presented in Eq. (9). Eq. (10) shows the derivative solution for each term, where n indicates the line number and m the column number.

$$\dot{J} = \begin{bmatrix} \dot{J_{11}} & \dot{J_{12}} & \dot{J_{13}} \\ \dot{J_{21}} & \dot{J_{22}} & \dot{J_{23}} \\ \dot{J_{31}} & \dot{J_{32}} & \dot{J_{33}} \end{bmatrix} \tag{9}$$

$$\dot{J_{nm}} = \frac{|J^{-1}|[C(\dot{J^{-1}})]^T{}_{nm} - \left[C(J^{-1})\right]^T{}_{nm}|\dot{J^{-1}}|}{|J^{-1}|^2} \tag{10}$$

Stewart Platform

As shown in Fig. 2, this mechanism consists of six limbs (with variable lengths d_1, d_2, d_3, d_4, d_5 and d_6) that are connected to a fixed base by six spherical joints (A_1, A_2, A_3, A_4, A_5 and A_6) and to a moving platform by six universal joints (B_1, B_2, B_3, B_4, B_5 and B_6). The position and the orientation of the moving platform are given by x, y, z, φ, θ and ψ.

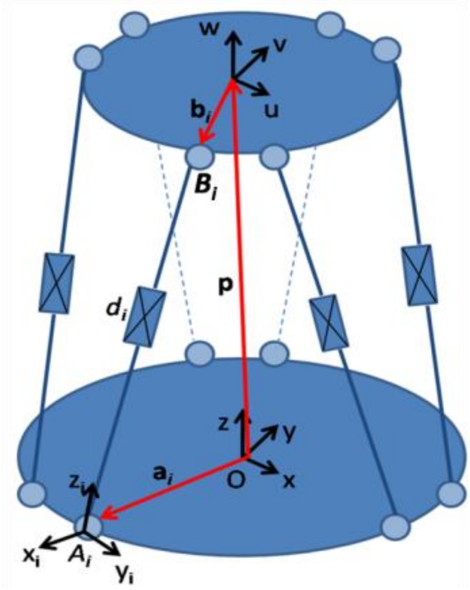

Figure 2 – Geometric scheme of a Stewart Platform.

The procedure to obtain the inverse and the direct kinematics is exactly the same one shown for the 3 DOF planar mechanism. Nevertheless, since the inverse jacobian expression of the Stewart Platform (Eq. 11) is much more complex than the one obtained for the planar mechanism, a completely analytical expression for the direct jacobian could not be computed. The inverse jacobian is obtained analytically, but its inverse (the direct jacobian) is numerically obtained for each time step.

$$J^{-1} = \begin{bmatrix} c\phi_1 s\theta_1 & s\phi_1 s\theta_1 & c\theta_1 & b_{1v}c\theta_1 - b_{1w}s\phi_1 s\theta_1 & b_{1w}c\phi_1 s\theta_1 - b_{1u}c\theta_1 & b_{1u}s\phi_1 s\theta_1 - b_{1v}c\phi_1 s\theta_1 \\ c\phi_2 s\theta_2 & s\phi_2 s\theta_2 & c\theta_2 & b_{2v}c\theta_2 - b_{2w}s\phi_2 s\theta_2 & b_{2w}c\phi_2 s\theta_2 - b_{2u}c\theta_2 & b_{2u}s\phi_2 s\theta_2 - b_{2v}c\phi_2 s\theta_2 \\ c\phi_3 s\theta_3 & s\phi_3 s\theta_3 & c\theta_3 & b_{3v}c\theta_3 - b_{3w}s\phi_3 s\theta_3 & b_{3w}c\phi_3 s\theta_3 - b_{3u}c\theta_3 & b_{3u}s\phi_3 s\theta_3 - b_{3v}c\phi_3 s\theta_3 \\ c\phi_4 s\theta_4 & s\phi_4 s\theta_4 & c\theta_4 & b_{4v}c\theta_4 - b_{4w}s\phi_4 s\theta_4 & b_{4w}c\phi_4 s\theta_4 - b_{4u}c\theta_4 & b_{4u}s\phi_4 s\theta_4 - b_{4v}c\phi_4 s\theta_4 \\ c\phi_5 s\theta_5 & s\phi_5 s\theta_5 & c\theta_5 & b_{5v}c\theta_5 - b_{5w}s\phi_5 s\theta_5 & b_{5w}c\phi_5 s\theta_5 - b_{5u}c\theta_5 & b_{5u}s\phi_5 s\theta_5 - b_{5v}c\phi_5 s\theta_5 \\ c\phi_6 s\theta_6 & s\phi_6 s\theta_6 & c\theta_6 & b_{6v}c\theta_6 - b_{6w}s\phi_6 s\theta_6 & b_{6w}c\phi_6 s\theta_6 - b_{6u}c\theta_6 & b_{6u}s\phi_6 s\theta_6 - b_{6v}c\phi_6 s\theta_6 \end{bmatrix} \quad (11)$$

