SHORT CRACKS BEHAVIOR IN THE ELASTOPLASTIC REGIME

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ABSTRACT

A quantitative elastoplastic analysis of the behavior of short cracks that depart from notch tips is proposed to calculate the stresses required to initiate and to propagate cracks in notched structural components. This analysis can model both fatigue and environmentally assisted cracking problems; can evaluate notch sensitivity in both cases; and can as well establish design or acceptance criteria for tolerable non-propagating crack-like defects in such cases. The mechanical behavior of short cracks is assumed driven by the applied stresses and by the stress gradient ahead the notch tip, and withstood by the material resistances to crack initiation and to long crack propagation by fatigue or EAC. In the elastoplastic case, the stress gradient ahead of the notch tip is quantified by a *J*-integral approach modified to consider the short crack behavior. The tolerable short crack predictions made by this model are evaluated by suitable fatigue and EAC tests of notched specimens specially designed to start non-propagating cracks from the notch tips, both under elastic and elastoplastic conditions.

KEYWORDS

Short cracks, notch sensitivity, fatigue cracking, environmentally assisted cracking, elastoplastic behavior, J-integral.

INTRODUCTION

Notches induce a linear elastic (LE) stress concentration factor K_t, but as their effects on fatigue can be smaller, in particular if K_t is high, they are often quantified by $K_f = 1 + q \cdot (K_t - 1)$, where $0 \leq q \leq 1$ is the notch sensitivity factor. For design purposes, *q*-values are normally estimated by fitting semi-empirical models to data from fatigue tests of notched components. However, *q* can also be analytically modeled by studying the behavior of short fatigue cracks that start at notch roots and grow for a while before stopping and becoming non-propagating. In particular, the model proposed in [1] estimates *q*-values using sound mechanical principles and well-defined mechanical properties, without the need for any additional data-fitting parameter. Moreover, it allows the notch sensitivity concept to be extended to environmentally-assisted cracking (EAC) problems as well, and its predictions have been validated under liquid metal embrittlement conditions by testing notched Al samples in a Ga environment [2], as well as under hydrogen embrittlement conditions by testing similar steel samples in aqueous H_2S environments [3]. This versatile notch sensitivity model is extended in this work to deal with elastoplastic (EP) problems using *J*-integral techniques properly adapted to consider the short crack behavior near the notch tips.

SHORT CRACKS UNDER LINEAR ELASTIC CONDITIONS

It is well known that fatigue cracks usually initiate at notch tips due to the stress concentration effects induced by the notch. A similar behavior occurs under EAC conditions as well. However, it is less well known that short cracks initiated at notch tips can stop to grow if the stress gradient ahead of the tip is steep enough. In fact, albeit such non-propagating cracks induce damage, they can be tolerated whenever the loading conditions cannot induce the higher local stresses needed to restart their growing process. This apparently odd behavior can be easily explained by the competition between the opposing effects of the decreasing stress σ ahead of the crack tip (due to the stress gradient that acts there) and of the increasing crack size a on its stress intensity factor (SIF) $K \approx \sigma\cdot\sqrt{(\pi\cdot a)}$, which can be seen as the crack driving force under LE conditions. To make the SIF compatible with a fatigue limit, El-Haddad, Topper, and Smith (ETS) redefined the SIF range acting in a Griffith plate by [4-5]

$$\Delta K = \Delta\sigma\cdot\sqrt{\pi(a+a_0)} \tag{1}$$

where $a_0 = (1/\pi)(\Delta K_0/\Delta S_0)$ is the short crack characteristic size, ΔK_{th0} is the long fatigue crack growth threshold and ΔS_{L0} is the fatigue limit at $R = \sigma_{min}/\sigma_{max} = 0$, both well defined material properties. This trick reproduces the correct limits $\Delta\sigma(a \to 0) \to \Delta S_0$ for very short cracks and $\Delta K(a >> a_0) \to \Delta K_{th0}$ for long cracks, as well as the data trend in a Kitagawa-Takahashi $\Delta\sigma\times a$ diagram by predicting that cracks do not grow whenever $\Delta\sigma \le \Delta K_{th0}/\sqrt{\pi(a + a_0)}$ [1-3].

