ON THE TOLERANCE TO SHORT FATIGUE CRACKS

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ABSTRACT

Most structural components are designed against fatigue crack initiation by procedures which do not recognize cracks, even when they should be tolerant as well to undetectable short cracks. As most of these procedures work well, they somehow are tolerant to such cracks, but standard design procedures cannot evaluate "how much tolerant". An acceptance criterion for such small cracks is proposed here, using sound mechanical concepts that require no adjustable parameters. The material is characterized by its basic fatigue resistances.

KEYWORDS

Notch sensitivity, tolerable short cracks, design against fatigue crack initiation.

INTRODUCTION

Modified stress concentration factors (SCF) $K_f = 1 + q\cdot(K_t - 1) = S_L(R)/S_{Lntc}(R)$ are commonly used for designing against fatigue crack initiation, where $0 \le q \le 1$ is the notch sensitivity factor [1], $K_t = \sigma_{max}/\sigma_n$ is the linear elastic SCF, σ_{max} and σ_{min} are the maximum and minimum LE stress at the notch root caused by σ_n, the nominal stress that would act there if the notch did not affect the stress field around it, and $S_L(R)$ and $S_{Lntc}(R)$ are the fatigue limits measured on standard and on notched test specimens (TS) at a given $R = \sigma_{min}/\sigma_{max}$ ratio. As such K_f are associated with the relatively fast generation of tiny non-propagating cracks at notch roots when $S_L(R)/K_t < \sigma_n < S_L(R)/K_f$ [2], then (i) fatigue crack initiation is caused by the notch tip stresses; (ii) the fatigue crack growth (FCG) of short cracks emanating from them is affected by the stress gradients around them too; and (iii) notch sensitivity is due to non-propagating cracks. Indeed, if a crack can start at a notch tip and then stop after a short growth, its driving force must decrease in spite of its growing size. As the cracking driving force depends on the crack size and on the stress acting on it, then the decrease of stresses ahead of the notch tip is more important than the crack growth contribution. Hence, the stress gradients ahead of the notch tips must be as important as their SCF for the notch sensitivity behavior [3-5].

ANALYSIS OF NOTCH EFFECTS ON THE FCG BEHAVIOR OF SHORT CRACKS

The FCG threshold of short cracks is smaller than the long crack threshold $\Delta K_{th}(R)$, or else the stress range $\Delta\sigma$ required to propagate them would be higher than the fatigue limit $\Delta S_L(R)$: as FCG is primarily controlled by the stress intensity factor (SIF) range, $\Delta K \propto \Delta\sigma\sqrt{\pi a}$, if short cracks with $a \to 0$ had the same $\Delta K_{th}(R)$ threshold of long cracks, their propagation by fatigue would require $\Delta\sigma \to \infty$, clearly a nonsense. Note that here the *short* cracks are *mechanical* and not *microstructural* small cracks, since material isotropy is assumed to model them, a hypothesis corroborated by the tests. The FCG threshold of short fatigue cracks under pulsating loads $\Delta K_0 = \Delta K_{th}(a, R = 0)$ can be modeled using the El Haddad-Topper-Smith (ETS) characteristic size a_0 [7], estimated from ΔK_0 and $\Delta S_0 = \Delta S_L(R = 0)$. This clever trick reproduces the Kitagawa-Takahashi plot trend [7], see Fig. 1, using a modified SIF range $\Delta K'$ to describe the FCG conditions of any crack, short or long, defined by:

$$\Delta K' = \Delta\sigma\sqrt{\pi(a + a_0)}, \text{ where } a_0 = (1/\pi)\left(\Delta K_0/\Delta S_0\right)^2 \quad (1)$$

