DINAME 2017 - Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics
A. T. Fleury, D. A. Rade, P. R. G. Kurka (Editors), ABCM, São Sebastião, SP, Brazil, March 5-10, 2017

Analysis of control strategies for autonomous motorcycles stabilization and trajectories tracking

Marília Maurell Assad[1], **Mauro Speranza Neto**[1], **Marco Antônio Meggiolaro**[1]

[1] Pontifícia Universidade Católica do Rio de Janeiro – Rua Marquês de São Vicente 255, Gávea, CEP 22451-900, Rio de Janeiro, RJ, Brasil, marilia.assad@gmail.com, msn@puc-rio, meggi@puc-rio

Abstract: Autonomous vehicles - defined as vehicles with carrying capacity of persons or property without the use of a human driver - are an interesting and recent problem, with increasing studies in the last twenty years. Besides being able to position itself globally and to predict trajectories, these systems have to deal with obstacles and traffic signals - a daily situation that requires a combination of sensors and control. Regarding this type of vehicles, a less explored option is the motorcycle: apart from the difficulties inherent in making a vehicle move independently, autonomous motorcycles have to be able to remain stable at any speed and trajectory. This work's main object of study is a small-scale electric motorcycle; represented by a linear model through a multibody approach: its four rigid bodies - wheels, chassis, handlebar and fork -, have separately a characteristic behavior and together they influence the dynamics of each other. This approach results in lower order models, easier to simulate and to apply classical or modern control strategies. The two-wheeled vehicle is considered an inverted pendulum with a mobile base and other simplifications are proposed, as constant displacement speed or small steering and yaw angles. Since this vehicle is naturally unstable, to ensure a follow-up course without overturning it is necessary to apply an adjusted control signal according to the motorcycle's behavior. The human driver does this unconsciously, setting the course through the handlebars and/or tilting the body to avoid falling. Once the autonomous system studied will not have the presence of a mechanical counterbalance, there remains only the first choice as a control strategy. Thus, this work analyzes the dynamic characteristics of the zero track vehicles and verifies the validity of different stability and path tracking control strategies of a motorcycle using as input only the steering of the handlebar.

Keywords: Autonomous vehicles, Motorcycle dynamics, Stability control, Trajectory control

NOMENCLATURE

Latin symbols

$\boldsymbol{C}$: virtual damping matrix
g: gravity
J: moment of inertia
$\boldsymbol{k}$: feedback gain vector
$\boldsymbol{K}$: stiffness matrix
$\boldsymbol{l}$: observer gain vector
$\boldsymbol{M}$: mass matrix
m: mass
$\boldsymbol{q}$: state variable vector
t: trail
v: velocity
x,y,z: distance in each plane
w: wheel base

Greek symbols

δ: steering angle
ε: caster angle
τ: torque
ϕ: roll angle
ψ: yaw angle

Subscripts

ff : relative to the front frame
fw : relative to the front wheel
m : relative to the main body
rf : relative to the rear frame
rw : relative to the rear wheel
s : relative to the steering body

INTRODUCTION

Motorcycles are an interesting study subject due to its unique dynamics: inherently unstable, they can achieve stability without a rider at certain speeds or settings, thanks to its geometry and gyroscopic effect. With respect to the mathematical modeling of this system, the most complete approach to describe its dynamic behavior is the multibody, where the motorcycle is interpreted as a combination of rigid bodies - including the wheels, bumpers, chassis, handlebar and fork driver - which have a characteristic behavior separately, but together influence the dynamics of one another.

An alternative step to obtain a more simplified multibody model is to consider only the main components of the vehicle, such as the driver, motorcycle's main body, wheels and front fork. This approach results in lower order models which are, therefore, easier to simulate and implement control strategies. Åström (2005), Keo (2011) and Tanaka (2009) adopt this path, which, briefly, consider the two-wheeled vehicle as an inverted pendulum mobile base; their work propose other simplifications, as a constant displacement speed or small steering and yaw angle. This approach was also chosen on this paper, since it easily allows to apply classical and modern control strategies (Sharp (2007) or Yi (2006)).

Regarding autonomous vehicles, a topic increasingly in vogue in recent years, it is important to comprehend the human driving behavior in order to successfully mimic it. The control exercised by motorcyclists is complex and divided into two main categories: the steering torque, applied at the motorcycle handlebar, and the driver's body roll movement, which cause a double reverse pendulum effect on the vehicle dynamics.

The effects of each controller is clear in a curve trajectory: in motorcycle competitions, the driver tilts its body in the same direction of the curve, which results in a torque that changes the rear wheel rotational axis, causing a arched trajectory. Examples of this application are in Keo and Yi, where an actuated inverted pendulum and a rotating mass reproduce the torque given by the driver.

The driving system also plays an important role in the motorcycle dynamics: to successfully perform a curve with only the steering torque, it is necessary to initially move the motorcycle's handlebar in the opposite direction of the desired movement and then steer it in the right path, a phenomenon known as *counter-steering*, as described by Fajans (2000). Guiding the vehicle in the opposite direction causes a centrifugal force that tilts the motorcycle in the desired direction, enabling a curved path. Interestingly, this method has the same operating principle of the previous one, without requiring a mass to cause the motorcycle inclination. Given that this work's object of study is a small-scale electric motorcycle, only the first category of control was applied, i.e., the stabilization and path tracking control are obtained solely by the steering torque.