DYNAMICS AND CONTROL STRATEGY

In this work, the limbs of both parallel mechanisms are modeled as pneumatic actuators. Fig. 3 shows a scheme of the valve/actuator model, while Equations 12 and 13 describe the dynamic model of this actuation system (Ogata, K., 2009).

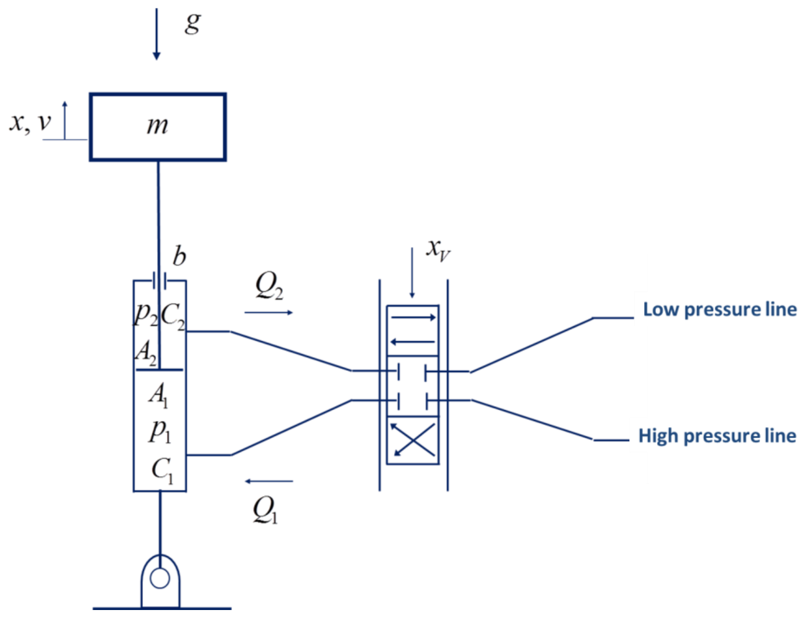

Figure 3 – Control strategy diagram.

$$\begin{cases} \dot{p}_1 = \frac{1}{C_1}(Q_1 - A_1 v) \\ \dot{p}_2 = \frac{1}{C_2}(A_2 v - Q_2) \\ \dot{v} = a = \frac{1}{m}(A_1 p_1 - A_2 p_2 - bv - mg) \end{cases} \tag{12}$$

$$\begin{cases} Q_1 = k_x x_V - k_p(p_1 - p_f) \\ Q_2 = k_x x_V + k_p(p_2 - p_f) \end{cases} \tag{13}$$

Rewriting Eq. 12 and 13 using the Laplace transform, representing them in terms of p_1, p_2 and a and isolating $a(s)$, the transfer functions of the dynamic actuator are obtained (Eq. 14).

$$a(s) = \frac{N_{xv}(s)}{D(s)} x_V(s) + \frac{N_{pv}(s)}{D(s)} p_f(s) - \frac{N_{gv}(s)}{D(s)} mg(s) \tag{14}$$

The terms $D(s)$, $N_{xv}(s)$, $N_{pv}(s)$ and $N_{gv}(s)$ are shown in Equations 15 to 18. Eq. 19 shows $a(s)$ in its complete form. The valve/actuator parameters were transformed into first order zeros, z_{xv}, z_{pv}, z_{1g} and z_{2g}, time constant, τ, damping factor, ζ, and natural frequency, ω, of the system.