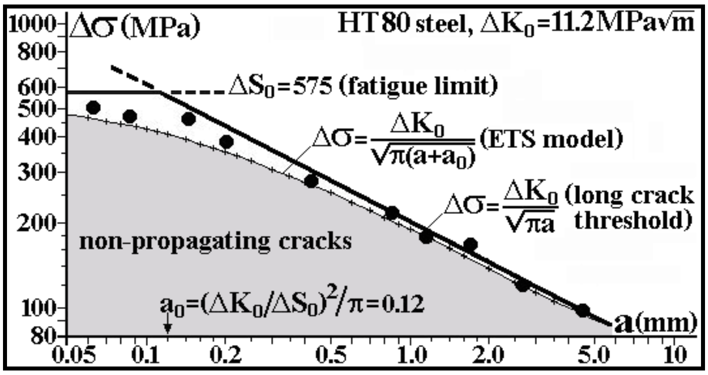

Fig. 1: Kitagawa-Takahashi plot describing the FCG of short and long cracks under pulsating loads ($R = 0$) in HT80 steel with $\Delta K_0 = 11.2$ MPa√m and $\Delta S_0 = 575$ MPa.

This $\Delta K'$ uses the Griffith's plate SIF, $\Delta K = \Delta\sigma\sqrt{(\pi a)}$. To deal with other geometries it should be rewritten using the non-dimensional geometry factor $g(a/w)$ of the cracked component:

$$\Delta K' = g(a/w)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\text{, where } a_0 = (1/\pi)\left[\Delta K_0/\left(g(a/w)\cdot\Delta S_0\right)\right]^2 \tag{2}$$

$\Delta\sigma(R = 0) \rightarrow \Delta S_0$ when $a \rightarrow 0$ if it is the notch root stress range. However, most listed $g(a/w)$ include the notch SCF. Hence, it is clearer to define the short crack characteristic size a_0 using $g(a/w) = \eta\cdot\varphi(a)$, where $\varphi(a)$ accounts for the stress gradient ahead of the notch tip and η includes all the remaining terms, such as the free surface correction:

$$\Delta K' = \eta\cdot\varphi(a)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}\text{, where } a_0 = (1/\pi)\left[\Delta K_0/\left(\eta\cdot\Delta S_0\right)\right]^2 \tag{3}$$

Operationally, it is more convenient to let the SIF range retain its original form, modifying the FCG threshold expression to become a function of the crack length a:

$$\Delta K_0(a) = \Delta K_0\cdot\sqrt{a/(a+a_0)} \tag{4}$$

The ETS equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant [8], a more general equation can be used introducing an adjustable parameter γ to fit experimental data:

$$\Delta K_0(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{5}$$

Equations (1) to (4) result from (5) if $\gamma = 2$. The bi-linear limit, $\Delta\sigma(a \le a_0) = \Delta S_0$ for short cracks and $\Delta K_0(a \ge a_0) = \Delta K_0$ for the long ones, is obtained if $g(a/w) = \eta\cdot\varphi(a) = 1$ and $\gamma \rightarrow \infty$. Most short crack FCP data is fitted by $\Delta K_0(a)$ curves with $1.5 \le \gamma \le 8$, but $\gamma = 6$ better reproduces classical q-plots based on data measured by testing semi-circular notched fatigue TS [3-5]. Using (5) as the FCP threshold, then any crack departing from a notch under pulsating loads should propagate if:

$$\Delta K = \eta\cdot\varphi(a/\rho)\cdot\Delta\sigma\sqrt{\pi a} > \Delta K_0(a) = \Delta K_0\cdot\left[1+(a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{6}$$

where $\eta = 1.12$ is the free surface correction. As fatigue depends on two driving forces, $\Delta\sigma$ and σ_{max}, to consider the σ_{max} (or the R) influence in short crack behavior, a_0 is redefined using the fatigue limit ΔS_R and the FCP threshold $\Delta K_R = \Delta K_{th}(a >> a_R, R)$ at the desired R:

$$a_R = (1/\pi)\left[\Delta K_R/\left(1.12\cdot\Delta S_R\right)\right]^2 \tag{7}$$

Likewise, the corresponding short crack FCP threshold should be re-written as:

$$\Delta K_R(a) = \Delta K_R \cdot \left[1 + (a_R/a)^{\gamma/2}\right]^{-1/\gamma} \tag{8}$$

