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Dynamic Models of Bicycles and Motorcycles using Power Flow Approach

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Abstract: *This paper presents a procedure for the determination of the analytical form of dynamic models for bicycles and/or motorcycles through the characterization of the power flow between its components (driver, handlebars and frame, suspension and wheels/tires, engine, transmission and brakes) including the influence of gyroscopic effect and the interactions between the longitudinal, lateral and vertical dynamics of those vehicles. From the kinematic relations associated to the velocities of the degrees of freedom of each part of the vehicle (driver, handlebar, frame, power train, suspension mechanisms and tires/wheels) their links are determined. Considering the power flow between the degrees of freedom and also between then and the subsystems elements, the equilibrium relations between the forces and torques are determined. Finally, taking the inertial, stiffness and damping effects of the various system components into consideration, the equations of motion or state equations that characterize the dynamics of the vehicles are analytically obtained, represented in any reference frame, local or global. This procedure is modular and can be applied to smaller model subsystems (e.g. with fewer components and degrees of freedom). These will later be coupled, also through the power flow, to generate the model of a new subsystem representing the dynamic characteristics of the former system and their interactions. This approach adopts the same basis, concepts and elements of the Bond Graph Technique, without its symbolic notation and graphical representation. As an illustration, the procedure is applied for modeling the longitudinal, vertical and lateral coupled dynamics of a motorcycle, with the purpose of analyzing their stability in the vertical and lateral plane.*

Keywords: *Bicycles and Motorcycles Dynamics, Kinematic Relations, Power Flow, Equilibrium Relations, Bond Graphs.*

1. INTRODUCTION

The development of analytical models for land vehicles with single track (motorcycles and bicycle), unstable by nature, has been handled by some few authors, mainly due to the extremely particular characteristics and to its complexity, as it involves diverse and different interactions among their subsystems (driver, handlebar and frame, suspension and wheels / tires, engine, transmission and brakes), besides a phenomenon of dynamical mechanical nature, which is the gyroscopic effect, which in itself is a complicating factor.

This paper presents the preliminary results of a procedure for the complete representation of such vehicles, starting from the analytical models of its subsystems' modules, simpler and easier to build and analyze, which can be coupled to build a system model involving more than one set of components, according to the analyst's interests, as long as certain compatibility constraints are met.

This procedure is based on the Bond Graph Technique and all its potential for the development of modules' models. However the formalism and the graphical representation of Bond Graphs are no longer needed once the model of one module is built, as a function of the power flow and of the causality relations among their internal elements and mainly as a function of the elements in the power input and power output ports. Thus the module can be coupled to other modules whose models were created observing such restrictions and therefore the analytical model of a system constructed from multiple subsystems may be easily established by the consistent combination in terms of power and causality, avoiding the necessity of apply the formal treatment of the Bond Graph Technique.

Figure 1 shows the various typical subsystems of a ground vehicle of motorcycle/bicycle type, and the variables that characterize the interaction between them. The developed modules employ exactly the structure and the input and output variables shown.

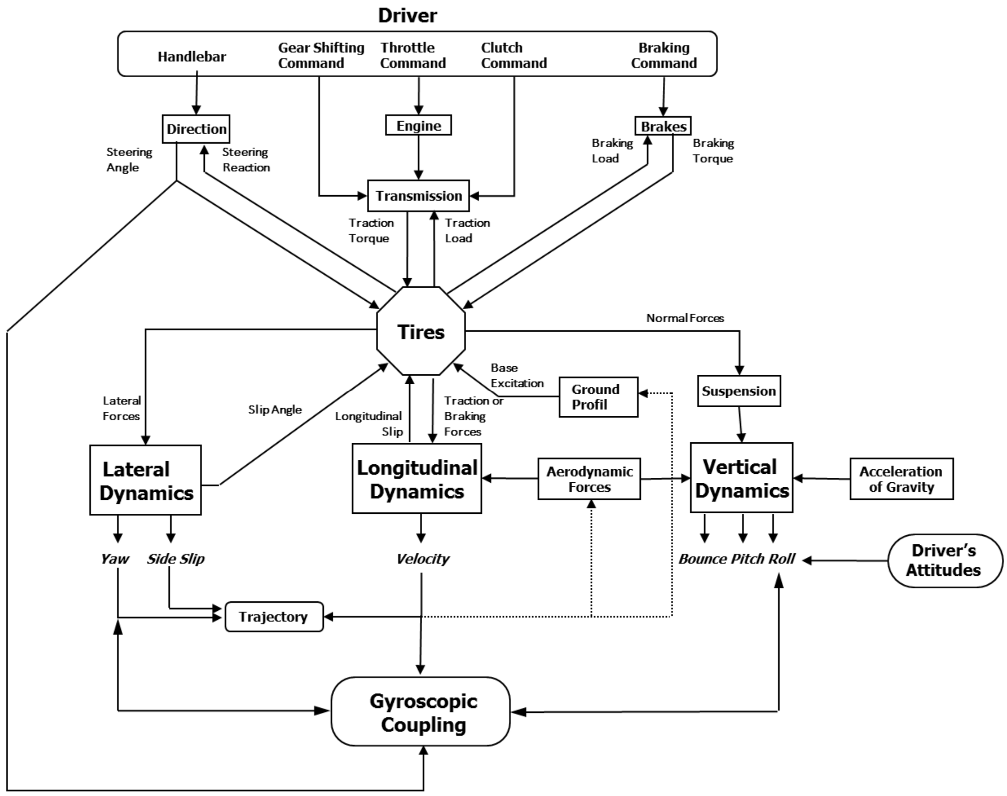


Figure 1. Ground Vehicle as a Dynamic System.
Subsystems Interaction in a Motorcycle.