MOTORCYCLE DYNAMICS

The motorcycle model, based on Meijaard (2007), is derived from the Whipple bicycle model: the vehicle is composed of two structures, or frames, joined by a revolution joint on the handlebars, with each frame supporting a free rotating axisymmetric wheel. The driver, originally simplified to a rigid extension of the main frame, is not considered on this paper; the front frame has uniformly distributed mass and wheels provide purely rotational movement, i.e., the sideslip and deformation of tires is not accounted in the model.

Roll, steer and yaw freedoms are allowed; longitudinal velocity is considered constant. To reduce the complexity of the model, the four rigid bodies have been simplified to two sets: the front frame includes the handlebar, fork and front wheel assembly; the rear frame is the main and rear wheel frame. The main system parameters are shown in "Fig. 1"; it is important to highlight the origin of the coordinate system – at the contact point between the rear wheel and the floor – and its direction, with the *z* axis positive in favor of gravity.

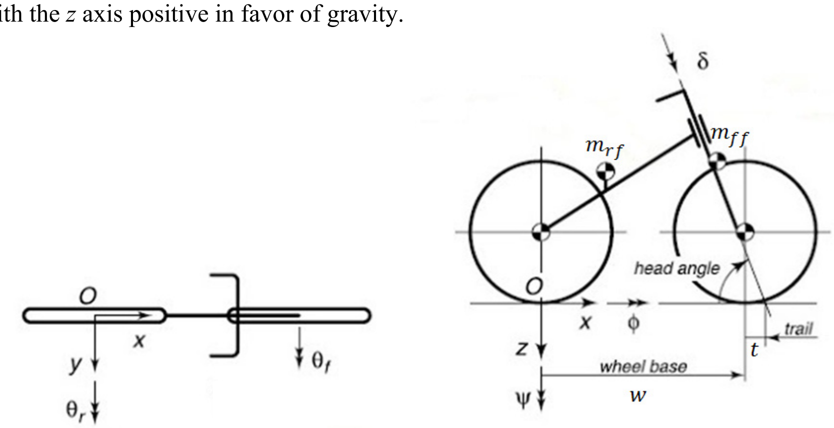


Figure 1 – Schematic of motorcycle's main parameters.

In order to validate the proposed model and test the suggested control strategies, an electric motorcycle was modeled using *SolidWorks.* The Duratrax450 is a 1/5 small-scale motorcycle, actuated by a *brushless* motor, which controls the vehicle speed, and steered by a servomotor, as seen in "Fig. 2 (a)". The motorcycle had its four main systems measured and weighted and the computational model, portrayed in "Fig. 2 (b)", gave its moments of inertia; the result is in "Tab. 1".

With this data, is possible to combine all four bodies into two main frames: the front frame, consisting of the front wheel and the steering body, and the rear frame, including the rear wheel and main body. Meijaard details this variables' manipulation, which makes explicit some important relations, such as the gyroscopic effect, i.e., torque about one axis due to angular acceleration about the other. The motorcycle model is then completely defined by 19 design parameters, including the vehicle's velocity; "Table 2" provides additional variables, dependent on main variables shown in "Tab. 1", whose development is explicit in Meijaard.

Table 1 – Duratrax450's main experimental data.

	Parameter	Symbol	Value
	Wheel base (m)	w	0.31
	Trail (m)	t	0.028
	Caster angle (rad)	ε	0.49
	Gravity (m/s²)	g	9.81
Rear wheel			
	Center of mass (m)	$(x_{rw} \quad y_{rw} \quad z_{rw})$	$(0 \quad 0 \quad -0.061)$
	Mass (kg)	m_{rw}	0.69
	Moments of inertia (kg.m²)	$\lfloor J_{rw_{xx}} \quad J_{rw_{yy}} \quad J_{rw_{zz}} \rfloor$	$[5.04 \quad 9.16 \quad 5.04]\,10^{-4}$
Main body			
	Center of mass (m)	$(x_m \quad y_m \quad z_m)$	$(0.14 \quad 0 \quad -0.11)$
	Mass (kg)	m_m	1.19
	Moments of inertia (kg.m²)	$\begin{bmatrix} J_{m_{xx}} & 0 & J_{m_{xz}} \\ 0 & J_{m_{yy}} & 0 \\ J_{m_{xz}} & 0 & J_{m_{zz}} \end{bmatrix}$	$\begin{bmatrix} 20.02 & 0 & -4.01 \\ 0 & 41.92 & 0 \\ -4.01 & 0 & 29.61 \end{bmatrix} 10^{-4}$
Steering body			
	Center of mass (m)	$(x_s \quad y_s \quad z_s)$	$(0.27 \quad 0 \quad -0.13)$
	Mass (kg)	m_s	0.13
	Moments of inertia (kg.m²)	$\begin{bmatrix} J_{s_{xx}} & 0 & J_{s_{xz}} \\ 0 & J_{s_{yy}} & 0 \\ J_{s_{xz}} & 0 & J_{s_{zz}} \end{bmatrix}$	$\begin{bmatrix} 4.17 & 0 & 1.94 \\ 0 & 4.58 & 0 \\ 1.94 & 0 & 1.89 \end{bmatrix} 10^{-4}$
Front wheel			
	Center of mass (m)	$(x_{fw} \quad y_{fw} \quad z_{fw})$	$(0.31 \quad 0 \quad -0.065)$
	Mass (kg)	m_f	0.12
	Moments of inertia (kg.m²)	$\lfloor J_{fw_{xx}} \quad J_{fw_{yy}} \quad J_{fw_{zz}} \rfloor$	$[1.63 \quad 3.19 \quad 1.63]\,10^{-4}$
Rear frame			
	Center of mass (m)	$(x_{rf} \quad y_{rf} \quad z_{rf})$	$(0.09 \quad 0 \quad -0.087)$
	Mass (kg)	m_{rf}	1.88
	Moments of inertia (kg.m²)	$\begin{bmatrix} J_{rf_{xx}} & 0 & J_{rf_{xz}} \\ 0 & - & 0 \\ J_{rf_{xz}} & 0 & J_{rf_{zz}} \end{bmatrix}$	$\begin{bmatrix} 3.41 & 0 & 2.19 \\ 0 & - & 0 \\ 2.19 & 0 & 12.23 \end{bmatrix} 10^{-3}$
Front frame			
	Center of mass (m)	$(x_{ff} \quad y_{ff} \quad z_{ff})$	$(0.29 \quad 0 \quad -0.10)$
	Mass (kg)	m_{ff}	0.25
	Moments of inertia (kg.m²)	$\begin{bmatrix} J_{ff_{xx}} & 0 & J_{ff_{xz}} \\ 0 & - & 0 \\ J_{ff_{xz}} & 0 & J_{ff_{zz}} \end{bmatrix}$	$\begin{bmatrix} 8.27 & 0 & 0.28 \\ 0 & - & 0 \\ 0.28 & 0 & 4.09 \end{bmatrix} 10^{-4}$