Since cracks that depart from notches are driven by the local $\Delta\sigma$ at their tips, if the geometry factors $g(a/w)$ used in their SIFs include K_t effects, as usual, they should be split into two parts: $g(a/w) = \eta\cdot\varphi(a)$, where $\varphi(a)$ quantifies stress gradient effects near the notch and tends towards K_t at its tip, $\varphi(a \to 0) \to K_t$, and η accounts for other effects that affect the SIF (like the free surface, e.g.). Moreover, since the SIF is a crack driving force, it should be material-independent. So, the a_0 effect on the short-crack behavior should be used to modify the fatigue crack growth (FCG) threshold instead of ΔK, making it a function of the crack size and of the fatigue limits, a trick that is quite convenient for operational purposes. In this way, the FCG threshold for pulsating loads $\Delta K_{th}(a, R = 0) = \Delta K_{th0}(a)$ is given by:

$$\frac{\Delta K_{th0}(a)}{\Delta K_{th0}} = \frac{\Delta\sigma\sqrt{\pi a}\cdot g(a/w)}{\Delta\sigma\sqrt{\pi(a+a_0)}\cdot g(a/w)} = \sqrt{\frac{a}{a+a_0}} \Rightarrow \Delta K_{th0}(a) = \Delta K_{th0}\left[1+(a_0/a)\right]^{-1/2} \tag{2}$$

However, since FCG depends both on ΔK and K_{max}, Eq. (2) should consider the K_{max} (or the R-ratio) effect on the short crack behavior. It can also be seen as just one of the models that obey the long-crack and the microcrack limits, including a data fitting parameter γ [1]. So, if $\Delta K_{thR} \equiv \Delta K_{th}(a >> a_R, R)$ is the FCG threshold for long cracks and $\Delta S_{LR} \equiv \Delta S_L(R)$ is the fatigue limit of the material, both measured (or estimated) at the desired R-ratio, then:

$$\Delta K_{thR}(a) = \Delta K_{thR}\cdot\left[1+(a_R/a)^{\gamma/2}\right]^{-1/\gamma}, \text{ where } a_R = (1/\pi)\cdot\left[\Delta K_{thR}/\left(\eta\cdot\Delta S_{LR}\right)\right]^2 \tag{3}$$

Eq. (3) reproduces the ETS model when $\gamma = 2$, and the bi-linear limits in Kitagawa-Takahashi diagrams as well, see [6] for details. For example, if a large steel plate with tensile strength $S_U = 600MPa$, $S_L = 200MPa$, and $\Delta K_{th0} = 9MPa\sqrt{m}$ works under $\Delta\sigma_n = 100MPa$ and $R = -1$, it can check if it is possible to replace a circular central hole with diameter $d = 20mm$ by an elliptical one with axes $2b = 20mm$ (perpendicular to $\Delta\sigma_n$) and $2c = 2mm$. Neglecting buckling, the circular hole has a safety factor $\phi_F = S_L/K_f\sigma_n = 200/150 \cong 1.33$ against fatigue crack initiation, since due to its large radius it has $K_f \cong K_t = 3$. However, since the elliptical hole tip radius is $\rho = c^2/b = 0.1mm \Rightarrow K_t = 1 + 2b/c = 21 \Rightarrow K_f = 1 + q\cdot(K_t - 1) = 7.33$ (according to Peterson's estimate, $q = (1 + \alpha/\rho)^{-1} = [1 + 0.185\cdot(700/600)/0.1]^{-1} \cong 0.32$ [6]), it would fail by classic SN procedures, since its $\sigma_a = K_f\cdot\sigma_n \cong 367MPa > S_L$. But since this K_f is larger than those typically obtained from notched coupons fatigue data, it is worthwhile to reevaluate this prediction.

Assuming $\Delta K_{th0}(a) = \Delta K_{th0}/[1 + (a_0/a)]^{-0.5}$ (by ETS), the steel fatigue limit $S_L = S_U/2$ (as usual), $\Delta S_{L0} = S_U/1.5$ (by Goodman) and $a_0 = (1/\pi)(\Delta K_{th0}/\eta\Delta S_{L0})^2 = (1/\pi)(1.5\Delta K_{th0}/1.12\cdot S_U)^2 \cong 0.13mm$, the SIF ranges $\Delta K_I(a)$ estimated for the two holes by the procedures developed in [1] are compared to the short-crack threshold $\Delta K_{th0}(a)$ in Fig. 1, where curve crossings define crack arrest, so the largest tolerable crack sizes. Hence, this model predicts that both the circular and the elliptical holes could support the nominal load range $\Delta\sigma_n$ without failing by fatigue.