In the following developed models, the driver's attitudes will not be considered, however, if necessary, the same approach can be applied and, once the representation of typical human movements in driving that vehicle is obtained, its inclusion in the appropriate degrees of freedom and its subsequent interaction as an additional subsystem are immediately established. The models will be described in the sequence considered as the most natural and easiest to understand, and in some subsystems the mathematical representations will be simplified or even omitted, since the interest of this article lies precisely in the description of the modules' determination procedure and in their interactions, from which the application of the formal treatment of Bond Graph Technique enables us to easily find the final analytical form of the model of each of the modules and of the subsystems formed by its combinations.

Figures 2 and 3 illustrate the motorcycle/bicycle multibody system with its typical bodies, the references adopted to describe their movements, their main angular velocities and geometric parameters, for identification of the notation used in the models' development. The other variables of the various subsystems will be defined in the following items, where necessary.

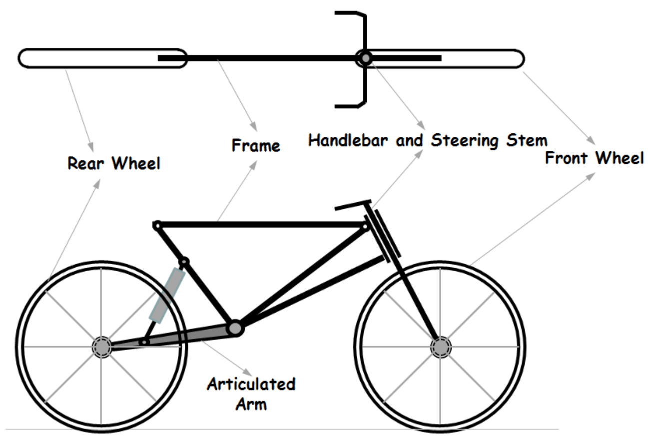


Figure 2. Motorcycle as a Multibody System.

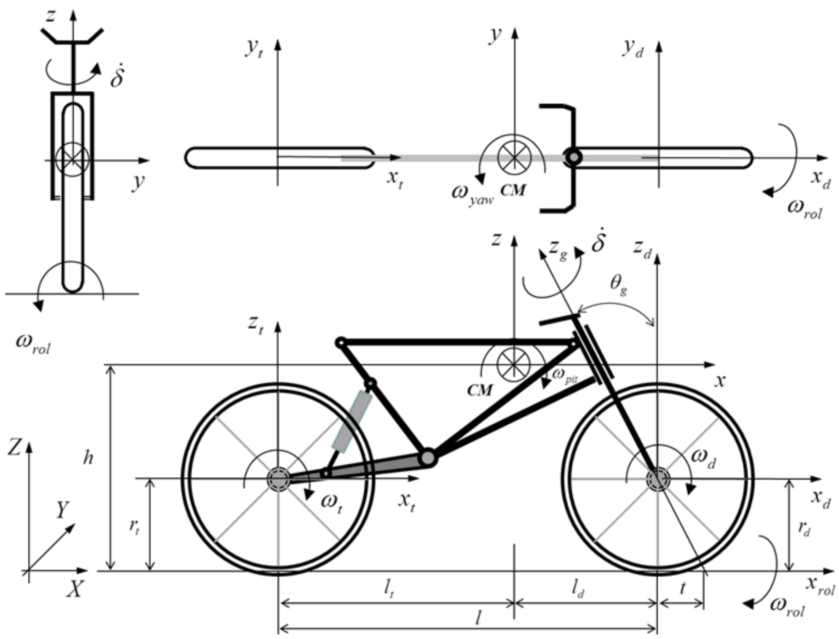


Figure 3. References, Angular Velocities and Geometric Parameters in the Motorcycle System.

2. LONGITUDINAL DYNAMICS

Initially the motorcycle's longitudinal dynamics model is presented, as it is the simplest, common to any vehicle and easy to understand. The Bond Graph (Fig. 4a) highlights the main system elements or components, the power flow among them (indicated by the half-arrow direction), and its cause and effect relations (represented by the effort sense through the vertical causal bar). The Power Flow Graph (Fig. 4b) is a compact way of representing the same system, in which only the components and their input and output variables are shown, such as the commands in the engine and in the brakes (δ_m, δ_d e δ_t), the inserted gear transmission ratio (N_i) and the variables of effort (torques and forces) and flow (angular and linear velocities) with the respective links of cause and effect in each of them.