(a) Small-scale electric motorcycle Duratrax450. **(b) Computational model by *SolidWorks*.**

Figure 2 – Test apparatus.

Table 2 – Duratrax450's additional parameters.

Parameter	Symbol	Value	Parameter	Symbol	Value
Total mass (kg)	m_t	2.13	Global moment of inertia about *x* axis (kg.m²)	J_{xx}	2.11×10^{-2}
Global centre of mass in *x* (m)	x_t	0.11	Global moment of inertia about *x* and *z* axis (kg.m²)	J_{xz}	2.41×10^{-2}
Global centre of mass in *z* (m)	z_t	-0.089	Global moment of inertia about *z* axis (kg.m²)	J_{zz}	4.83×10^{-2}
Perpendicular distance from front frame centre of mass to steering axis (m)	f	0.079	Front frame moment of inertia about *x* axis (kg.m²)	$J_{\varepsilon x}$	5.11×10^{-4}
Front wheel gyroscopic coefficient	S_f	3.80×10^{-3}	Front frame moment of inertia about *z* axis (kg.m²)	$J_{\varepsilon z}$	6.64×10^{-4}
Global gyroscopic coefficient	S_t	2.46×10^{-2}	Front frame moment of inertia about steer axis (kg.m²)	$J_{\varepsilon\varepsilon}$	5.27×10^{-4}
Static moment	S_u	1.99×10^{-2}			

Given that the motorcycle is considered to be freely rolling forward on the *xy* plane, with constant speed, and approximately parallel to the global *x* axis – i.e., small roll, yaw and steer angles, as well as small displacement on the *y* axis –, the governing linear equations of motion are the ones referring to two lateral degrees of freedom: the rightward lean of the rear frame (ϕ) and rightward steer of handlebars (δ). The forces acting on the system are: gravity in each center of mass, the ground reactions to the front wheel and the torque applied to the steering system, T_δ, considered positive when moving the handlebars to the right, and applied negatively on the rear frame. The lean and steer equations are described, respectively, as the first and second lines of "Eq. 1".

$$\begin{bmatrix} J_{xx} & fJ_{xz}+J_{\varepsilon x} \\ fJ_{xz}+J_{\varepsilon x} & f^2J_{zz}+2fJ_{\varepsilon z}+J_{\varepsilon\varepsilon}\end{bmatrix}\begin{bmatrix}\ddot{\phi}\\ \ddot{\delta}\end{bmatrix}+v\begin{bmatrix} 0 & S_f\cos(\varepsilon)+f(S_t-m_tz_t)+J_{xz}\dfrac{\cos(\varepsilon)}{w} \\ -fS_t-S_f\cos(\varepsilon) & fS_u+(fJ_{zz}+J_{\varepsilon z})\dfrac{\cos(\varepsilon)}{w}\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\delta}\end{bmatrix}+$$
$$+\left(g\begin{bmatrix} m_tz_t & -S_u \\ -S_u & -S_u\sin(\varepsilon)\end{bmatrix}+v^2\begin{bmatrix} 0 & (S_t-m_tz_t)\dfrac{\cos(\varepsilon)}{w} \\ 0 & (S_u+S_f\sin(\varepsilon))\dfrac{\cos(\varepsilon)}{w}\end{bmatrix}\right)\begin{bmatrix}\phi\\ \delta\end{bmatrix}=\begin{bmatrix}0\\ T_\delta\end{bmatrix} \tag{1}$$

The previous expression allows a parallel with a mechanical mass-spring-damper system; the matrix **M** is symmetric, positive-definite and gives the kinect energy of the system; the "virtual" damping matrix **C** represents gyroscopic torques due to tilting ($\dot{\phi}$) and steering ($\dot{\delta}$) and inertial reactions to yaw acceleration; the stiffness matrix **K** has a system independent of longitudinal speed ($\mathbf{K_0}$) which represents changes in potential energy and a system ($\mathbf{K}_2$) who is due to gyroscopic and centrifugal effects. The dynamical system may also be written in state-space representation; the numerical result to "Tab. 1" data is in "Eq. 2".

