AN INTEGRAL METHOD TO PREDICT NON-PROPORTIONALITY FACTORS AND EQUIVALENT RANGES IN MULTIAXIAL HISTORIES

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ABSTRACT

This work studies an approach to evaluate equivalent stress and strain ranges in non-proportional (NP) histories, called the Moment Of Inertia (MOI) method. The MOI method assumes that the path contour in the deviatoric stress or strain diagram is a homogeneous wire with unit mass. The center of mass of such wire gives then the mean component of the path, while the moments of inertia of the wire can be used to obtain the equivalent stress or strain ranges. The MOI method is an alternative to convex enclosure methods, such as the Minimum Ball or Maximum Prismatic Hull methods, without the need for computationally-intensive search algorithms or adjustable parameters. The MOI method can deal with an arbitrarily shaped history without losing information about such shape, as opposed to a convex enclosure method. Therefore, it can be successfully used even in highly non-convex stress or strain NP history paths such as cross or star-shaped paths, which result in poor predictions when convex enclosure methods are used. The MOI method is relatively simple, intuitive, and easy to implement and to compute, therefore it should be considered as an alternative engineering tool to deal with NP histories. Coupled with a multiaxial rainflow algorithm, it is able to deal with very long variable amplitude histories, which would be too computationally intensive for incremental plasticity or convex enclosure methods to obtain stress or strain ranges. In this work, the MOI method is also generalized to calculate as well the non-proportionality factor F_{np} of a loading history, using an alternative sub-space of the deviatoric stresses or strains. Experimental results for 12 different multiaxial histories prove the effectiveness of the MOI method not only to predict the associated fatigue lives, but also to predict the observed non-proportionality factors.

KEYWORDS

Degree of non-proportionality, equivalent ranges, Multiaxial fatigue, Multiaxial loading, Non-proportional loading.

INTRODUCTION

Multiaxial fatigue lives can be calculated from equivalent stress (or strain) ranges and their mean components [1]. However, estimating such ranges and mean values for non-proportional (NP) loading cycles is not an easy task. These components are traditionally estimated by convex circular, ellipsoidal or prismatic enclosures of the entire history path in stress or strain diagrams. However, enclosing surface methods are not suited for complex NP histories, since they do not account for path shape dependence of fatigue damage.

Consider a periodic load history formed by repeatedly following a given loading path domain $\mathcal{D}$, where $\mathcal{D}$ contains all points from the stress or strain variations along one of its periods. Assume that two out-of-phase shear stresses τ_B and τ_{B2} act parallel to the critical plane, where the crack will most likely initiate. Both τ_B and τ_{B2} influence the growth of shear cracks along the critical plane. To calculate the maximum shear stress range $\Delta\tau_{max}$ at the critical plane, it is necessary to draw the path $\mathcal{D}$ of the stress history along a $\tau_B \times \tau_{B2}$ diagram, as shown in Fig. 1.

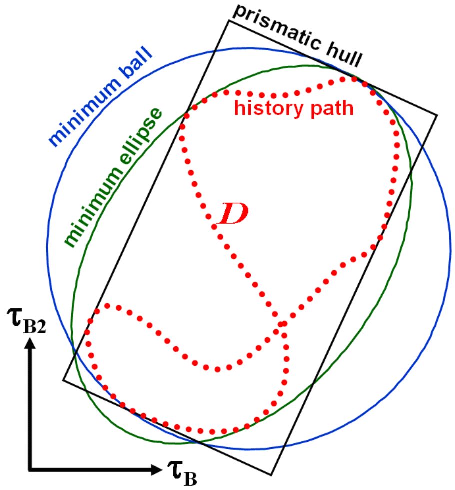

Fig. 1: Periodic (or single) stress history path *D* in a $\tau_B \times \tau_{B2}$ diagram, enclosed in surfaces such as circles (balls), ellipses and rectangular prisms