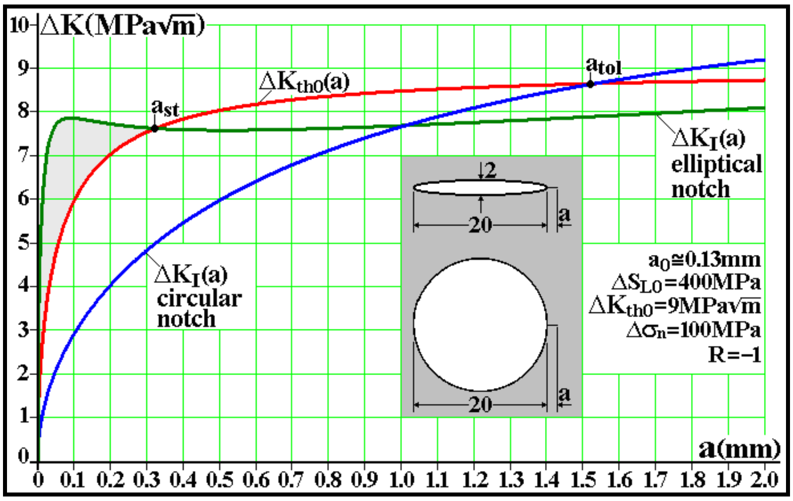

Figure 1: Cracks should not initiate at the circular hole (which tolerates cracks $a_{tol} < 1.52mm$), while the crack that initiates at the elliptical hole tip should stop at $a_{st} \cong 0.33mm$.

SHORT CRACKS UNDER ELASTOPLASTIC CONDITIONS

Under contained elastoplastic conditions around crack tips, which invalidate the use of SIFs to quantify the local crack driving forces, the non-propagating crack problem can be modeled using the J-integral approach [7-8], as originally proposed in [5]. However, since like in the LE case short fatigue cracks present higher FCG rates than long cracks in the EP case as well, it is operationally convenient to modify their $J_{th}(a)$ propagation threshold to consider the effects of the short crack characteristic size a_0 when accounting for their peculiar behavior near EP notch tips. In the LE case, the size-dependent threshold $J_{th}(a)$ must of course be given by $K_{th}(a)/E'$, where $E' = E$ or $E' = E/(1-\nu^2)$ for plane stress or plane strain limit conditions. In this way, $J_{th}(a)$ can then be easily compared with the crack driving force quantified by J when modeling the EP short crack behavior.

If the stresses controlled by J grow proportionally to the load P applied on the cracked piece, then for a Ramberg-Osgood material with strain-hardening coefficient H and exponent h, it can be shown [6] that the crack driving force J is given in clear engineering notation by:

$$J = J_{el} + J_{pl} = K_I^2/E' + \left[(P/P_{pc})S_Y\right]^{(1+h)/h}\left[(w-a)/H^{1/h}\right]\hbar(a/w,h) \tag{4}$$

where $K_I(P)$ is the SIF applied on the cracked piece (as if it remained LE), P_{pc} is the plastic collapse load, S_Y is the yielding strength, w is the cracked piece width, $w - a$ is its residual ligament, and $\hbar$ is a non-dimensional function that depends on the cracked piece geometry and on the strain-hardening exponent, which in practice can nowadays be easily calculated in most finite element (FE) codes. To model the short crack behavior, like its LE analog $K_{th}(a)$, the size-dependent crack propagation threshold $J_{th}(a)$ must include the a_0 effect:

$$J_{th}(a) = J_{th}/(1 + a_0/a) \tag{5}$$

Hence, like in the LE case, EP cracks grow whenever their driving force J is higher than their size-dependent threshold J_{th}(a), a material property that can be properly measured, and stop otherwise. Cracks that depart from a notch tip can be much affected by the notch stress gradient when their size is small or similar to the notch tip radius ρ, so they can start and then stop after growing for a while, becoming thus non-propagating. Figure 2 e.g. shows J for a crack that departs from a notch with $\rho = 1mm$ and stops at a size $a = 1.8mm$, when its J become smaller than its threshold $J_{th}(a)$, becoming non-propagating. If P remains fixed, notice that this cracked component would then tolerate cracks with size $1.8 < a < 9mm$.