$$\begin{bmatrix}\ddot{\phi}\\ \ddot{\delta}\\ \dot{\phi}\\ \dot{\delta}\end{bmatrix}=\begin{bmatrix} -0.93v & -3.5v & 91.0 & -30.0v^2-2.7 \\ 8.1v & -6.4v & -26.0 & 12.0v^2+100.0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{\phi}\\ \dot{\delta}\\ \phi\\ \delta\end{bmatrix}+\begin{bmatrix}-177.0\\ 1.5\times10^3\\ 0\\ 0\end{bmatrix}T_\delta \tag{2}$$

It is known that, for certain speed ranges, the motorcycle is able to remain stable without the aid of a driver; thus it is interesting to analyze the vehicle stability with zero input torque. The four poles of this system can be divided in three categories: the smaller real value is associated with capsize mode, in which the motorcycle falls sideways, with the roll and steering angles slowly increasing; imaginary eigenvalues are associated with the weave mode, in which the rear frame oscillates around the steering body; the last eigenvalue, real and with larger module, is known as the wobble mode, in which the steering shaft oscillates and aligns quickly with the motorcycle body in the direction of its movement, in a tractrix-like movement. The weave and wobble modes are significant at high speeds (up to 10 m/s), since these oscillatory effects can lead to serious instability.

Given the maximum speed of the electric motorcycle is of 15 m/s, the numerical result to the eigenvalues variation with speed increase is in "Fig. 3". The analysis of eigenvalues position in an uncontrolled configuration shows that the Duratrax450 is not self-stabilizing, since its weave mode never reach the stable region; most likely this phenomenon is due to, mainly, its geometric design, with a steering body much lighter than the rear frame.

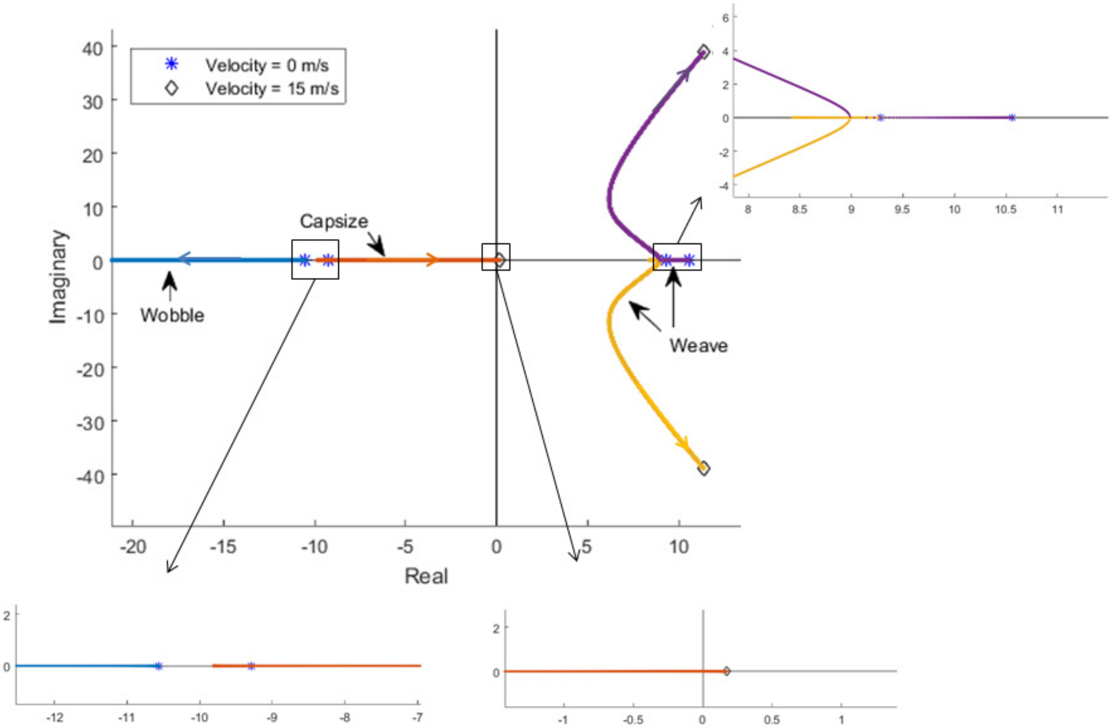


Figure 3 – Poles variation with speed increase.

STABILIZATION CONTROL STRATEGIES

The first step to control an autonomous motorcycle is to balance it so it will not fall down. In this work, two strategies to keep the motorcycle straight are analyzed: an ideal state feedback control and a state feedback control with an observer, since not all state variables are measurable.

