INCREMENTAL DAMAGE CALCULATION FOR VHCF UNDER NON-PROPORTIONAL MULTIAXIAL LOADING

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ABSTRACT

In this work, the novel concept of nested damage surfaces, introduced by the authors, is used to predict fatigue damage under high-cycle fatigue. The proposed Incremental Fatigue Damage (IFD) model follows Miner's rule, integrating differentials of fatigue damage until reaching unity or any other user-defined critical value. Since damage is continuously integrated as the loading is applied, the method does not require cycle identification and counting, which are challenging and ill-defined tasks under non-proportional multiaxial loadings. Damage memory is stored through internal material variables and nested "damage surfaces" in stress space. Such surfaces can be calibrated according to any traditional high-cycle fatigue damage rule, such as multiaxial generalizations of Wöhler's curves, Findley's equation, or elastic versions of Fatemi-Socie's or Smith-Watson-Topper's models. The IFD predictions are validated for uniaxial variable amplitude loading histories.

KEYWORDS

Incremental fatigue damage; Damage integration; Multiaxial fatigue; Non-proportional loading.

INTRODUCTION

Wetzel and Topper proposed the first uniaxial Incremental Fatigue Damage (IFD) model a long time ago [1]. IFD models aim to calculate damage as a continuous variable, without the need to define or count cycles, and outside the framework of Continuum Damage Mechanics (CDM). Wetzel used each element of a discretized stress-strain model not only to evaluate plastic strains, but also the consequent fatigue damage, storing in this way the damage memory required for a correct damage integration in cyclic histories. Fatigue damage integration is continuously carried out without waiting for each hysteresis loop to close. Chu [2] outlined the generalization of Wetzel's model to multiaxial NP loadings, however indirectly requiring cycle detection, thus limiting its advantages. Stefanov proposed other IFD methods [3], however they do not properly take into account the "damage memory" effect without the need for heuristic calibration routines. Instead of integrating fatigue damage itself, other methods integrate strain energy or energy-based damage parameters [4], eventually giving good results under low-cycle fatigue; however, such elastoplastic energy methods are limited to ductile materials that display measurable plastic deformation, preventing their use in most high-cycle applications where damage results from elastic cycles.

Instead of integrating strain energy or energy-based damage parameters, the IFD approach integrates fatigue damage itself. As a result, it follows Miner's rule, integrating differentials of fatigue damage until reaching the 1.0 (or any other) critical value. No cycle detection or counting is required, since damage is continuously integrated as the loading is applied. This approach is based on the derivative of the normal stress σ with respect to damage D, called here *generalized damage modulus* $\mathcal{D}_\sigma$, which for uniaxial histories can be defined as

$$\mathcal{D}_\sigma \equiv d\sigma/dD \quad \Rightarrow \quad D = \int dD = \int (1/\mathcal{D}_\sigma) \cdot d\sigma \tag{1}$$

From Eq. (1), damage D can be continously integrated as long as the instantaneous value of $\mathcal{D}_\sigma$ is known along a stress path with infinitesimal increments $d\sigma$. But this is not a trivial task for multiaxial non-proportional (NP) variable-amplitude loading (VAL) histories (which require damage integration along a general multiaxial load path), because $\mathcal{D}_\sigma$ depends not only on the current stress state but also on the previous loading history. So, IFD models need to allow $\mathcal{D}_\sigma$ to vary as a function of the stress level and of the existing state of damage [5].

IFD APPROACH WITH NESTED DAMAGE SURFACES

Alternatively to rheological models, a direct analogy between IFD and incremental plasticity has been proposed by the authors [6] to store damage memory, using internal material variables. In this IFD model, the current damage state is stored as a five-dimensional (5D) vector $\vec{D}' \equiv [D_1\ D_2\ D_3\ D_4\ D_5]^T$, a purely mathematical internal variable that allows 5D *damage* increments $d\vec{D}'$ to be more easily represented as a function of the 5D deviatoric stress increments $d\vec{s}'$ and the current $\mathcal{D}_\sigma$, in a multiaxial generalization of Eq. (1) called *damage evolution rule*. The scalars D_1 through D_5 are signed damage quantities associated with each of the directions of the 5D deviatoric stress vector $\vec{s}'$.