$$D(s) = (ms + b)(C_1 s + k_p)(C_2 s + k_p) + A_1^2(C_2 s + k_p) + A_2^2(C_1 s + k_p) \tag{15}$$

$$N_{xv}(s) = ((A_1 C_2 + A_2 C_1)s + (A_1 + A_2)k_p)k_x s \tag{16}$$

$$N_{pv}(s) = ((A_1 C_2 - A_2 C_1)s + (A_1 - A_2)k_p)k_p s \tag{17}$$

$$N_{gv}(s) = (C_1 s + k_p)(C_2 s + k_p)s \tag{18}$$

$$a(s) = \frac{k_{xv}(s+z_{xv})s}{(\tau s+1)(s^2+2\zeta\omega s+\omega^2)} x_v(s) + \frac{k_{pv}(s+z_{pv})s}{(\tau s+1)(s^2+2\zeta\omega s+\omega^2)} p_f(s) - \frac{k_{gv}(s+z_{1g})(s+z_{2g})s}{(\tau s+1)(s^2+2\zeta\omega s+\omega^2)} mg(s) \tag{19}$$

The control strategy based on linear accelerations and angular velocities of the moving platform of parallel mechanisms is as shown in the diagram of Figure 4. The position, velocity and linear accelerations as well as the orientations, angular velocities and accelerations form the set of data that describes the desired path. This information passes through the inverse kinematics models of the mechanisms to obtain the desired velocities and accelerations of the actuators. The error signal $e\ddot{\mathbf{q}}$ and their integrals $e\dot{\mathbf{q}}$ and $e\mathbf{q}$ are handled in the controller, used in the actuating system model to generate the actual actuators state ($\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$). With the direct jacobian model, the actual moving platform state is obtained ($\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$). The effective linear accelerations and angular velocities of the moving platform are measured by an inertial unit to then be compared with the desired values.

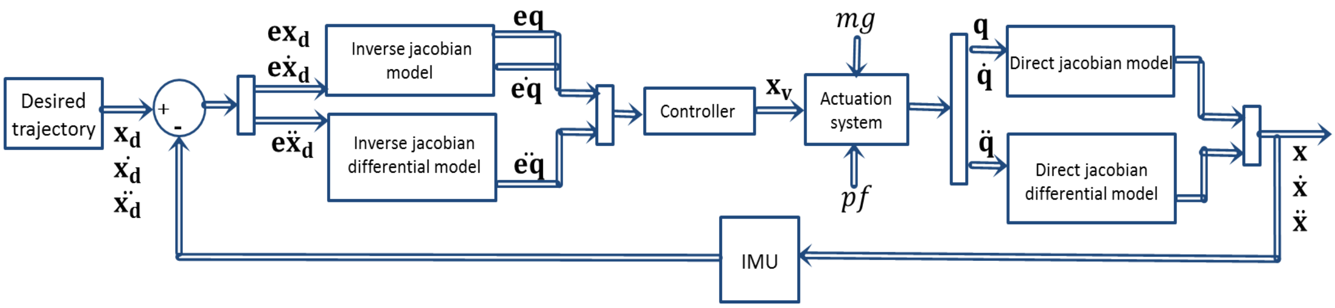

Figure 4 – Control strategy diagram.

SIMULATION AND RESULTS

The actuating system closed loop is now simulated to estimate the order of magnitude of the gains to be used in the controller. Fig.5 shows the time response results of the piston acceleration for different values of gain, K_a. Table 1 shows the parameters used in the simulations. The inputs of pressure (p_f) and load (mg) on the actuators were 0,05 kgf/mm^2 and 1 kgf, respectively.