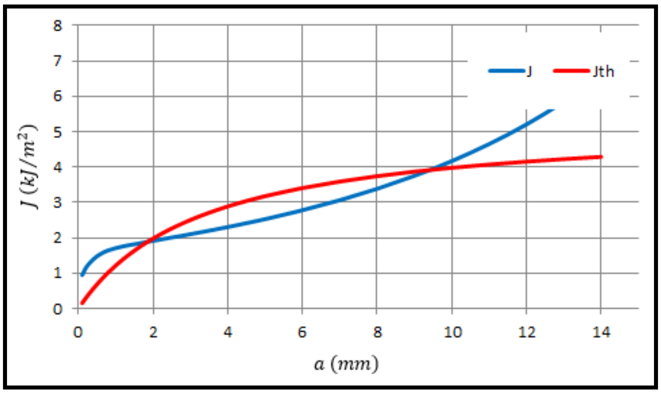

Figure 2: EP crack that starts at a notch tip and then stops after growing for 1.8mm.

EXPERIMENTAL RESULTS

J and *K* of DC(T) specimens [9] are quantified in 2D Abaqus/CAE FE code, considering EP conditions near the notch tip as needed. The short crack model predictions are verified by EAC testing AISI 4140 steel samples, whose crack initiation and long crack growth thresholds under EAC in a solution of H_2S are S_{EAC} = 332MPa and $K_{EACth} = 34.2MPa\sqrt{m}$, measured by NACE procedures [10]. Then two $w = 60mm$ DC(T)s with a notch of length $b = 15mm$ and tip radius $\rho = 2mm$ are tested under two LE load levels, $P = 6750N$ and $P = 8250N$. According to the short crack models presented above, the first should generate a non-propagating crack whereas the second should break, as indeed they did, see Fig. 3-5.

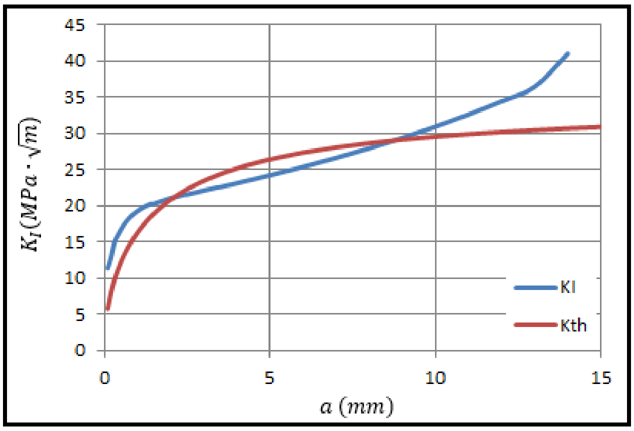

Figure 3: *SIF*×*a* and $K_{th}(a)\times a$ analyses for the LE DC(T) under EAC and $P = 6750$N.

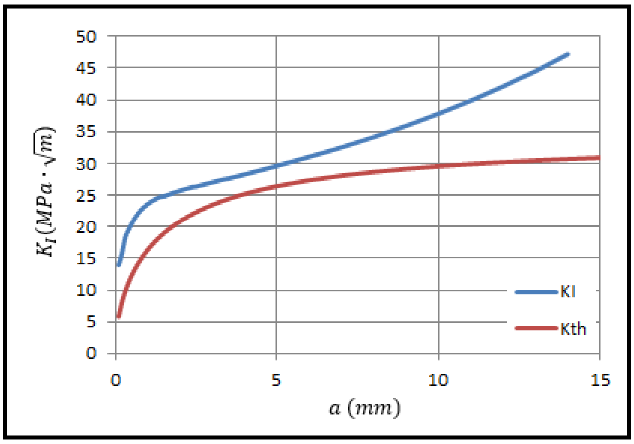

Figure 4: *SIF*×*a* and $K_{th}(a)\times a$ analyses for the LE DC(T) under EAC and $P = 8250N$.

Figure 5: LE specimens after 30 days under EAC in aqueous hydrogen sulfide environment.

In the sequence, two other similar DC(T) specimens, one with notch tip radius $\rho = 0.2mm$ and loaded by $P = 3.1kN$ and the other with $\rho = 0.3mm$ and $P = 6kN$, were tested under EP conditions around the notch tip in the same aggressive environment. Considering their size-dependent threshold $J_{th}(a)$ and their crack driving forces J, both specimens should withstand the loads without breaking, as indeed they did, see Fig. 6-8.