According to Tada, the SIF of a crack with size a that departs from a circular hole of radius ρ is given within 1% by [9]:

$$\begin{cases} K_I = 1.1215 \cdot \sigma\sqrt{\pi a} \cdot \varphi(x), \;\; x \equiv a/\rho \\ \varphi(x) = \left[1 + \dfrac{0.2}{(1+x)} + \dfrac{0.3}{(1+x)^6}\right] \cdot \left[2 - 2.354\left(\dfrac{x}{1+x}\right) + 1.206\left(\dfrac{x}{1+x}\right)^2 - 0.221\left(\dfrac{x}{1+x}\right)^3\right] \end{cases} \tag{9}$$

Hence, $\varphi(x = 0) = 3$ reproduces the SCF of circular holes, exactly as it should. It also gives $\varphi(x \to \infty) = 1/1.1215\sqrt{2} \cong 0.63$. Since to propagate by fatigue any crack its SIF must be larger than its threshold, then for pulsating loads:

$$\Delta K_I = \Delta\sigma\sqrt{\pi a} \cdot \eta \cdot \varphi(a/\rho) > \Delta K_{th}(a) = \Delta K_0 \cdot \left[1 + (a_0/a)^{\gamma/2}\right]^{-1/\gamma} \tag{10}$$

The stress gradient factor $\varphi(a/w)$ cannot affect a_0, otherwise it would not be a material property. The FCG criterion can thus be rewritten using two dimensionless functions, one related to the notch stress gradient $\varphi(a/w)$, and the other $g(\Delta S_0/\Delta\sigma, a/\rho, \Delta K_0/\Delta S_0\sqrt{\rho}, \gamma)$ which includes the applied stress range $\Delta\sigma$, and depends on the crack size, on the notch radius ρ, on the fatigue resistances ΔK_0 and ΔS_0, and on the data fitting exponent γ (if it is used):

$$\varphi(a/\rho) > \frac{(\Delta S_0/\Delta\sigma) \cdot \left[\Delta K_0/(\Delta S_0\sqrt{\rho})\right]}{\left[(\eta\sqrt{\pi a/\rho})^\gamma + \left[\Delta K_0/(\Delta S_0\sqrt{\rho})\right]^\gamma\right]^{1/\gamma}} \equiv g\left(\frac{\Delta S_0}{\Delta\sigma}, \frac{a}{\rho}, \frac{\Delta K_0}{\Delta S_0\sqrt{\rho}}, \gamma\right) \tag{11}$$

Therefore, if $x \equiv a/\rho$ and $\kappa \equiv \Delta K_0/\Delta S_0\sqrt{\rho} = \eta\cdot\sqrt{(\pi a_0/\rho)}$, a fatigue crack departing from a Kirsch hole under pulsating loads grows whenever $\varphi(x)/g(\Delta S_0/\Delta\sigma, x, \kappa, \gamma) > 1$. Figure 2 plots some φ/g functions for several fatigue strength to loading stress range ratios $\Delta S_0/\Delta\sigma$ as a function of the normalized crack length $x = a/\rho$ assuming a small notch root radius compared to the short crack characteristic size $\rho \cong 1.40a_0$, and a material with $\kappa = 1.5$ and $\gamma = 6$ [5].

Analyzing Fig. 2, it should be noted that for high stress ranges $\Delta\sigma$, the strength to load ratio $\Delta S_0/\Delta\sigma$ is small, and the corresponding φ/g curve is always higher than 1, so cracks can initiate and grow from this Kirsch hole border without stopping during this process. On the other hand, small stress ranges with load ratios $\Delta S_0/\Delta\sigma \geq K_t = 3$ have $\varphi/g < 1$, thus such loads cannot initiate a fatigue crack from this hole border. Moreover, small enough cracks introduced there by any other means cannot propagate at such low loads. Intermediate load ranges can initiate and propagate a fatigue crack from this hole border, until the decreasing φ/g ratio reaches 1, where the crack stops and becomes non-propagating because the stress gradient ahead of its small root is sharp enough to eventually force $\Delta K_I(a) < \Delta K_{th}(a)$. Finally, the curve tangent to the $\varphi/g = 1$ line identifies the smallest (pulsating) stress range that can cause crack initiation and propagation without arrest from the notch border by fatigue alone. Hence, by definition, it is associated with this hole fatigue SCF K_f. Note also that the abscissa of its tangency point gives the largest non-propagating crack size that can arise from it by fatigue alone, a_{max}. For any other ρ/a_0, γ, and κ combination, K_f and a_{max} can always be found by solving the system