State feedback

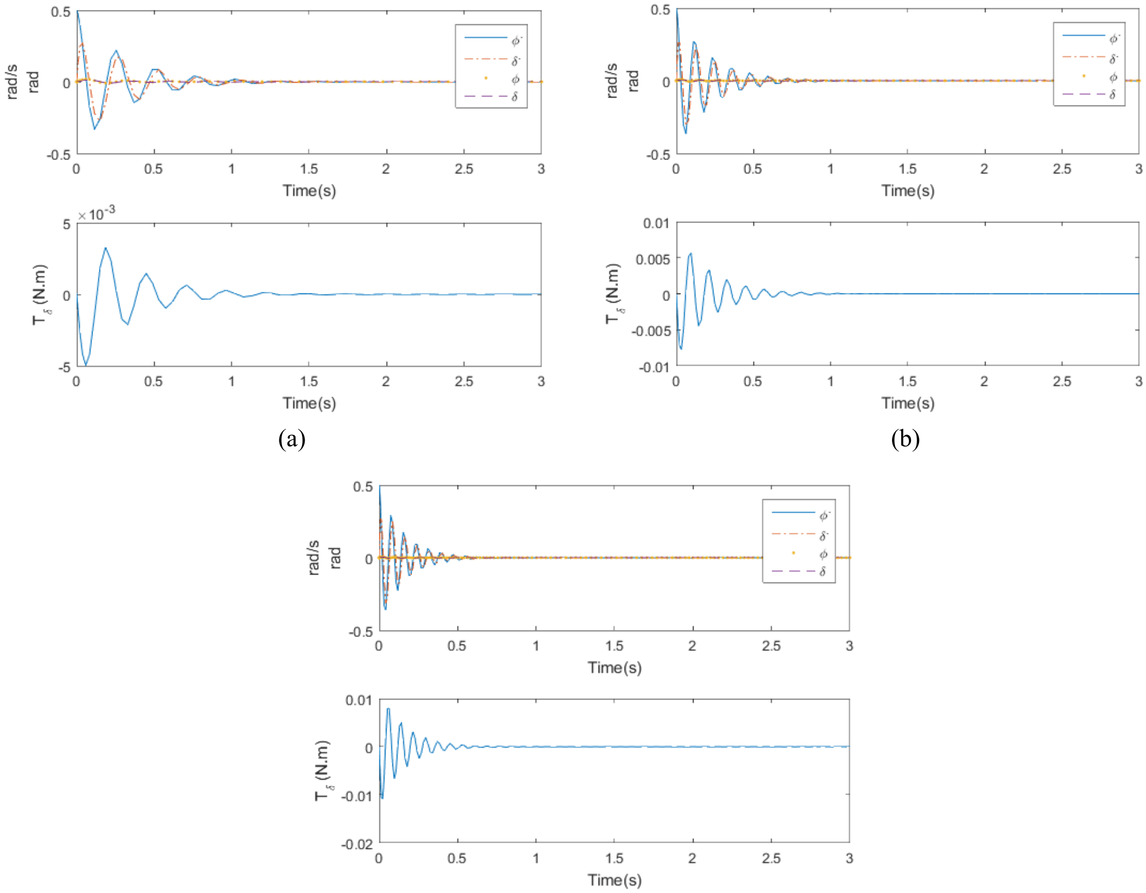
Considering all state variables ideally measurable, the state space model output of the motorcycle systems becomes the identity matrix, and the input control is in "Eq. 3". To verify if the state feedback can be made in this new system, the controllability matrix must have full rank; this system, for any speed range, is completely controllable.

$$u = -\boldsymbol{kX} = -\begin{bmatrix} k_{\dot{\phi}} & k_{\dot{\delta}} & k_{\phi} & k_{\delta} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\delta} \\ \phi \\ \delta \end{bmatrix} \tag{3}$$

Since the capsize and wobble eigenvalues are already stable – negative real poles – the main objective of the state feedback is to change the complex part of the weave eigenvalues. "Table 3" brings the gain vector for three speeds and "Fig. 4" illustrates the simulation results, considering an initial state of $\begin{bmatrix} 0.5 & 0 & 0 & 0 \end{bmatrix}$; is worth mentioning that the maximum torque provided by the servomotor is 0.32 N.m.

Table 3 – State feedback gain to three different speeds.

Speed (m/s)	Desired eigenvalues	State feedback gain
5	$\begin{bmatrix} -0.68 & -3.1 \pm 24.0i & -42.0 \end{bmatrix}$	$\begin{bmatrix} -8.2\times10^{-5} & 8.3\times10^{-3} & -4.3\times10^{-2} & 0.35 \end{bmatrix}$
10	$\begin{bmatrix} -0.18 & -4.6 \pm 52.0i & -82.0 \end{bmatrix}$	$\begin{bmatrix} 1.2\times10^{-5} & 1.2\times10^{-2} & -2.9\times10^{-2} & 0.98 \end{bmatrix}$
15	$\begin{bmatrix} -0.1 & -6.4 \pm 80.0i & -122.0 \end{bmatrix}$	$\begin{bmatrix} 1.2\times10^{-5} & 1.7\times10^{-2} & -2.8\times10^{-2} & 2.07 \end{bmatrix}$



(c)

Figure 4 – Simulation to stabilization state feedback at: (a) 5 m/s; (b) 10 m/s; (c) 15 m/s.

State feedback with observer

In practice, not all state variables are available for feedback: only the steer angle (δ), given by the servomotor, and the roll angular speed ($\dot{\varphi}$), measured by a gyrometer, are directly measurable. Therefore, "Eq. 4" brings the observed state, where l is the observer gain vector, and the input control is in "Eq. 5", a state feedback of the observed state. To verify if the full order state observer can be made in this new system, the observability matrix must have full rank; again, for any speed range, the system is completely observable.