The motor is represented just by its performance curve, obtained, for example, from experimental testing on a dynamometer, linking the command (δ_m) issued by the driver with the input angular velocity (ω_m) and the output torque (T_m). To address as fully as possible the power consumption in the system, losses in the transmission and wheel bearings (b_T *and* b_D) were included, as well as the slip of the rear tire (in traction), which may be modeled by any relation of interest, linear or non-linear, the front tire/wheel load and resistive forces (F_R) due to aerodynamic drag, to the slopes (route angle θ_{via}) and to the friction in the tire rolling (μ_r). The brake subsystem is not detailed, only the braking commands and the torques generated in the front and rear wheels are considered. Table 1 at the end of the article lists the parameters used in all treated models.

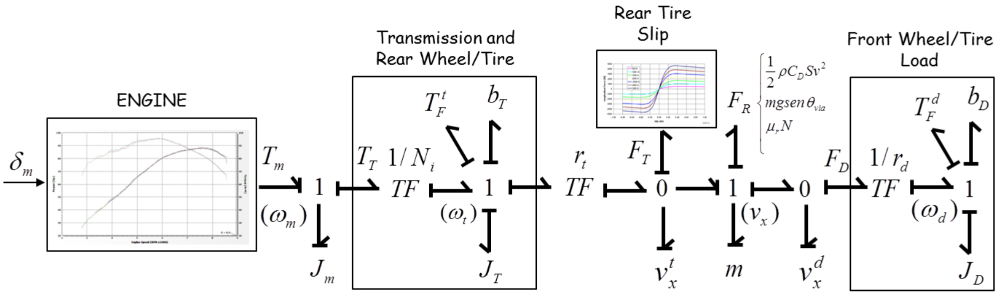
Neglecting the slip on the rear tire in traction, there is no independent longitudinal tire force generated by the slip in this element, and F_T is directly related to T_T, as well as ω_t and v_x. Assuming also no wheel locking during any braking condition, that is, that the maximum frictional force (μN) will not be exceeded in any tire, from the relations associated to the representations of Fig. 4, properly handled, the mathematical model that characterizes the longitudinal dynamics of a motorcycle (or bicycle), having as input the imposed commands on the propulsion element and on the brakes, and as outputs the acceleration, velocity and displacement of the vehicle, is given by

$$\begin{cases} \dfrac{dv_x}{dt} = \dfrac{1}{m_E}\left(\dfrac{N_i}{r_t} T_m(\delta_m, \dfrac{N}{r_t} v_x) - b_E v_x - \dfrac{1}{2}\rho C_D S v_x^2 - (\mu_r + \mu\delta_d) N_d - (\mu_r + \mu\delta_t) N_t - mg\, sen\theta_{via} \right) \\ \dfrac{dx}{dt} = v_x \end{cases} \tag{1}$$

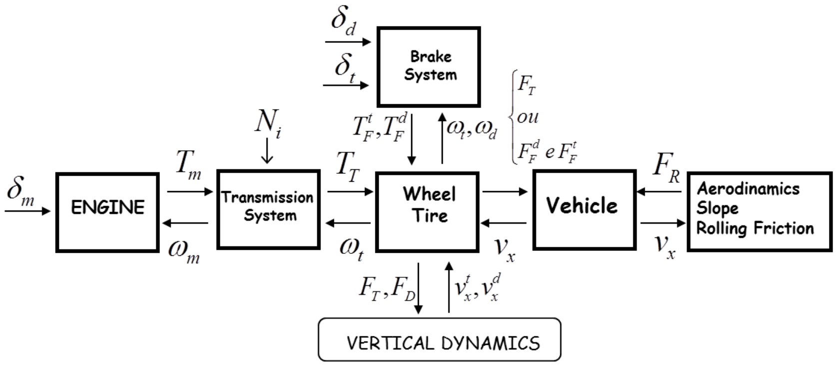
where the equivalent mass and dissipation are given by

$$\begin{cases} m_E = m + \dfrac{N_i^2}{r_t^2} J_m + \dfrac{1}{r_t^2} J_T + \dfrac{1}{r_d^2} J_D \\ b_E = \dfrac{N_i \dot{N}_i}{r_t^2} J_m + \dfrac{N_i^2}{r_t^2} b_m + \dfrac{1}{r_t^2} b_T + \dfrac{1}{r_d^2} b_D \end{cases} \tag{2}$$

and complementary relations that enables determining the normal forces (N_d and N_t) at the contact of the tires with the ground at each instant of time will be obtained from the interaction between models of Longitudinal and Vertical Dynamics, which will be treated below. Note that if the rolling friction on the tires is neglected and there is no braking, the normal forces do not interfere on the vehicle behavior in traction (speed boost), in this model which does not consider rear tire slip. It should be noted that models including the discarded effects can be easily obtained from the presented treatment.



(a) Bond Graph



(b) Power Flow Graph.

Figure 4. Model for Longitudinal Dynamics.