The search for an effective range using the deviatoric stress path started with the pioneering work of Dang Van [2], who studied various methods to define and calculate it. Since then, several "enclosing surface methods" have been proposed [3-7], which try to find circles, ellipses or rectangles that contain the entire load path (in the 2D case). In a nutshell, in the 2D case, the Minimum Ball (MB) method [3] searches for the circle with minimum radius that contains $\mathcal{D}$; the minimum ellipse (ME) methods [4-6] search for an ellipse with semi-axes *a* and *b* that contains $\mathcal{D}$ with minimum area $\pi \cdot a \cdot b$ or minimum norm $(a^2 + b^2)^{1/2}$; and the maximum prismatic hull (MPH) methods [5, 7] search among the smallest rectangles that contain $\mathcal{D}$ the one with maximum area or maximum diagonal (it's a max-min search problem). The value of $\Delta\tau_{max}$ in Fig. 1 would either be assumed as the value of the circle diameter, or twice the ellipse norm, or the length of the enclosing rectangle diagonal. If the history path was represented in a $\gamma_B \times \gamma_{B2}$ shear strain diagram, these exact same methods would result in estimates for the maximum shear strain range $\Delta\gamma_{max}$.

Enclosing surface methods can be useful to estimate the equivalent stress (or strain) amplitude associated with NP loading paths. However, such methods have several issues, in special regarding information loss. Enclosing surface algorithms do not take into account the actual loading path, but only the convex hulls associated with them. All paths that share the same convex hull share as well the same enclosing surface for a given method, even though they might lead to different equivalent amplitudes and fatigue lives. This issue is addressed by a method to calculate equivalent and mean components that takes into account the actual loading path, not only its convex hull. This method is presented next.

THE MOMENT OF INERTIA (MOI) METHOD

The Moment Of Inertia (MOI) method calculates alternate and mean components of complex NP load histories. To accomplish that, the history must first be represented in a 2D subspace of the transformed 5D Euclidean stress or strain space. The MOI method assumes that the 2D path/domain $\mathcal{D}$, represented by a series of points *(X, Y)* from the stress or strain variations along it, is analogous to a homogeneous wire with unit mass. Note that *X* and *Y* can have stress or strain units, but they are completely unrelated to the directions *x* and *y* usually associated with the material surface. The mean component of $\mathcal{D}$ is assumed, in the MOI method, to be located at the center of gravity of this imaginary homogeneous wire shaped as the history path. Such center of gravity is located at the perimeter centroid (X_c, Y_c) of $\mathcal{D}$, calculated from contour integrals along the entire path

$$X_c = \frac{1}{p} \cdot \oint X \cdot dp, \quad Y_c = \frac{1}{p} \cdot \oint Y \cdot dp, \quad p = \oint dp \tag{1}$$

where *dp* is the length of an infinitesimal arc of $\mathcal{D}$ and *p* is the path perimeter (Fig. 2).

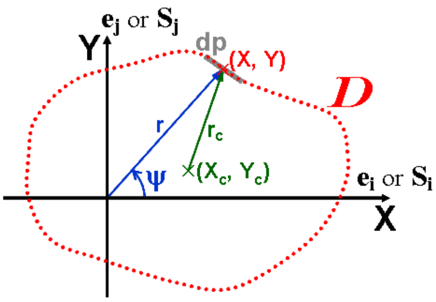

Fig. 2: Load history path, assumed as a wire with unit mass in the deviatoric 2D space

The MOI method is so called because it makes use of the mass moments of inertia (MOI) of such homogeneous wire. These moments are first calculated with respect to the origin *O* of the diagram, assuming the wire has unit mass, resulting in

$$I_{XX}^{O} = \frac{1}{p} \cdot \oint Y^2 \cdot dp, \quad I_{YY}^{O} = \frac{1}{p} \cdot \oint X^2 \cdot dp, \quad I_{XY}^{O} = -\frac{1}{p} \cdot \oint X \cdot Y \cdot dp \tag{2}$$

Then, the mass moments of inertia of such unit mass wire, with respect to its center of gravity (X_c, Y_c), are obtained. They are computed from the moments of inertia of the path $\mathcal{D}$ with respect to its perimeter centroid (X_c, Y_c), which are easily obtained from the parallel axis theorem, assuming a unit mass:

$$I_{XX} = I_{XX}^{O} - Y_c^2, \quad I_{YY} = I_{YY}^{O} - X_c^2, \quad I_{XY} = I_{XY}^{O} + X_c \cdot Y_c \tag{3}$$

The MOI method simply assumes that the deviatoric stress or strain ranges, $\Delta S \equiv \Delta\sigma_{Mises}$ or $\Delta e \equiv \Delta\varepsilon_{Mises}$, depend on the mass moment of inertia I_{ZZ} with respect to the perimeter centroid, perpendicular to the *X*-*Y* plane. This is physically sound, since history paths further away from their perimeter centroid PC will contribute more to the effective range and amplitude, which is coherent with the fact that wire segments further away from the center of gravity of an imaginary homogeneous wire contribute more to its MOI. Note that the use of integral parameters to evaluate NP paths is not new, it was already used e.g. in [8] to estimate the non-proportionality factor. But the use of a moment of inertia analogy to obtain effective ranges is a novel idea, a true alternative for the existing enclosing surface methods.