A field of ($M + 1$) nested iso-damage (or damage) surfaces is then defined in the 5D deviatoric space, see Fig. 1, in a framework to provide internal material variables that can store damage memory. Each damage surface has a *constant* user-defined radius $r_{\sigma i}$, while the radius differences between consecutive surfaces are defined as $\Delta r_{\sigma i} = r_{\sigma i+1} - r_{\sigma i}$. The innermost damage surface is called the fatigue limit surface, while the outermost is the failure surface, defined respectively for $i = 1$ and $i = M + 1$. The radius $r_{\sigma 1}$ of the fatigue limit surface can be calibrated to become arbitrarily small, in case the studied material does not present a fatigue limit. These radii $r_{\sigma i}$ are user-defined stress levels used in the discretization and non-linear interpolation of the damage curve, calibrated e.g. from the component's Wöhler/ Basquin's curve, Findley's equation, or elastic versions of Fatemi-Socie's or Smith-Watson-Topper's models. More complex stress-life equations can be used in the $r_{\sigma i}$–based calibration, e.g. using Haibach's slope correction for very high cycle lives [7].

The *damage backstress* vector $\vec{\beta}'_\sigma$ is here defined as the location of the center of the current fatigue limit surface, which can be decomposed as the sum of *M damage backstresses* $\vec{\beta}'_{\sigma 1}$, $\vec{\beta}'_{\sigma 2}$, …, $\vec{\beta}'_{\sigma M}$ that describe the relative positions between centers of consecutive damage surfaces, as illustrated in Fig. 1 for a 2D deviatoric stress space. *Damage memory* is stored here by the current arrangement among these *damage surfaces*. No *damage* occurs if the 5D stress increment $d\vec{s}'$ happens inside the *fatigue limit surface*. The *accumulated damage* D is then equal to the integral of the scalar norm $|d\vec{D}'|$ of the 5D damage increments, i.e.

$$D = \int dD = \int |d\vec{D}'| \tag{2}$$

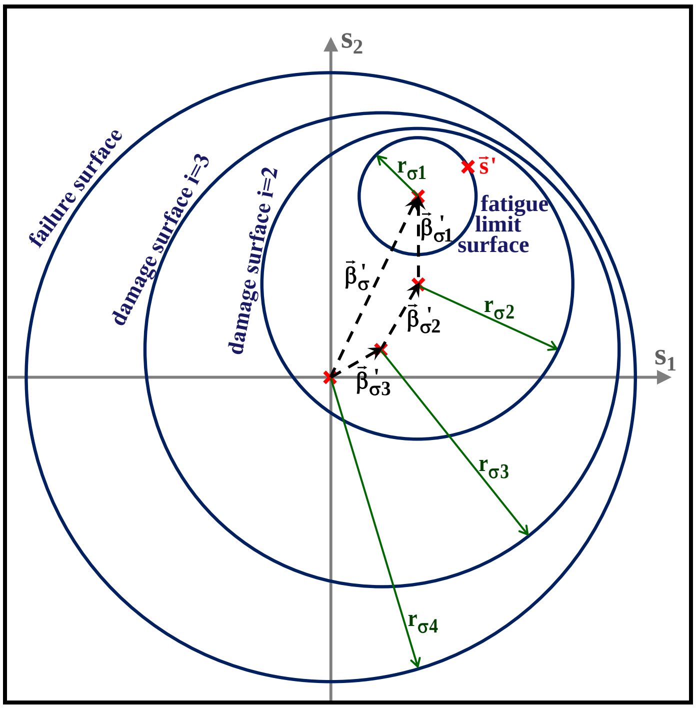

Fig. 1: Fatigue limit, damage, and failure surfaces in the $s_1 \times s_2$ deviatoric space for $M = 3$, showing the damage backstress vector $\vec{\beta}'_{\sigma}$ that defines the location of the fatigue limit surface center, and its components $\vec{\beta}'_{\sigma1}$, $\vec{\beta}'_{\sigma2}$, and $\vec{\beta}'_{\sigma3}$ that describe the relative positions between the centers of consecutive surfaces.