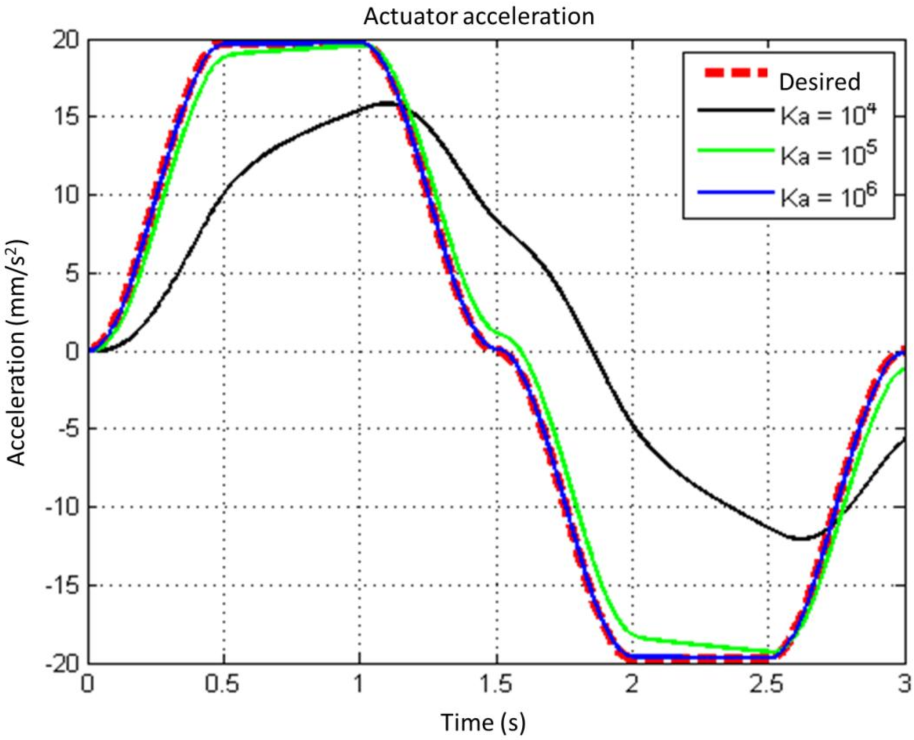

Figure 5 – Actuator acceleration time response.

3 DOF planar mechanism

The kinematic model based on the differential of the jacobian is compared with the jacobian model by the integration of its outputs. As an input, a vertical smooth step is given. Eq. (20) shows the input function $f_1(t)$ given on the coordinate z of the moving platform. Figures 6 and 7 show the response of the limbs by integrating (with the corresponding boundary conditions) the speed of the actuator obtained by the jacobian model (blue) and by integrating twice the acceleration given by the model based on the differential of the jacobian (red).

$$f_1(t) = \begin{cases} 10t^2, for\ t < 1; \\ 20(t-1) + 10 - 10(t-1)^2, for\ 1 \geq t < 2; \\ 20, for\ t \geq 2; \end{cases} \tag{20}$$

Table 1 – Simulation parameters.

Identification	Symbol	Value
Inferior chamber area (mm^2)	A_1	201
Superior chamber area (mm^2)	A_2	134
Pneumatic capacitance of the inferior chamber (mm^5/kgf)	C_1	$3{,}0 \times 10^{-11}$
Pneumatic capacitance of the superior chamber (mm^5/kgf)	C_2	$3{,}0 \times 10^{-11}$
Actuator damping factor (kgf/s)	b	35,0
Valve displacement coefficient (mm^2/s)	k_x	0,10
Valve pressure coefficient (mm^5/kgf.s)	k_p	0,10
Actuating system time response (s)	τ	0,30
Actuating system damping factor (-)	ξ	1000
Actuating system natural frequency (rad/s)	ω_n	13,09