From the perpendicular axis theorem, which states that $I_{ZZ} = I_{XX} + I_{YY}$, and from a dimensional analysis, it is found that

$$\frac{\Delta\sigma_{Mises}}{2} \text{ or } \frac{\Delta\varepsilon_{Mises}}{2} = \sqrt{3 \cdot I_{ZZ}} = \sqrt{3 \cdot (I_{XX} + I_{YY})} \qquad (4)$$

CALCULATION OF THE NON-PROPORTIONALITY FACTOR F_{np}

To account for NP hardening effects, it is necessary to correctly evaluate the non-proportionality factor F_{np}. The NP factor F_{np} can be estimated from the load history path. Several methods have been proposed to estimate F_{np}. E.g. Itoh [8] estimated F_{np} using a contour integral definition along the path. This Itoh-Socie method searches for the direction of maximum strain in the path, and then it performs an integral average along the entire path of the absolute value of the strain components perpendicular to such direction.

The Moment Of Inertia (MOI) method is now used to evaluate F_{np}. To accomplish that, consider the 2D projection of the deviatoric stress or strain history mentioned before. Now, to calculate the directions suffering larger stress or strain magnitudes, the load history path $\mathcal{D}$ can once again be imagined as a homogeneous wire with unit mass, as it was assumed before to calculate the Mises ranges. This is physically sound, since the mass moments of inertia I_{XX} and I_{YY} of such wire in the horizontal (X) and vertical (Y) directions are a measure of how much the path stretches in the Y and X directions, respectively.

If the path crosses more than once some direction ψ, then it is reasonable to assume that the point with maximum magnitude r among them is the one that better represents the contribution of the Mises stresses or strains in this direction. This means that the MOI equations to compute F_{np} must be evaluated only for the enclosing hull (which is not necessarily convex) defined by the outer perimeter of the entire history path. Note that this hull must be computed for the entire history (since the specimen was virgin up to a certain point in time) to be able to account for all non-proportional hardening suffered along the specimen life until now (the previously presented MOI method for the range and mean calculations, on the other hand, do not make use of any hull, and they are computed for each rainflow-counted cycle, not for the entire history).

The MOI I_e in the direction ψ_e of the maximum projected deviatoric strain e_{max} from the history is then obtained, resulting in

$$I_e = \frac{I_{XX} + I_{YY}}{2} + \frac{I_{XX} - I_{YY}}{2} \cos 2\psi_e + I_{XY} \sin 2\psi_e \quad \Rightarrow \quad F_{np} = \sqrt{2 \cdot I_e} / e_{max} \qquad (5)$$

EXPERIMENTAL EVALUATION OF THE MOI AND ENCLOSING SURFACE FATIGUE LIFE PREDICTIONS

The MOI and enclosing surface estimates of effective ranges are now used to reproduce the multiaxial fatigue lives of 304 stainless steel specimens tested by Itoh et al. [8]. Thirteen periodic histories are studied, represented by the block loadings shown in Fig. 3 for Cases 0 through 12. The multiaxial fatigue lives are calculated using the Smith-Watson-Topper (SWT) model in Bannantine-Socie's critical plane approach [1], searching for the plane where the damage parameter $\sigma_{max} \cdot \Delta\varepsilon/2$ is maximized. The material properties used in these calculations are:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{1754}\right)^{1/0.276} , \quad (\sigma_{max}\frac{\Delta\varepsilon}{2})_{max} = \frac{757^2}{E}(2N)^{2b} + 30.5 \cdot (2N)^{b+c} \tag{6}$$