If a given stress state $\vec{s}'$ is on the fatigue limit surface with a normal unit vector $\vec{n}'_{\sigma}$, and if its infinitesimal increment $d\vec{s}'$ is in the outward direction, then $d\vec{s}'^T \cdot \vec{n}'_{\sigma} > 0$ and a fatigue damage increment is obtained from a *damage evolution rule*:

$$d\vec{D}' = (1/\mathcal{D}_{\sigma}) \cdot (d\vec{s}'^T \cdot \vec{n}'_{\sigma}) \cdot \vec{n}'_{\sigma} \cdot f_{MS}(\vec{\sigma}) \cdot f_{NP}(\vec{\beta}'_{\sigma}, \vec{n}'_{\sigma}) \tag{3}$$

where $f_{MS}(\vec{\sigma})$ is a scalar *mean stress function* of the current 6D stress $\vec{\sigma}$ to account for mean/maximum-stress effects, which can be defined e.g. from Goodman's or Gerber's $\sigma_a\sigma_m$ relations; and $f_{NP}(\vec{\beta}'_{\sigma}, \vec{n}'_{\sigma})$ is an optional *NP function* to account for the influence of the non-proportionality of the load path on the resulting damage. Depending on the material, the mean stress function $f_{MS}(\vec{\sigma})$ could be based on the current hydrostatic stress σ_h or on the normal stress perpendicular to the critical plane where the microcrack should initiate. Except for the failure surface (which never translates), during this damage process the fatigue limit and all damage surfaces suffer translations calculated from the increments

$$d\vec{\beta}'_{\sigma i} = \begin{cases} \mathcal{d}_{\sigma i} \cdot \vec{v}'_{\sigma i} \cdot dD, & \text{if } |\vec{\beta}'_{\sigma i}| < \Delta r_{\sigma i} \\ 0, & \text{if } |\vec{\beta}'_{\sigma i}| = \Delta r_{\sigma i} \end{cases} \tag{4}$$

where $\mathcal{d}_{\sigma i}$ are material coefficients calibrated for each surface, and $\vec{v}'_{\sigma i}$ are the *damage surface translation directions* adapted e.g. from Jiang-Sehitoglu's translation rule [8] used in plasticity, resulting in the adapted expression

$$\vec{V}'_{\sigma i} = \vec{n}'_{\sigma} \cdot \Delta r_{\sigma i} - (|\vec{\beta}'_{\sigma i}| / \Delta r_{\sigma i})^{\chi_{\sigma i}} \cdot \vec{\beta}'_{\sigma i} \tag{5}$$

where $\chi_{\sigma i}$ are fitting exponents for each surface. The current generalized damage modulus $\mathcal{D}_{\sigma}$ is then obtained from the consistency condition that guarantees that the current stress state is never outside the fatigue limit surface:

$$\mathcal{D}_{\sigma} = \left(\sum_{i=1}^{M} \mathcal{d}_{\sigma i} \cdot \vec{V}'^{T}_{\sigma i} \right) \cdot \vec{n}'_{\sigma} \tag{6}$$

allowing the calculation of the evolution of the damage vector $\vec{D}'$ from Eq. (3). The (scalar) accumulated damage D is then obtained from Eq. (2). This formulation can deal with any multiaxial stress history, proportional or NP, and eliminates the need to define or count cycles and find equivalent ranges. If Jiang-Sehitoglu's translation rule is used in the IFD formulation, then a procedure analogous to the one in [8] could be adopted to calibrate the radius $r_{\sigma i}$ and the coefficient $\mathcal{d}_{\sigma i}$ from each damage surface i.

Finally, to account for mean-stress effects, a simple function inspired on Fatemi-Socie's damage parameter could be adopted in Eq. (3), which in a uniaxial case would simply become

$$f_{MS}(\vec{\sigma}) \equiv (1 + \alpha_{MS} \cdot \sigma_x / S_{Yc})^{B_{MS}} \tag{7}$$

where α_{MS} and B_{MS} are material-dependent parameters and σ_x is the current (instantaneous) normal stress. But since the IFD approach does not involve cycle detection or counting, the mean or peak stress values during a cycle (which require the definition of cycles and knowledge of future stress values) cannot be used in $f_{MS}(\vec{\sigma})$. Thus, only current/ instantaneous stress values such as σ_x can be used in $f_{MS}(\vec{\sigma})$.

NUMERICAL EVALUATION

To evaluate the prediction capabilities of the proposed IFD model, traditional cycle-based fatigue damage calculations are compared with continuous IFD predictions on a material with Basquin's equation constant *772.5*MPa and exponent -0.09, subjected to the uniaxial history $\sigma_x = \{0 \rightarrow 300 \rightarrow -300 \rightarrow 300 \rightarrow -300 \rightarrow 300\}$MPa. To consider mean stress effects, Eq. (7) is adopted using $\alpha_{MS} = 0.4$ and $B_{MS} = 1$. The IFD calculations assume Jiang-Sehitoglu's translation rule with $M = 16$ damage surfaces, calibrated from the same Basquin equation used in the cycle-based calculations following an analogous procedure from [8].