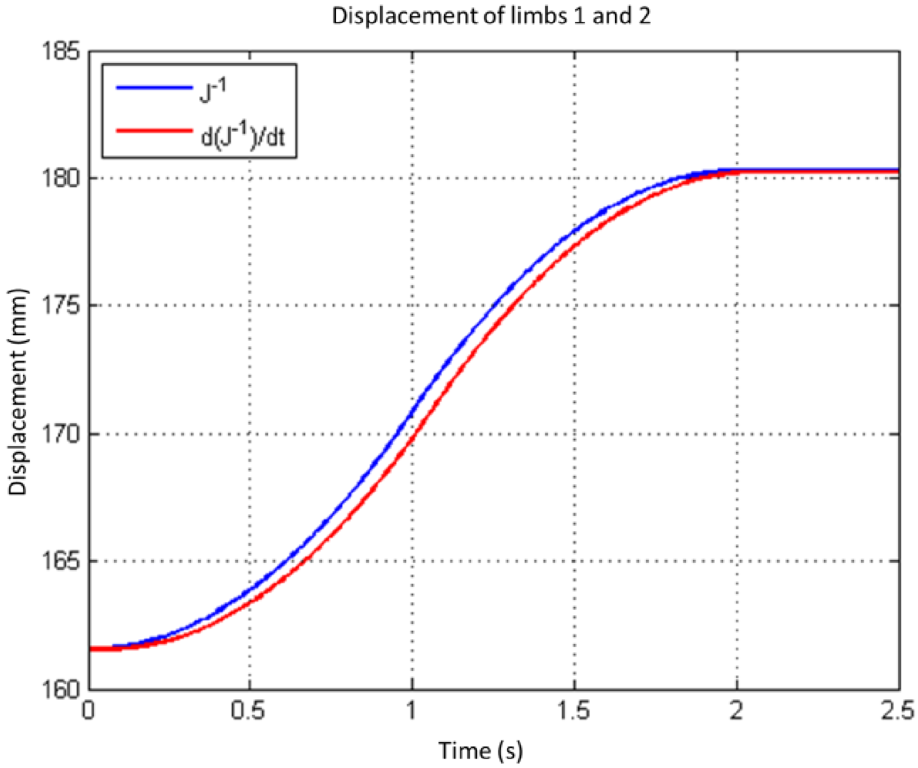

Figure 6 – Displacement of limbs 1, 2 in both methods.

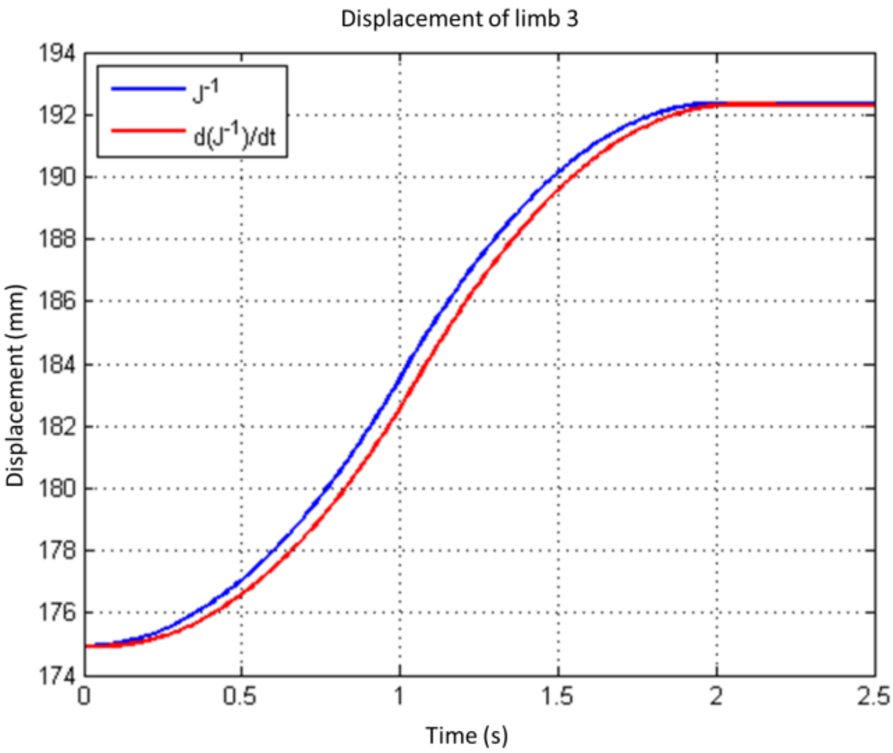

Figure 7 – Displacement of limb 3 in both methods.

For the closed loop control strategy shown (with the dynamic model of the actuator and the kinematic models of the mechanisms) and using the same parameters the time response of the moving platform is obtained with two different desired trajectories: sinusoidal movement on the *z* direction (Fig.8) and a dual smooth pulse of acceleration (Fig.9).