$$E = 197{,}000\, MPa, \quad b = -0.0886, \quad c = -0.303$$

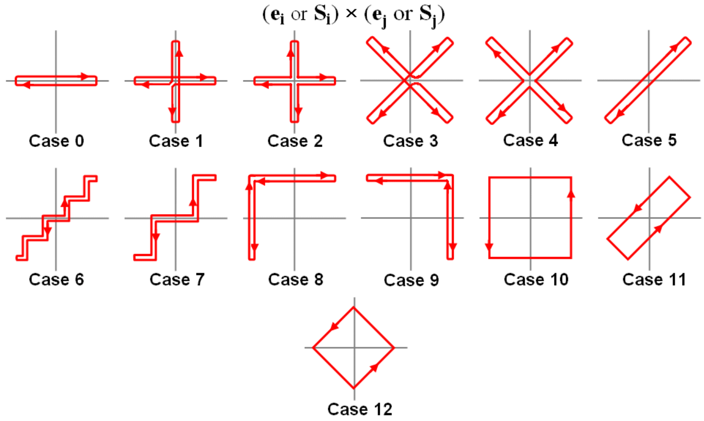

Fig. 3: History paths used in the experimental validation of the equivalent range predictions

Table 1 shows the experimental fatigue lives and the associated MOI, MB and MPH method life predictions for each one of the 12 loading histories. The MOI method outperforms the MB and MPH method for all cases. The MOI method also outperforms the Itoh et al. method in the calculation of F_{np} for all 12 experiments, as seen in Fig. 4.

path / N	experim.	MOI	MB	MPH
Case 0	7100	7085	7085	7085
Case 1	2800	3379	3379	1150
Case 2	4200	4462	4462	1504
Case 3	820	640	640	229
Case 4	900	858	858	304
Case 5	3200	3557	3557	3557
Case 6	2600	2332	2393	2177
Case 7	1700	1590	1751	1453
Case 8	470	604	856	572
Case 9	660	604	856	572
Case 10	320	329	949	329
Case 11	1200	1073	2241	1073
Case 12	710	689	2023	689

as if 90º out of phase (MPH, Cases 1–4)

as if proportional (MB, Cases 8–12)

Table 1: Fatigue life *N* (in cycles) experimentally measured and predicted using the Smith-Watson-Topper damage model and the Moment Of Inertia (MOI), Minimum Ball (MB) and Maximum Prismatic Hull (MPH) methods. Note that Cases 1-4 consider 2 cycles per block (e.g. the measured life for Case 1 was 1,400 blocks, and thus shown as 2,800 cycles).

CONCLUSIONS

The Moment Of Inertia (MOI) method is able to efficiently predict equivalent ranges in multiaxial loading histories. Since it is not based on path enclosures, it deals better with path shape dependence issues. The method can also be applied to calculate non-proportionality factors. Experimental results for 12 different multiaxial histories collected from comprehensive studies proved the effectiveness of the MOI method to predict the associated fatigue lives, when compared to the existing enclosing surface methods.

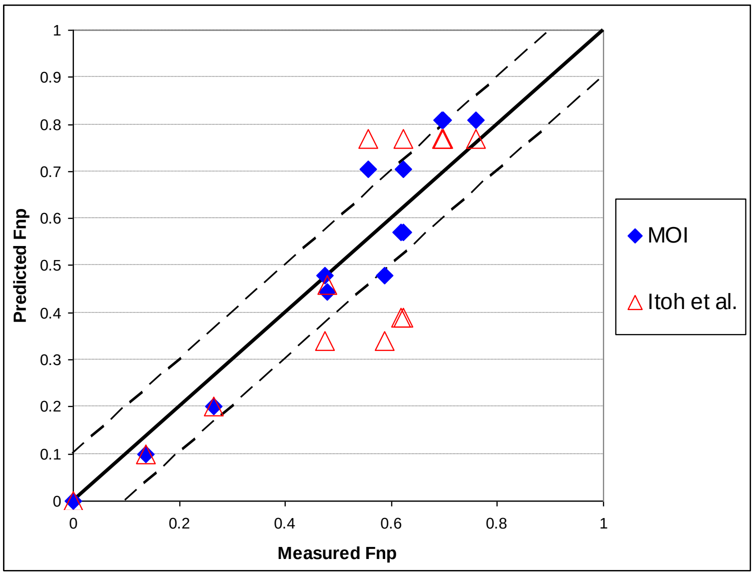

Fig. 4: Measured and predicted F_{np} from the MOI and Itoh-Socie's methods, for a 304 stainless steel at strain range levels between *0.7%* and *0.8%*.

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