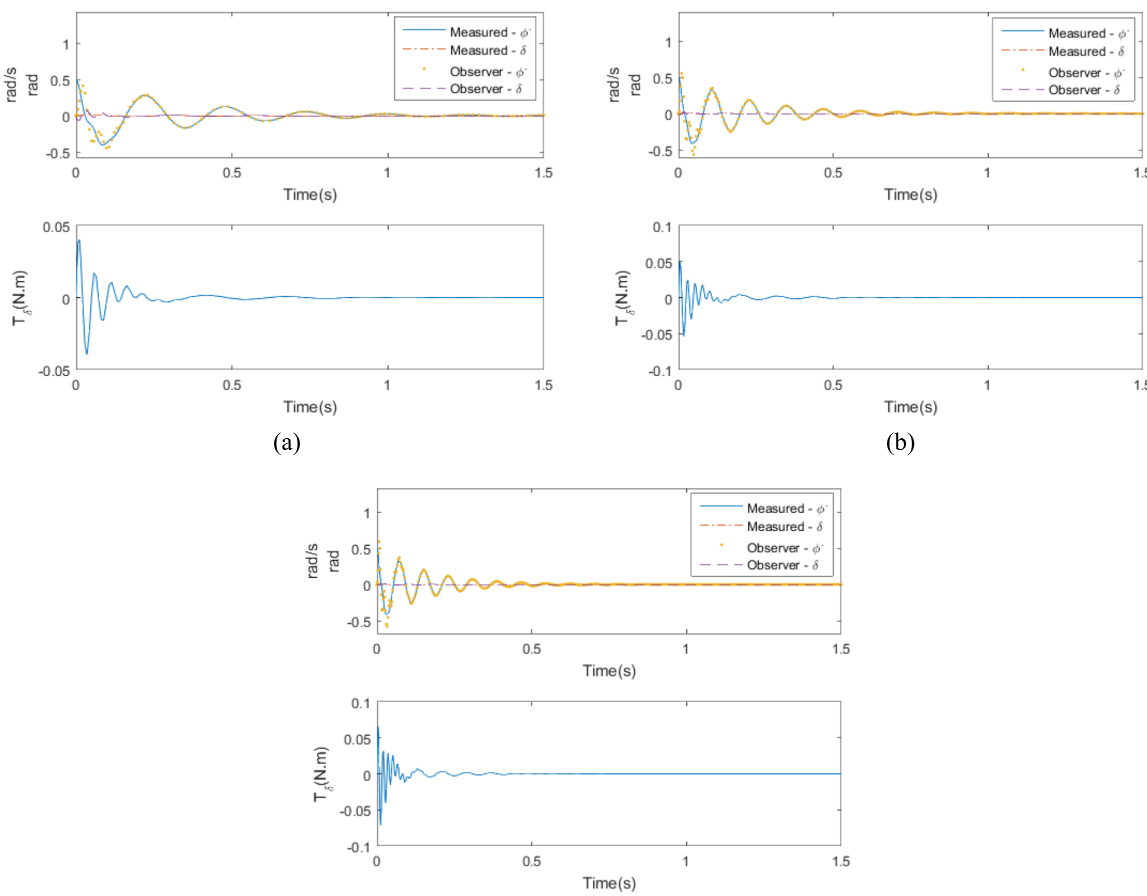
$$\dot{\tilde{\mathbf{X}}} = \mathbf{A}\tilde{\mathbf{X}} + \mathbf{l}\mathbf{C}\left(\mathbf{X} - \tilde{\mathbf{X}}\right) + \mathbf{B}\mathbf{u} \tag{4}$$

$$u = -\mathbf{k}\tilde{\mathbf{X}} \tag{5}$$

Since the observer and state feedback gain are independent, it is possible to choose two different sets of desirable eigenvalues; the poles of the observer are usually chosen so that its response is faster than the system's, roughly five times faster, i.e., multiplied by five. The feedback poles will follow the previous method's design, keeping the open loop eigenvalues stable, meaning the feedback gain will be the same in "Tab. 3". "Table 4" brings the observer gain vector for three speeds and "Fig. 5" illustrates the simulation results, considering an initial state of $\begin{bmatrix}0.5 & 0 & 0 & 0\end{bmatrix}$ only to the measured state space.

Table 4 – Observer gain to three different speeds.

Speed (m/s)	Desired eigenvalues	Observer gain vector
5	$\begin{bmatrix}-3.39 & -15.7 \pm 121.9i & -210.8\end{bmatrix}$	$\begin{bmatrix}5.22 & -773.25 \\ -650.36 & 22.03 \\ 27.84 & -32.06 \\ -42.80 & 203.89\end{bmatrix}$
10	$\begin{bmatrix}-0.89 & -22.8 \pm 262.4.0i & -410.1\end{bmatrix}$	$\begin{bmatrix}-26.25 & -3.04\times10^{3} \\ -1.89\times10^{3} & 1.19\times10^{3} \\ 12.26 & -18.40 \\ -16.95 & 409.79\end{bmatrix}$
15	$\begin{bmatrix}-0.51 & -32.1 \pm 398.7i & -612.0\end{bmatrix}$	$\begin{bmatrix}-44.75 & -6.84\times10^{3} \\ -2.96\times10^{3} & 2.75\times10^{3} \\ 9.85 & -12.06 \\ -9.87 & 611.86\end{bmatrix}$



(c)

Figure 5 – Simulation to stabilization state feedback with observer at: (a) 5 m/s; (b) 10 m/s (c)15 m/s.