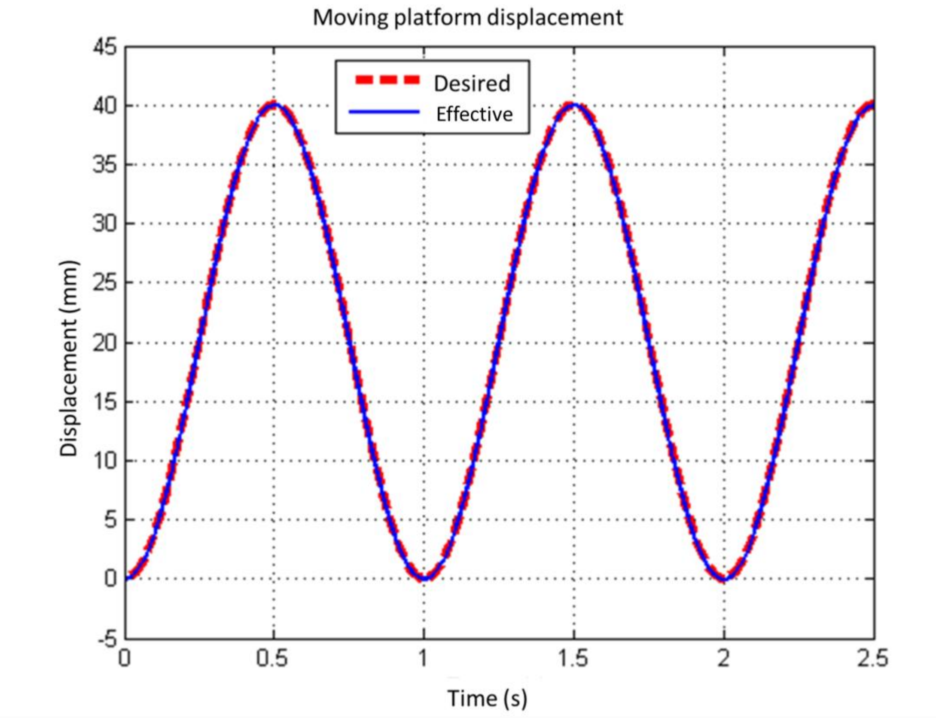

Figure 8 – Displacement of the moving platform for a sinusoidal input.

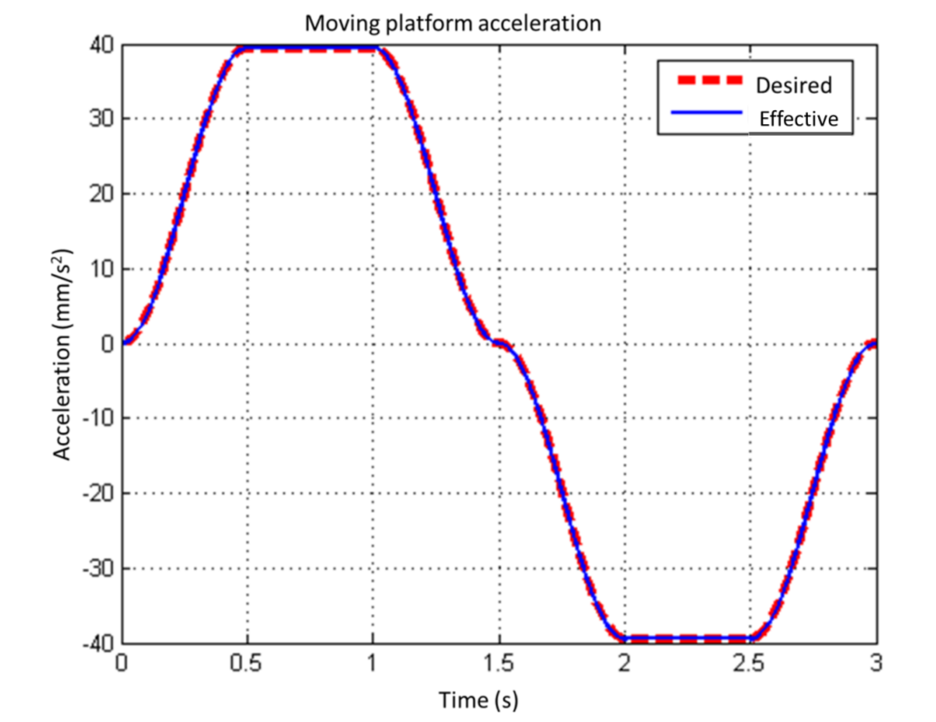

Figure 9 – Displacement of the moving platform for a dual smooth pulse.