Figure 2(left) shows the hysteresis loops $\sigma_x \times D_1$, where D_1 is the first component of the 5D damage vector $\vec{D}'$. Notice in this figure that damage components such as D_1 can become negative, as a result of an unloading process. This is not an issue, since $\vec{D}'$ is just an internal variable used to calculate the actual fatigue damage. Indeed, the accumulated damage D is obtained from the integral of the norm of the infinitesimal increments $|d\vec{D}'|$, see Eq. (2). It is important to note that this loading example is linear elastic, without any significant macroscopic plasticity; the non-linear shape of the stress $\times$ damage hysteresis loops is just a consequence of the non-linearity of Basquin's (or Wöhler's) damage equation.

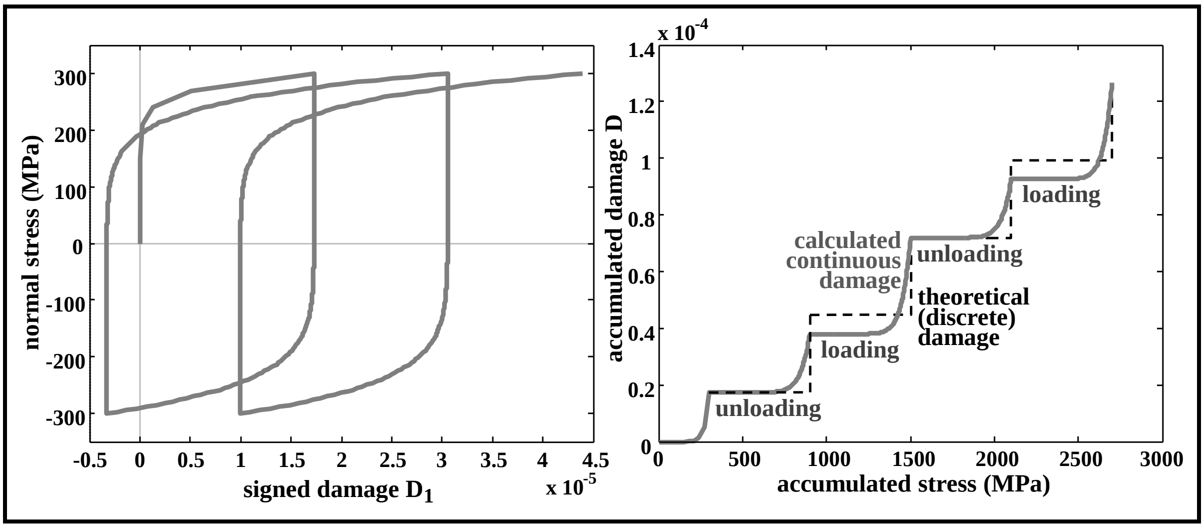

Fig. 2: Stress × damage hysteresis loops (left) and resulting accumulated damage (right) for a mean stress function based on Fatemi-Socie's damage parameter.

Figure 2(right) shows the resulting accumulated damage *D* as a function of an accumulated stress, defined as the integral of the norm of the infinitesimal deviatoric increments $d\vec{s}'$. The depicted *theoretical damage* is calculated in the traditional (discrete) way after each of the three rainflow-counted half-cycles $\{0 \rightarrow 300\}$, $\{300 \rightarrow -300\}$ and $\{-300 \rightarrow 300\}$MPa. Notice how the continuous IFD calculations almost exactly reproduce, at the end of each full cycle, the discrete predictions. Nevertheless, a larger damage increment is predicted by the IFD during the loading half-cycle than during unloading, see Fig. 2(right). This prediction is not physically unsound, since most of the microplasticity happens towards the end of each half-cycle, where $\sigma_x \rightarrow 300$MPa during loading and $\sigma_x \rightarrow -300$MPa during unloading in this example. Such a difference in damage increment causes the stress × damage hysteresis loops from Fig. 2(left) to remain open, which resembles but has nothing to do with a ratcheting problem, and has no physical inconsistency since $\vec{D}'$ is just an internal variable.