TRAJECTORY CONTROL STRATEGY

The second step to control an autonomous motorcycle is to make it follow a desired trajectory. Since the previous model only considered the roll and steer angles and speeds, it needs to be adjusted to include the yaw angle (ψ) and lateral speed. “Equation 6” shows the yaw angular speed, “Eq. 7”, the rear wheel lateral velocity and “Eq. 8”, the new state space model, all based on Meijaard and considering small displacements and angles. With the altered model, described in the “Eq. 8”, the state feedback strategy is applied at the system, to make a lane change with the motorcycle.

$$\dot{\psi} = \frac{cos(\varepsilon)}{w}\left(t\dot{\delta} + v\delta\right) \tag{6}$$

$$\dot{y} = v\psi \tag{7}$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\delta} \\ \dot{\phi} \\ \dot{\delta} \\ \dot{\psi} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -0.93v & -3.5v & 91.0 & -30.0v^2 - 2.7 & 0 & 0 \\ 8.1v & -6.4v & -26.0 & 12.0v^2 + 100.0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0.079 & 0 & 2.9v & 0 & 0 \\ 0 & 0 & 0 & 0 & v & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\delta} \\ \phi \\ \delta \\ \psi \\ y \end{bmatrix} + \begin{bmatrix} -177.0 \\ 1.5\times10^3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} T_\delta \tag{8}$$

State feedback

Considering all new state variables ideally measurable and the path desired y_d, the input control becomes "Eq. 9". In this new system, two eigenvalues are added at the origin; the main objective of the state feedback is to change the positive complex eigenvalues and to make those zero eigenvalues negative. Unlike the previous strategies, the desired eigenvalues are the same to every speed simulated, in order to guarantee an equal time response, given by the dominant eigenvalue.

"Table 5" brings the gain vector for three speeds and "Fig. 6", a comparison between the path traveled in each velocity; "Fig. 7" illustrates the simulation results, with zero initial condition and a desired trajectory of lane change with one meter of lateral displacement. The simulation results make clear the *countersteering* phenomenon, in which the motorcycle is initially move to the opposite side to which you want to move. The adjustment of controller's gains has respected the system torque limit, which translates to a higher stabilization time than expected.

$$u = y_d - \boldsymbol{kX} = y_d - \begin{bmatrix} k_{\dot{\phi}} & k_{\dot{\delta}} & k_{\phi} & k_{\delta} & k_{\psi} & k_{y} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\delta} \\ \phi \\ \delta \\ \psi \\ y \end{bmatrix} \tag{9}$$

Table 5 – State feedback gain to three different speeds.

Speed (m/s)	Desired eigenvalues	State feedback gain
5	$[-1 \ -5 \ -10 \ -15 \ -20 \ -25]$	$[-3.98\times10^{-3} \ \ 2.55\times10^{-2} \ \ -0.31 \ \ 1.11 \ \ -0.28 \ \ -3.91\times10^{-2}]$
10	$[-1 \ -5 \ -10 \ -15 \ -20 \ -25]$	$[4.47\times10^{-2} \ \ 7.01\times10^{-3} \ \ -9.75\times10^{-2} \ \ 2.03 \ \ -0.14 \ \ -9.88\times10^{-3}]$
15	$[-1 \ -5 \ -10 \ -15 \ -20 \ -25]$	$[7.6\times10^{-2} \ \ -1.34\times10^{-2} \ \ -5.42\times10^{-2} \ \ 3.10 \ \ -9.50\times10^{-2} \ \ -4.30\times10^{-3}]$

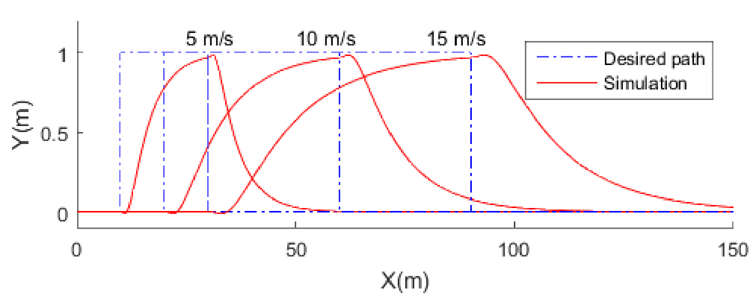
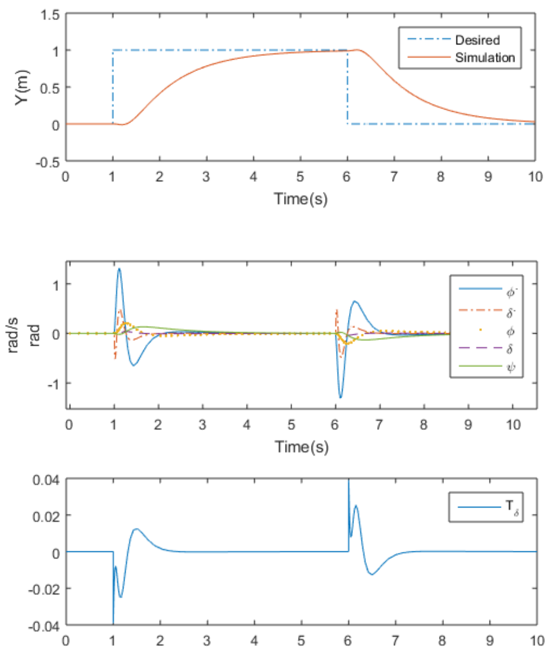
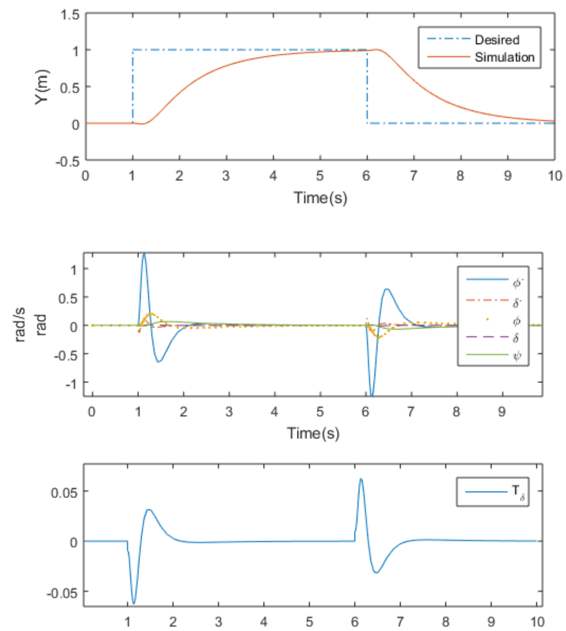