3. VERTICAL DYNAMICS

Figure 5 shows the physical model, the Bond Graph and the Power Flow Graph for a motorcycle vertical dynamics, considering the typical geometries of the front and rear suspensions. If a physical model is adopted in which there is no geometry of suspensions, i.e. the lines of action of the springs and dampers are vertical, aligned with the displacement of the non-suspended masses, and there is no dissipation in the tires, the resulting Power Flow Graph is that presented in Fig. 5c, since this representation does not need the "internal" detailed description of the system components. In that case, as

$$l_a^t = l_b^t = l_s^t = l_t \quad and \quad l_a^d = l_d \quad and \quad \alpha_t = \alpha_d = 0^o \quad and \quad b_{pd} = b_{pt} = 0 \tag{3}$$

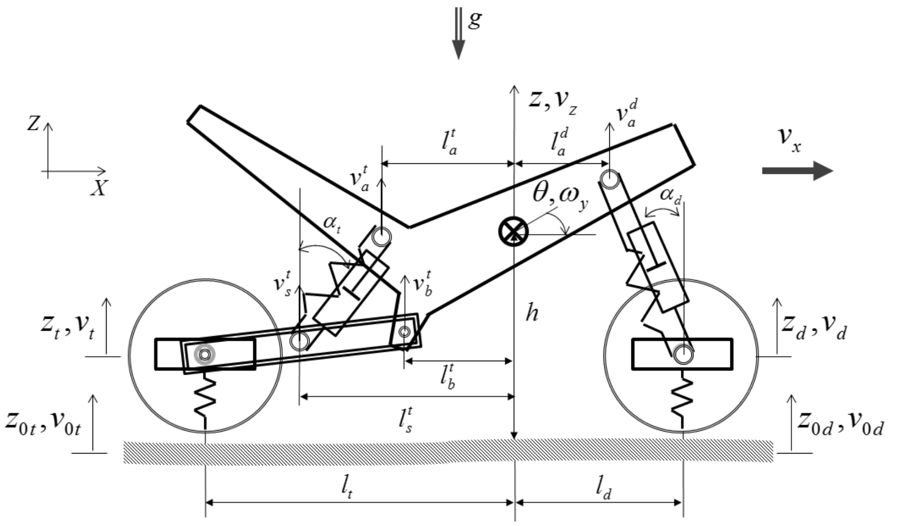
the mathematical model in the form of equations of motion used to represent the vertical dynamics in the XZ plane of a motorcycle on passive (conventional) suspension moving on a road with uneven ground, ripples or roughness, relating the inputs (F_z, M_y, F_d, F_t, z_{0d}, z_{0t}) with their degrees of freedom (z, θ, z_d, z_t) is given by

$$M\begin{bmatrix} \ddot{z} \\ \ddot{\theta} \\ \ddot{z}_d \\ \ddot{z}_t \end{bmatrix} + B\begin{bmatrix} \dot{z} \\ \dot{\theta} \\ \dot{z}_d \\ \dot{z}_t \end{bmatrix} + K\begin{bmatrix} z \\ \theta \\ z_d \\ z_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} F_z \\ M_y \\ F_d \\ F_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ k_{pd} & 0 \\ 0 & k_{pt} \end{bmatrix}\begin{bmatrix} z_{0d} \\ z_{0t} \end{bmatrix} \tag{4}$$