Stewart Platform

The kinematic model based on the differential of the jacobian is compared to the jacobian model by the integration of its outputs. As an input, a vertical smooth step is applied on the coordinate z of the moving platform. Fig. 10 shows the response of the moving platform when entering with ($\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$) on the inverse kinematic model and then entering its outputs ($\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$) on the direct kinematic model to obtain the moving platform variables to compare that output with the inputs of the inverse kinematic model.

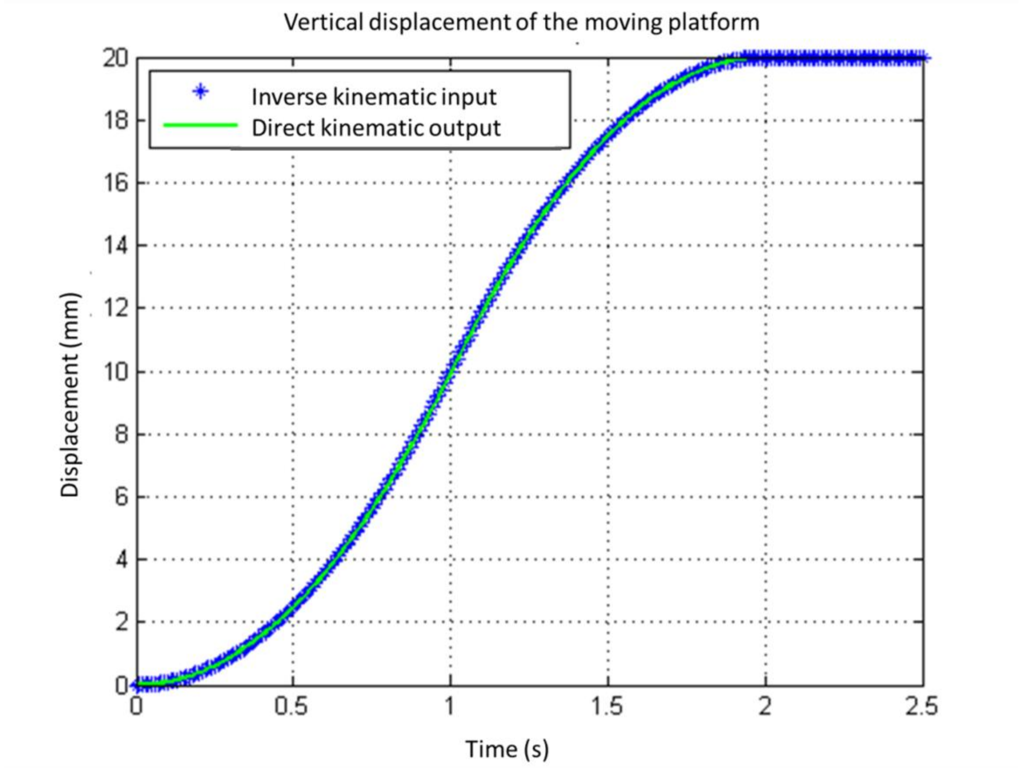

Figure 10 – Vertical displacement of the moving platform.

For the closed loop control strategy shown (with the dynamic model of the actuator and the kinematic models of the mechanisms) and using the same parameters, the time response for the moving platform is obtained with two different desired trajectories: sinusoidal movement on the *z* direction (Fig.11) and a smooth step on the *z* direction (Fig.12).