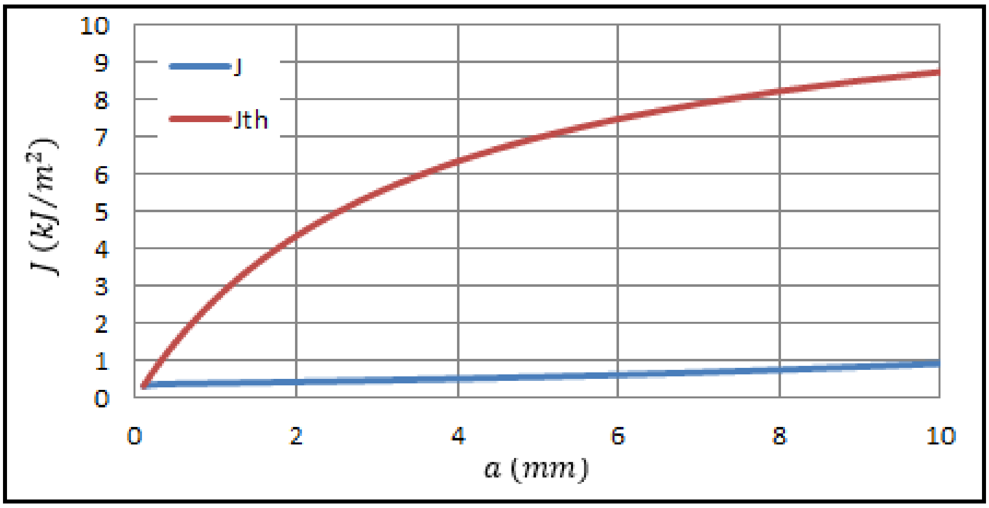

Figure 6: J and $J_{th}(a)$ calculated for the DC(T) loaded by $P = 3.1kN$ under EP conditions.

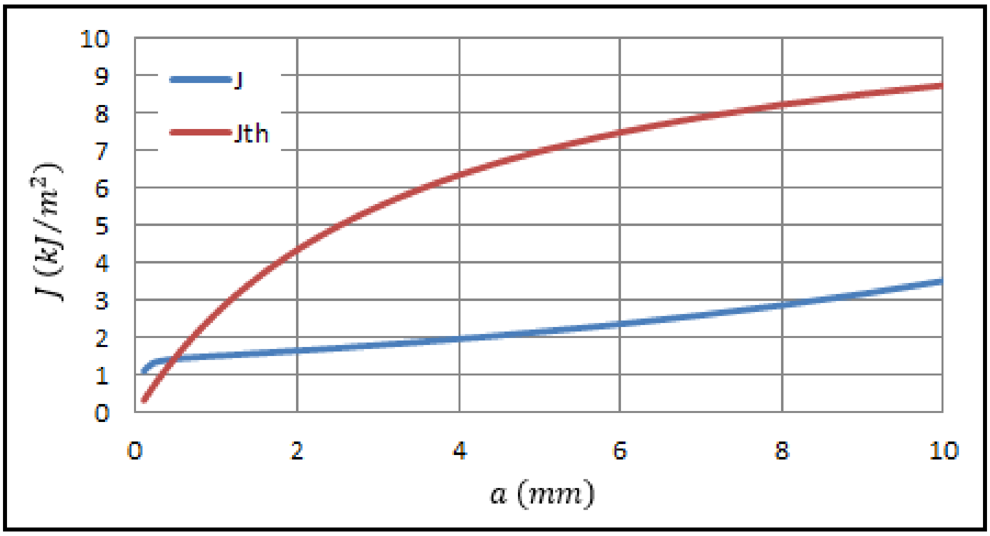

Figure 7: J and $J_{th}(a)$ calculated for the DC(T) loaded by $P = 6kN$ under EP conditions.

Figure 8: EP specimens after 30 days under EAC in aqueous hydrogen sulfide environment.

CONCLUSION

The proposed model is able to evaluate the propagating or non-propagating behavior of short cracks that depart from notch tips under linear elastic and elastic-plastic load regimes for any geometry and applied load. Hence, it can be used to assess the size of tolerable short cracks or crack-like defects in practical applications. This work presents experimental evidence to support this claim under stress corrosion cracking conditions, but a similar mechanics is also valid for modeling the behavior of short fatigue cracks.

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