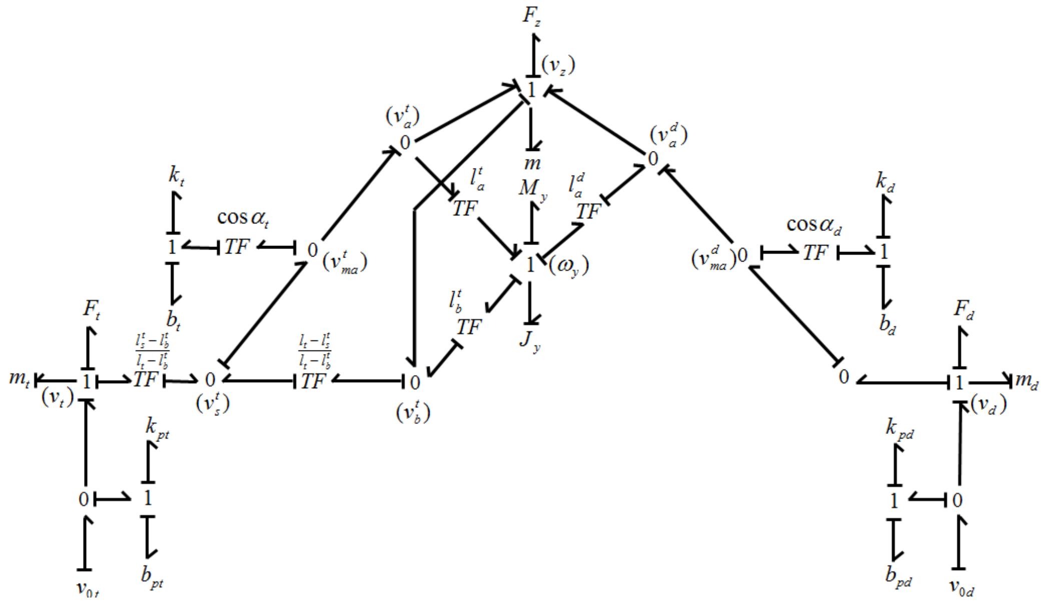
where

$$M = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & J_y & 0 & 0 \\ 0 & 0 & m_d & 0 \\ 0 & 0 & 0 & m_t \end{bmatrix}, \quad B = \begin{bmatrix} (b_d + b_t) & (l_t b_t - l_d b_d) & -b_d & -b_t \\ (l_t b_t - l_d b_d) & (l_t^2 b_t + l_d^2 b_d) & l_d b_d & -l_t b_t \\ -b_d & l_d b_d & b_d & 0 \\ -b_t & -l_t b_t & 0 & b_t \end{bmatrix}, \quad K = \begin{bmatrix} (k_d + k_t) & (l_t k_t - l_d k_d) & -k_d & -k_t \\ (l_t k_t - l_d k_d) & (l_t^2 k_t + l_d^2 k_d) & l_d k_d & -l_t k_t \\ -k_d & l_d k_d & k_d + k_{pd} & 0 \\ -k_t & -l_t k_t & 0 & k_t + k_{pt} \end{bmatrix} \tag{5}$$

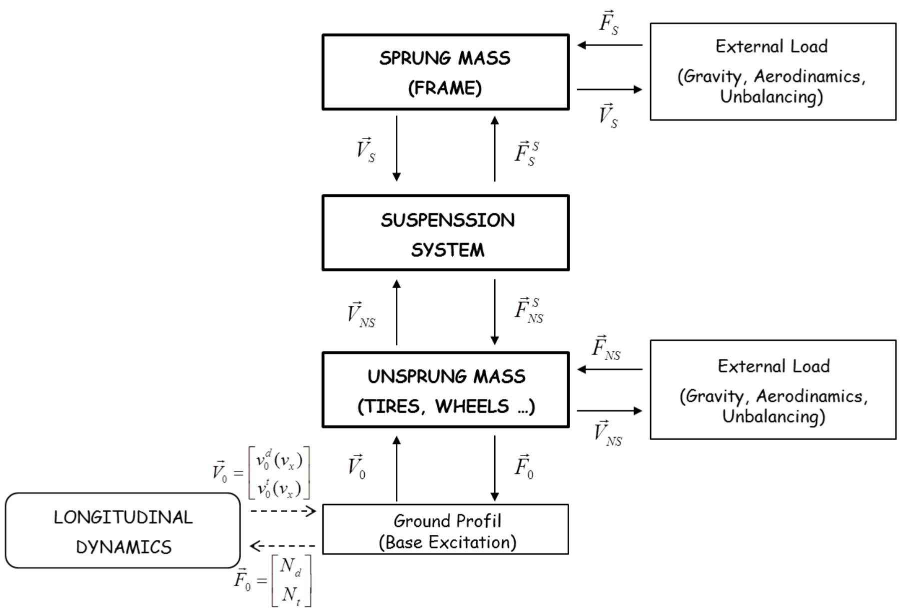
and $z_{0d}(t) = \int v_{0d}(t)\, dt$ and $z_{0t}(t) = \int v_{0t}(t)\, dt$ represent the ground conditions.



(a) Physical Model with Suspensions' Geometry.



(b) Bond Graph with Suspensions' Geometry.

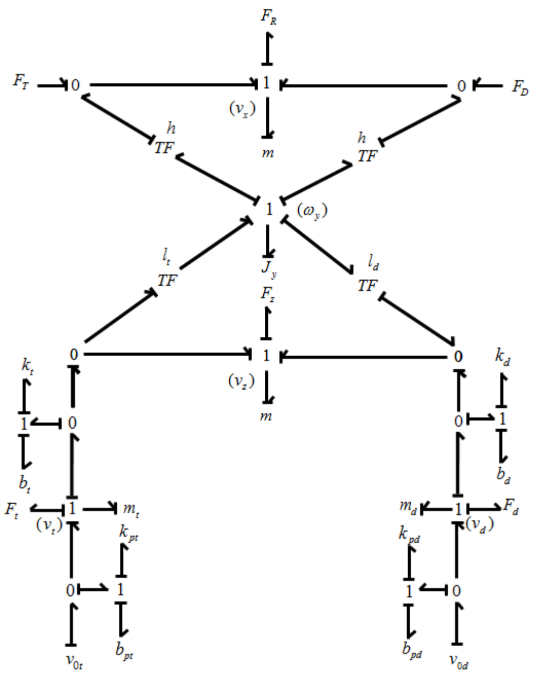


(c) Power Flow Graph.

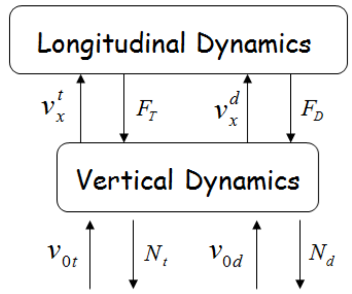
Figure 5. Vertical Dynamics Model.

4. INTEGRATION OF LONGITUDINAL AND VERTICAL DYNAMICS

Figure 6a shows a Bond Graph for the coupling/integration of Longitudinal Dynamics (not detailed) and Vertical Dynamics (represented by the vehicle without geometry of suspensions, according to the equations in the previous item). The link between the two models is given by the longitudinal velocities of the tires' contact points with the ground, in which the traction and/or braking forces (F_T e F_D) are applied. Figure 6b shows the Power Flow Graph of both coupled subsystems and its coupling variables.