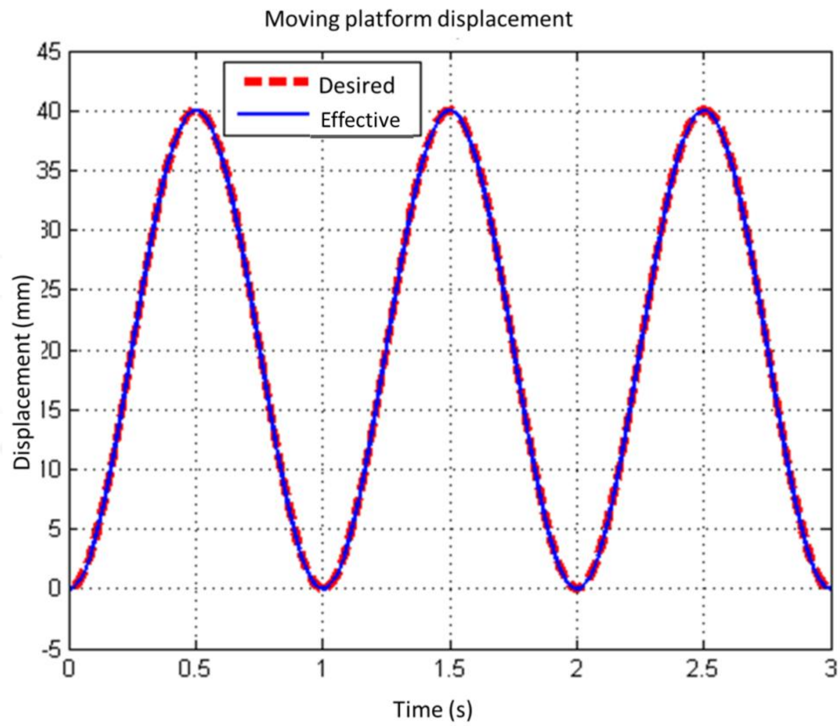

Figure 11 – Vertical displacement of the moving platform with a sinusoidal input.

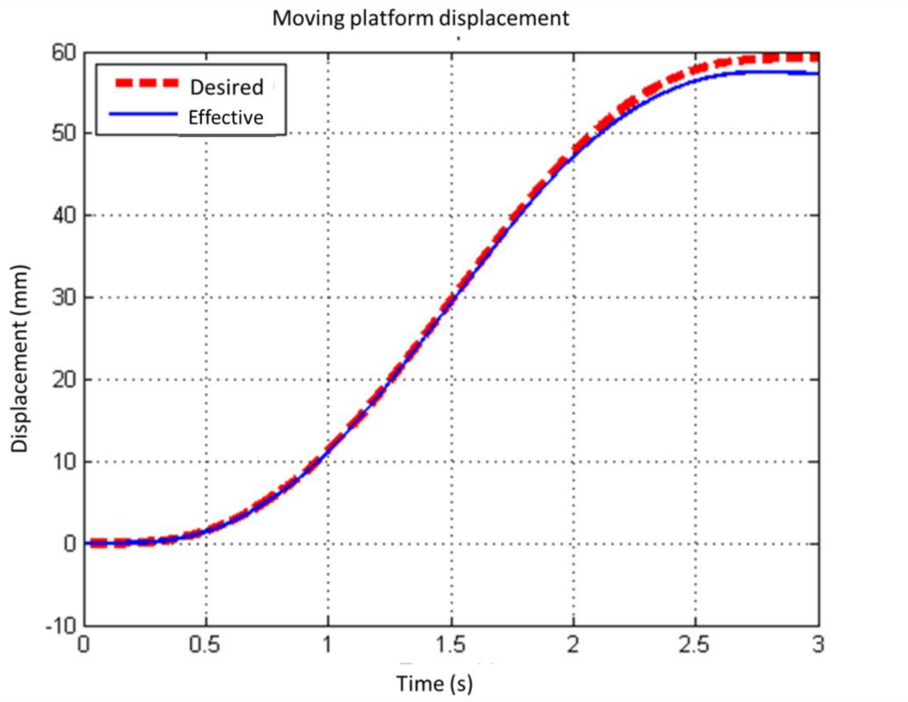

Figure 12 – Vertical displacement of the moving platform with a smooth step input.

CONCLUSIONS AND FUTURE WORK

In this work, a non-conventional control method for a parallel manipulator was presented, along with the analytical solution for the direct kinematics of a 3 DOF planar parallel mechanism and a quasi-analytical solution for the direct kinematics of the Stewart Platform, a 6 DOF parallel mechanism. The results based on the simulation with the closed-loop control diagram show the possibility of using this kind of control strategy in most types of mechanisms (parallel or not).

The ongoing work focuses on implementing these models in an experimental platform to verify these dynamic responses in real environments. One pneumatic actuated Stewart Platform has already been built for this purpose, along with the implementation of these methods to other types of mechanisms, either serial or hybrids ones, and on the detailed study of the dynamics of the pneumatic actuators used in the built Stewart Platform.

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