$$\begin{cases} \varphi/g = 1 \\ \partial(\varphi/g)/\partial x = 0 \end{cases} \Rightarrow \begin{cases} \varphi(x_{max}) = g(x_{max}, K_f, \kappa, \gamma) \\ \partial\varphi(x_{max})/\partial x = \partial g(x_{max}, K_f, \kappa, \gamma)/\partial x \end{cases} \tag{12}$$

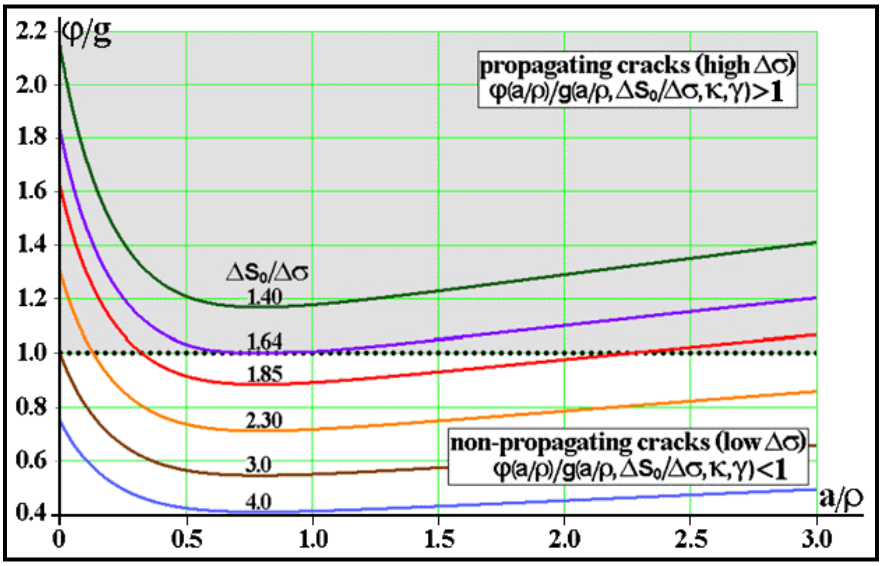

Fig. 2: Cracks that can start from the border of a (small) circular hole may propagate by fatigue and then stop if their $\varphi/g < 1$.

Kirsch holes induce relatively mild stress gradients. Larger holes compared with the short crack characteristic size, with radii $\rho >> a_0$, are associated to small κ values and do not induce short crack arrest. That is a nice way to mechanically interpret the notch sensitivity concept. In other words, if the system $\{\varphi/g = 1, \partial(\varphi/g)\partial x = 0\}$ is solved for a given γ and several tip radii using $\kappa \equiv \Delta K_0/\Delta S_0\sqrt{\rho}$, then the notch sensitivity factor q is obtained by:

$$q(\kappa,\gamma) \equiv [K_f(\kappa,\gamma)-1]/(K_t-1) \tag{13}$$

This approach has 4 advantages: (i) it is an analytical procedure; (ii) it considers the effect of the fatigue resistances to crack initiation and propagation on q; (iii) it can use the exponent γ used to modify the original ETS model to better fit short FCG data; and (iv) it can be easily extended to other notch geometries. For example, the SIF of cracks that depart from a semi-elliptical notch with semi-axles b and c, with b collinear with the crack a and perpendicular to the (nominal) stress σ_n, can be described by:

$$\Delta K_I = \eta \cdot F(a/b, c/b) \cdot \Delta\sigma\sqrt{\pi a} \tag{14}$$

where $\eta = 1.1215$ is the free surface correction factor and $F(a/b, c/b)$ is the geometrical factor associated to the notch stress concentration effect. Such notches SCF K_t is given by [15]:

$$K_t = (1+2b/c)\cdot\left[1+0.1215/(1+c/b)^{2.5}\right] \tag{15}$$

Using $s = a/(a+b)$ expressions for $F(a/b, c/b)$ were obtained in [3] by fitting a series of finite elements (FE) analyses for several types of semi-elliptical notches:

$$\begin{cases} F(a/b,c/b) \equiv f(K_t,s) = K_t\sqrt{\left[1-\exp(-K_t^2\cdot s)\right]/(K_t^2\cdot s)}, \; c \le b \\ F(a/b,c/b) \equiv f'(K_t,s) = K_t\sqrt{\dfrac{1-\exp(-K_t^2\cdot s)}{K_t^2\cdot s}}\cdot\left[1-\exp(-K_t^2)\right]^{-s/2}, \; c \ge b \end{cases} \tag{16}$$

Traditional notch sensitivity estimates like Peterson's $q = (1+\alpha/\rho)^{-1}$ suppose that it depends only on the notch tip radius ρ. The model proposed here, on the other hand, recognizes that q depends on ρ, ΔS_0, and ΔK_0 (and in γ, if it is used). Experimental evidence that supports this model predictions are presented in [4-5].

INFLUENCE OF SHORT CRACKS ON THE FATIGUE STRENGTH

Traditional *SN* and εN fatigue design methods assume crack-free pieces, but it is impossible to guarantee that they are free of cracks smaller than the detection threshold of the non-destructive method used to identify them. Nevertheless, most structural components are still designed against fatigue crack initiation using procedures that do not recognize such small cracks. Hence, their *infinite* life predictions become unreliable when they are introduced by any means during manufacture or service, and not quickly detected and properly removed. But while large cracks may be easily detected, small cracks may pass unnoticed. Therefore, structural components that must last for very long fatigue lives should be designed to be tolerant to undetectable short cracks, since continuous work cannot be guaranteed if any of the cracks they might have can somehow propagate during their service lives. Despite self-evident, this requirement is still not included in most fatigue design routines used in practice. Indeed, most long-life designs just intend to maintain the service stresses at the structural component critical point below its fatigue limit, $\Delta\sigma < \Delta S_L(R)/\varphi_F$, where φ_F is a suitable safety factor. However, in spite of not recognizing any cracks, most long-life designs work just fine. This means that they are somehow tolerant to undetectable or to functionally admissible short cracks. But the question "how much tolerant" cannot be answered by *SN* or εN procedures alone. Such problem can be avoided by adding short crack concepts to their "infinite" life design criteria which, in its simplest version, may be given by [5]:

$$\Delta\sigma < \Delta K_R \Big/ \varphi_F \left\{\sqrt{\pi a}\cdot g(a/w)\cdot\left[1+(a_R/a)^{\gamma/2}\right]^{1/\gamma}\right\}, \text{ where } \Delta K_R(a) = \Delta K_R\cdot\left[1+(a_R/a)^{\gamma/2}\right]^{-1/\gamma}$$

$$\text{and } a_R = (1/\pi)\left[\Delta K_R/\eta\Delta S_R\right]^2 \quad (17)$$

As the fatigue limit $\Delta S_R = \Delta S_L(R)$ considers the effect of the microstructural defects inherent to the material, this equation complements it by describing the tolerance to small cracks in actual service conditions. The usefulness of this criterion is well illustrated by a practical example, as follows. Due to a rare manufacture problem, a lot of an important component was delivered with tiny surface cracks, causing some unexpected failures. To quantify the effect of such tiny cracks in that piece fatigue strength, knowing that its rectangular cross-section has *2* by *3.4mm*, one assumes that it has $S_L = 246MPa$ and that it is made from steel with $S_U = 990MPa$. This measured fatigue limit is about $S_U/4$, whereas it would be traditionally estimated by $S_L \cong S_U/2 = 495MPa$. This difference may be due to a surface finish factor $k_{sf} = 0.5$, a value between those proposed by Juvinall for $S_U = 1GPa$ steels with cold-drawn ($k_{sf} = 0.45$) and machined surfaces ($k_{sf} = 0.7$) [10]. The surface finish should not affect cracks, but as this difference could be due to other factors too, like tensile residual stresses, the only safe option is to use $S_L = 246MPa$ to evaluate the tiny crack effects. Therefore, from Goodman one gets $S_L(R) = S_R = S_L S_U(1 - R)/[S_U(1 - R) + S_L(1 + R)]$ for $R > -1$ (or $\sigma_m > 0$), for example. The FCG threshold ΔK_R is also needed to model short crack effects, but if data is not available, as in this case, it must be estimated e.g. by $\Delta K_R(R \le 0.17) = \Delta K_0 = 6MPa\sqrt{m}$ *and* $\Delta K_R(R > 0.17) = 7\cdot(1 - 0.85R)$ [11]. This practice increases the predictions uncertainty, but it is the only option available. Besides, it tends to be conservative. Moreover, it assumes that $\Delta K_R(R < 0) \cong \Delta K_0$, a safe estimate too (except if the load history contained severe compressive underloads which might accelerate the crack, not the case here). Using the SIF of an edge cracked strip of width *w* loaded in mode I, then the tolerable stress ranges under pulsating loads shown in Fig. 3 are estimated within a fatigue safety factor φ_F as:

$$\Delta\sigma_0 \le \frac{\Delta K_0\big/\left(\varphi_F\sqrt{\pi a}\right)}{\left[0.752+2.02\frac{a}{w}+0.37\left(1-\sin\frac{\pi a}{2w}\right)^3\right]\sec\frac{\pi a}{2w}\sqrt{\frac{2w}{\pi a}\tan\frac{\pi a}{2w}}\;\left[1+\left(\frac{a_0}{a}\right)^{\gamma/2}\right]^{1/\gamma}} \quad (18)$$

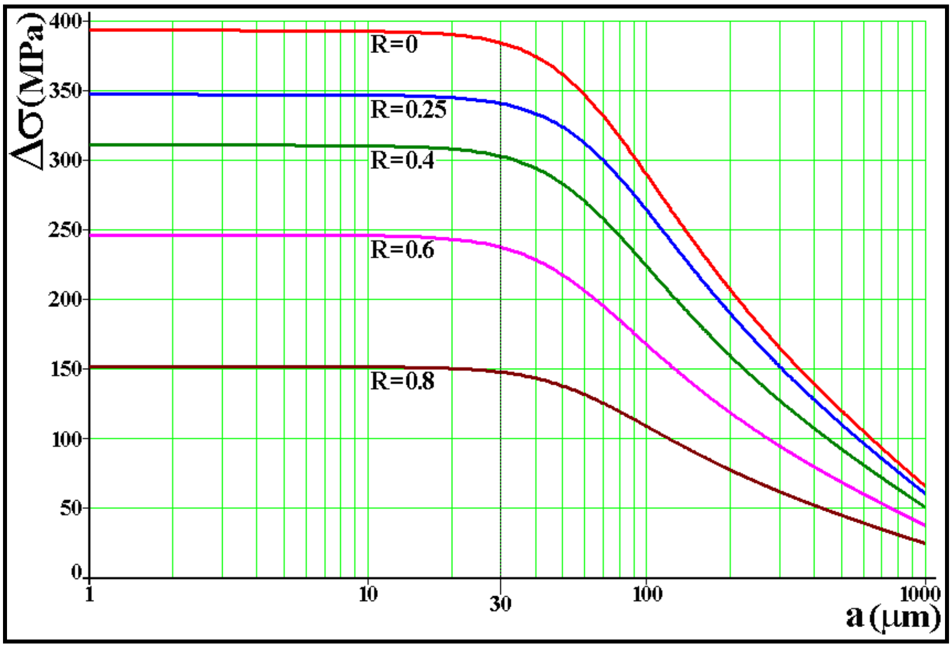

Fig. 3: Stress ranges tolerable under several *R*–ratios by the analyzed component considering it contains an edge crack of size *a*, for $w = 3.4mm$, $a_0 = 59\mu m$, $\gamma = 6$, and $\varphi_F = 1.6$.

Conclusions

A generalized ETS parameter was used to model the short crack size dependence of the FCG threshold and to estimate the notch sensitivity from the FCG behavior of short non-propagating cracks that may initiate from their tips.

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