(a) Bond Graph.



(b) Power Flow Graph.

Figure 6. Model for the Integration of Longitudinal and Vertical Dynamics.

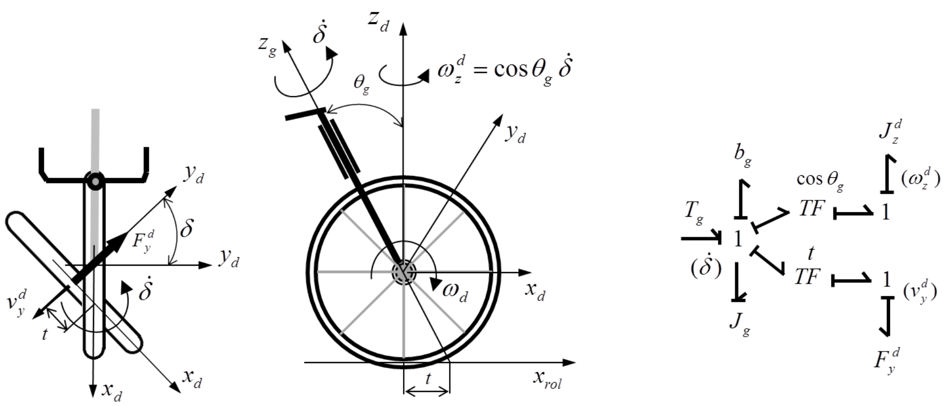
Disregarding the influence of the suspensions and the tires' deformations and dissipations in the vertical direction, the normal forces on the tire contact with the ground are obtained from the models of Fig. 6 and Fig. 4 combined, and are given by

$$\begin{cases} N_d(t) = \dfrac{mg(\cos\theta_{via}(l_t + \mu\delta_t h) - \sin\theta_{via}\ h) - \dfrac{N_i}{r_t} T_m h - \dfrac{1}{2}\rho\, C_D\, S v_x^2 h}{(l_t + l_d + \mu\,(\delta_t - \delta_d)h)} \\ N_t(t) = \dfrac{mg(\cos\theta_{via}(l_d - \mu\delta_d h) + \sin\theta_{via}\ h) + \dfrac{N_i}{r_t} T_m h + \dfrac{1}{2}\rho\, C_D\, S v_x^2 h}{(l_t + l_d + \mu\,(\delta_t - \delta_d)h)} \end{cases} \tag{6}$$

5. LATERAL DYNAMICS

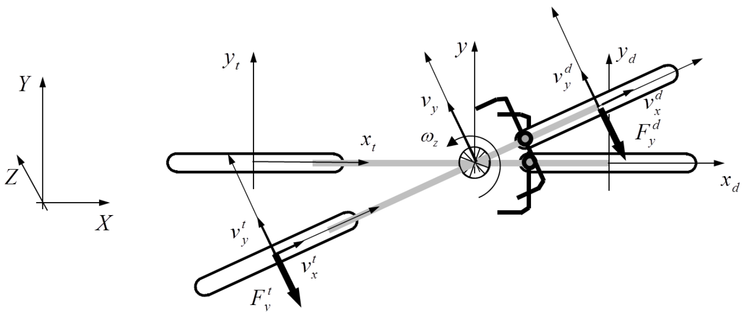
Figure 7 illustrates the physical model and the main variables of the handlebar subsystem and the Bond Graph which enables us to obtain the equations that describe its rotational movement (δ) as a function of the driver's torque (T_g), including the inertia (J_g) and the dissipation in the fork bearing (b_g), the front wheel inertia (J_z^d) in the vertical direction, and the restoring torque due to the trail and to the front lateral force (F_y^d) generated in the tire contact with the ground. This subsystem is responsible for the excitation of the Lateral Dynamics and for the directional control of the motorcycle/bicycle, especially if driver's attitudes (position of his body in relation to the vehicle) are not considered.

Figure 8 shows the physical model, the variables, the parameters and the Bond Graph for the lateral dynamics treatment, including its interaction with the longitudinal dynamics, since such a problem is usually described and solved in the vehicle local reference, requiring that the equations governing their movements are properly corrected due to the adoption of a non-inertial frame. The model considers only yaw rate of the chassis as significant, disregarding the influence of its roll and pitch rates.