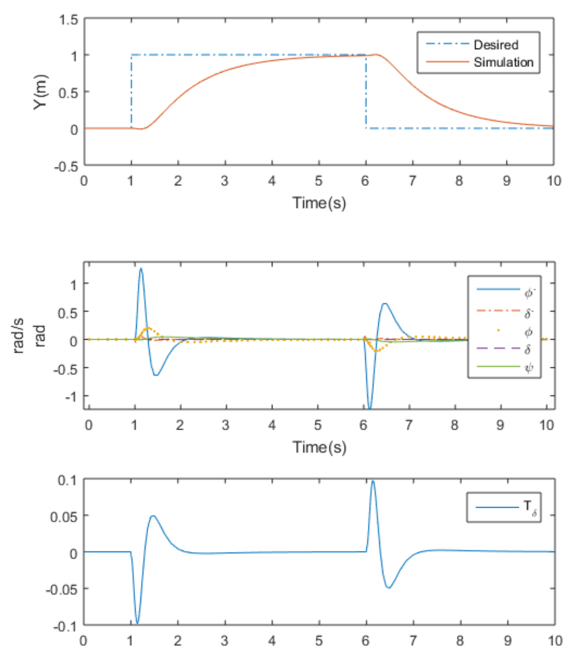
Figure 6 – Comparison between trajectory control at: (a) 5 m/s; (b) 10 m/s; (c) 15 m/s.



(a)



(b)



(c)

Figure 7 – Simulation to trajectory state feedback at: (a) 5 m/s; (b) 10 m/s; (c) 15 m/s.

CONCLUSIONS

In this work the linearized model of a small-scale electric motorcycle was equated through a multibody approach, considering its main four rigid bodies - wheels, chassis, handlebar and fork. The physical system was also modelled in a computational simulator in order to provide all moments of inertia and other minor parameters.

Additional simplifications were brought into the system to facilitate its analysis: the two-wheeled vehicle is considered an inverted pendulum with a mobile base, with constant longitudinal velocity and small steering, roll and yaw angles.

Three control strategies are proposed to act on the system natural instability and replace a human driver. The usual approach is to maintain balance and set the trajectory through the handlebars and tilting the body to avoid falling. Since the small scale motorcycle does not have the presence of a mechanical counterbalance, there remains only the steering system a control input. Thus, this work analyzed the stability and path tracking control strategies of a motorcycle using as input only the steering of the handlebar.

Numerical simulations with the complete system were carried out to prove the modern control strategies' validity: a state space feedback controller was capable to successfully keep the motorcycle upright; since only two of the state variables are measurable by the instrumentation system, another space feedback was proposed, also successful, with an state observer to estimate the missing variables values and optimize the control algorithm.

At last, a state feedback was proposed to make the motorcycle follow a desired trajectory, i.e., a lane change path. The state space model was adjusted to include two new variables regarding the lateral dynamics of the system and simulations confirmed the validity of this strategy.

As future works, the authors intend to improve the dynamic motorcycle model, so as to make lateral displacement, yaw and roll motions of the bicycle closer to reality. The authors also intend to investigate other control strategies to the proposed problem, such as a linear quadratic regulator, in order to minimize the path tracking error.

ACKNOWLEDGEMENTS

This work had the financial support of Brazilian government organization CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico).

REFERENCES

Assad, M.M., Implementação e Validação Experimental de Estratégias de Controle para Estabilização e Acompanhamento de Trajetórias de Motocicletas Autônomas, D.Sc.thesis, DEM, PUC-Rio, Rio de Janeiro, RJ, Brazil, to be published.

Åström, K. J., Klein, R. E., Lennartson, A. Bicycle dynamics and control: adapted bicycles for education and research. Control Systems, IEEE, v. 25, n. 4, p. 26-47, 2005.

Fajans, J. Steering in bicycles and motorcycles. American Journal of Physics, v. 68, n. 7, p. 654-659, 2000.

Keo, L. et al. Experimental results for stabilizing of a bicycle with a flywheel balancer. In: Robotics and Automation (ICRA), 2011 IEEE International Conference on. IEEE, 2011. p. 6150-6155.

Martins, G.N., Speranza Neto, M. and Meggiolaro, M.A., 2016, "Dynamic models of bicycles and motorcycles using power flow approach", CONEM 2016, IX Congresso Nacional de Engenharia Mecânica, Fortaleza, Brazil.

Meijaard, J. P., et al. Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. Vol. 463. No. 2084. The Royal Society, 2007.

Sharp, R. S. Motorcycle steering control by road preview. Journal of dynamic systems, measurement, and control, v. 129, n. 4, p. 373-381, 2007.

Tanaka, Y., Murakami, T. A study on straight-line tracking and posture control in electric bicycle. Industrial Electronics, IEEE Transactions on, v. 56, n. 1, p. 159-168, 2009.

Yi, J. et al. Trajectory tracking and balance stabilization control of autonomous motorcycles. In: Robotics and Automation, 2006. ICRA 2006. Proceedings 2006 IEEE International Conference on. IEEE, 2006. p. 2583-2589.

RESPONSIBILITY NOTICE

The authors are the only responsible for the material included in this paper.