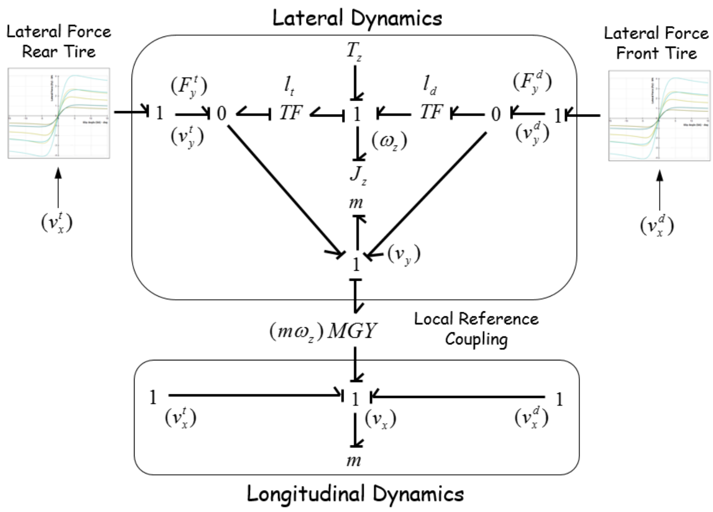


(a) Physical Model and Variables. **(b) Bond Graph.**

Figure 7. Handlebar Subsystem.



(a) Physical Model and Variables.



(b) Bond Graph

Figure 8. Lateral Dynamics

6. GYROSCOPIC COUPLING

Figure 9 shows the Bond Graph for the coupling of a rigid body's angular velocities on local reference frame, for a symmetric body, a simple way to treat the gyroscopic effect and its influence on a vehicle movement such as a motorcycle/bicycle. All couplings of this nature that are crucial for the understanding and the analysis of this system, were represented by the Fig. 9 structure, including the necessary relationships among the variables involved in the various bodies/subsystems where such an influence exists (frame, rear wheel, front wheel and handlebar).

From the Bond Graph of Fig. 9, the Euler equations can be readily determined,

$$\begin{bmatrix} \sum T_x \\ \sum T_y \\ \sum T_z \end{bmatrix} = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} + \begin{bmatrix} (J_z - J_y)\omega_z\omega_y \\ (J_x - J_z)\omega_z\omega_x \\ (J_y - J_x)\omega_y\omega_x \end{bmatrix} \tag{7}$$

and the problem of representation, understanding, interpretation, analysis and simulation of the gyroscopic effect can be solved in the transient and in the permanent regimes for each body associated with this structure or for a combination of bodies, as will be addressed below.

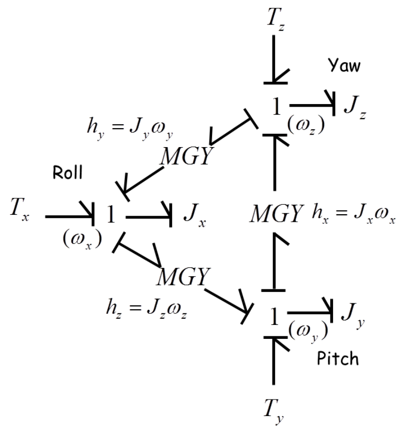


Figure 9. Bond Graph for the Rigid Body Coupling of Angular Velocities on a Local Reference Frame.

7. INTEGRATION OF LONGITUDINAL, VERTICAL AND LATERAL DYNAMICS

Figure 10 shows the integration of the several subsystems previously treated with the main motorcycle's structures (chassis and front and rear wheels) represented by Bond Graph in the previous item, where variables are associated with the transfer of power among them. With the modules developed and tested separately, the "assembly" of Fig. 10 enables us to visualize the complete system, as well as to determine analytically its mathematical model by the oriented manipulation of the equations of each one of its components or combinations of them. Furthermore, it is relatively simple to create any sub-model, starting from the union of the modules' models of interest.

We note that the link between the chassis and the wheels rolls is explained in the model of Fig. 10, and it would also be possible to explain the influence of its yaw forcing the wheels in that direction, what however was not included in order to simplify the representation.

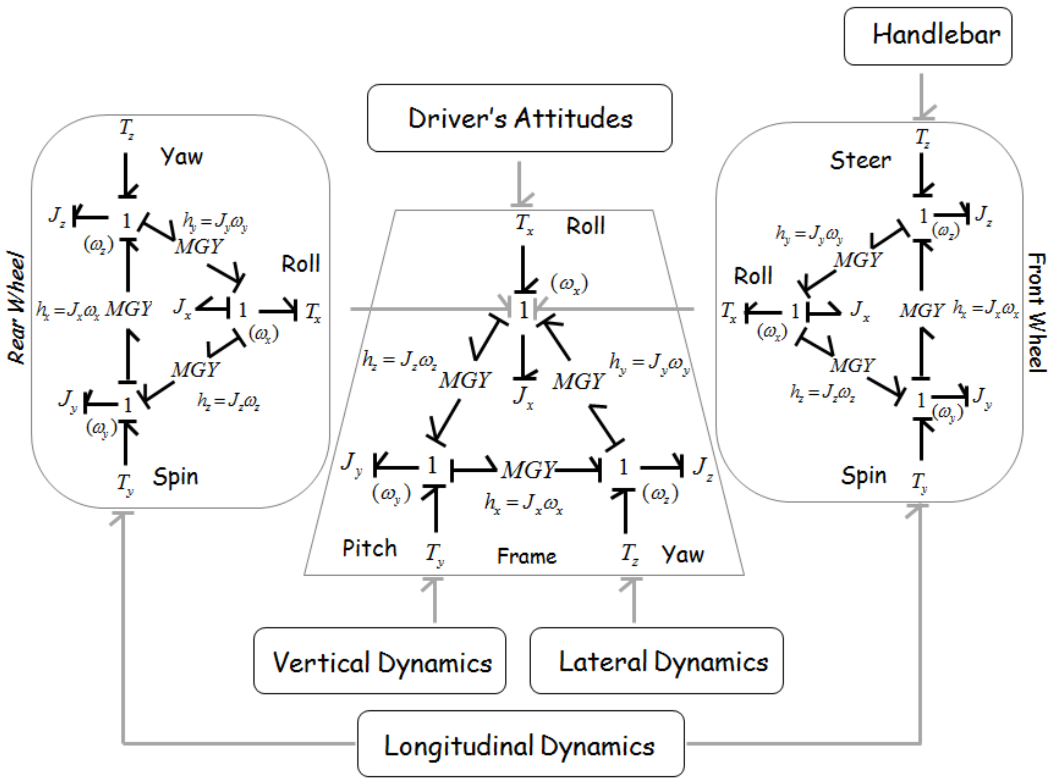


Figure 10. Mixed Bond and Power Flow Graph for Motorcycle System with Coupling for Rotational Movements.

8. CONCLUDING REMARKS

In order to simplify the presentation of the models, transformations of reference were purposefully omitted. It should be considered that these operations are implicit in the relationships among some of the variables of the various subsystems, or in some cases that the hypothesis of small rotation angles according to the nature of the expected movement was adopted, as in the case of the frame pitch over the suspensions.

The full model of the motorcycle system based on the presented methodology is being established, but tests with the modules and some of their combinations have been or are being carried out, and the results indicate that the procedure adopted is consistent and well-founded.

Table 1 lists the parameters (with the adopted symbols and their SI units) employed in the developed mathematical models, whose numerical values are being determined in such a way that the system dynamics can be resolved and analyzed.

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10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.

Table 1. Parameters used in dynamic models of the motorcycle/bicycle system.

Parameter	***Symbol***	***Unit***
Total mass (sprung + unsprung) – Longitudinal Dynamics	m	*kg*
Sprung mass (frame, handlebar + driver) – Vertical Dynamics	m	*kg*
Mass (unsprung) of the set of front wheel-tire	m_d	*kg*
Mass (unsprung) of the set of rear wheel-tire	m_t	*kg*
Efective radius of front wheel-tire (with ground contact)	r_d	*m*
Efective radius of rear wheel-tire (with ground contact)	r_t	*m*
Transmission ratio (i = gear)	N_i, $_{i=1...n}$	--
Transmission time variation rate	$\dot{N}_i$	*1/s*
Distance from the front axle to the center of mass (bike + driver)	l_d	*m*
Distance from the rear axle to the center of mass (bike + driver)	l_t	*m*
Distance between axles	$l = l_d + l_t$	*m*
Height of mass center (bike + driver) from the ground	h	*m*
Coefficient of rolling resistance on the tires	μ_r	--
Coefficient of tire-ground static friction (adhesion limit)	μ	--
Coefficient of dissipation in front wheel bearing	b_D	*Nm/rad/s*
Coefficient of dissipation in rear wheel bearing and transmission	b_T	*Nm/rad/s*
Vehicle pitch moment of inertia	J_y	$kg.m^2$
Stiffness of the suspensions springs	k_d e k_t	*N/m*
Damping in suspensions' shock absorbers	b_d e b_t	*N/m/s*
Stiffness of the tires	k_{pd} e k_{pt}	*N/m*
Damping in the tires	b_{pd} e b_{pt}	*N/m/s*
Cross-sectional area of the vehicle + driver body (in a position)	S	m^2
Drag coefficient of the vehicle + driver body (in a position)	C_D	--
Specific mass for the air (at sea level)	ρ	kg/m^3
Angle of the handlebar axis in relation to the vertical (head angle)	θ_g	*rad*
Trail	t	*m*
Acceleration of gravity	g	m/s^2
Engine spin moment of inertia	J_m	$kg.m^2$
Rear wheel/tire spin moment of inertia	J_T	$kg.m^2$
Front wheel/tire spin moment of inertia	J_D	$kg.m^2$
Route angle	θ_{via}	*rad*
Equivalent Mass	m_E	*kg*
Equivalent coefficient of dissipation	b_E	*N/m /s*
Engine coefficient of dissipation	b_m	*Nm/rad/s*
Distance from the rear superior anchorage point to the center of mass	l_a^t	*m*
Distance from the front superior anchorage point to the center of mass	l_a^d	*m*
Distance from the articulation of inferior suspension arm to the center of mass	l_b^t	*m*
Distance from the rear inferior anchorage point to the center of mass	l_s^t	*m*
Angle of the rear suspension axis in relation to the vertical	α_t	*rad*
Angle of the front suspension axis in relation to the vertical	α_d	*rad*
Fork moment of inertia	J_g	$kg.m^2$
Coefficient of dissipation in the fork bearing	b_g	*Nm/rad/s*
Moment of inertia of front wheel/tire in vertical direction	J_z^d	$kg.m^2$
Vehicle yaw moment of inertia	J_z	$kg.m^2$
Vehicle roll moment of inertia	J_x	$kg.m^2$