The damage memory provided by the fatigue limit and damage surfaces is able to deal with VAL, exactly reproducing rainflow-based uniaxial calculations, but without the need for any cycle detection or counting. Figure 3 shows, for a VAL history with zero mean stress, the agreement between the proposed IFD approach (using e.g. $f_{MS}(\vec{\sigma}) \equiv 1 + 0.4 \cdot \sigma_x / S_{Yc}$) and traditional SN calculations, which is almost exact after every full loading-unloading cycle (but not at every half-cycle, as discussed before regarding Fig. 2). The agreement is as good as the quality of the calibration of the damage surface parameters to the adopted damage model. For VAL under high mean stress levels, higher order $f_{MS}(\vec{\sigma})$ equations need to be adopted for an accurate damage prediction, as mentioned before.

CONCLUSIONS

In this work, an Incremental Fatigue Damage model based on nested damage surfaces was reviewed and applied to high-cycle fatigue. The method does not require cycle identification and counting, a major advantage for multiaxial problems. The proposed method is not a Continuum Damage Mechanics approach, since it does not rely on macroscopic properties such as the progressive loss of elastic stiffness. The IFD predictions were validated for selected uniaxial variable amplitude loading histories.

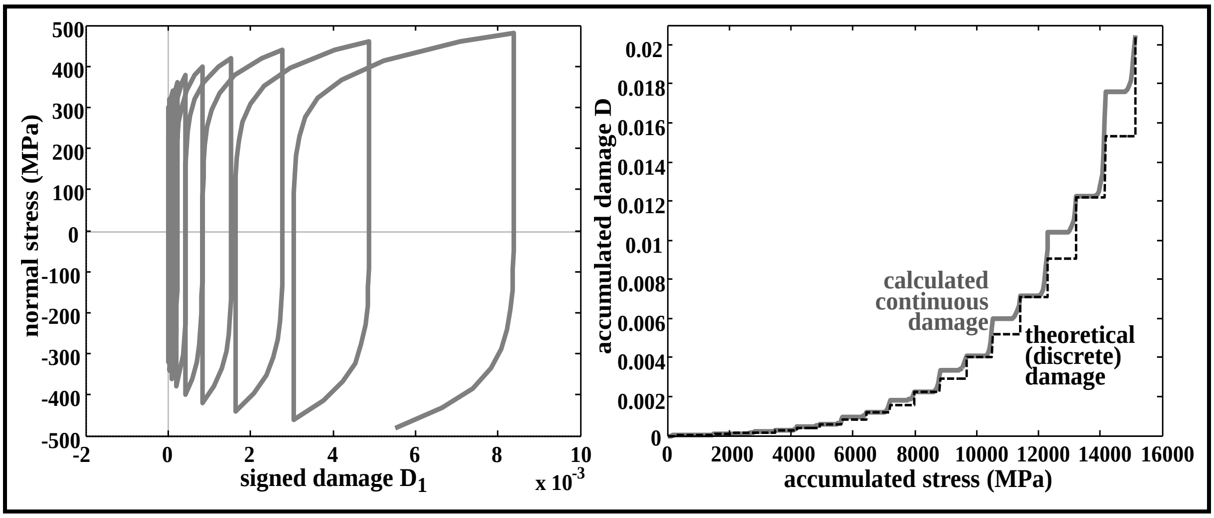

Fig. 3: Stress × damage hysteresis loops (left) and resulting accumulated damage (right) for a VAL history with zero mean stress.

REFERENCES

[1] Wetzel, R.M.:
A Method of fatigue damage analysis
Ph.D. Thesis, U Waterloo (1971)

[2] Chu, C.C.:
A new incremental fatigue method
ASTM STP 1389 (2000), pp. 67-78

[3] Stefanov, S.H.
A curvilinear integral method for multiaxial fatigue life computing under non-proportional, arbitrary or random stressing
Int J Fatigue 15 (1993), pp. 467-472

[4] Nowack, H.; Baum, C.; Ott, W.; Buczynski, A.; Glinka, G.:
Achievements of the incremental multiaxial fatigue prediction method EVICD
Proc. 5th Int Conf Low Cycle Fatigue, Germany (2003), pp. 277-282

[5] Kreiser, D.; Jia, S.X.; Han, J.J.; Dhanasekar, M.:
A nonlinear damage accumulation model for shakedown failure
Int J Fatigue 29 (2007), pp. 1523-1530

[6] Meggiolaro, M.A.; Castro, J.T.P.; Wu, H.:
A Multiaxial Incremental Fatigue Damage Formulation using Nested Damage Surfaces
Frattura e Integrità Strutturale 37 (2016), pp. 138-145

[7] Haibach, E.:
Modified linear damage accumulation hypothesis accounting for a decreasing fatigue strength during increasing fatigue damage
LBF TM Nr.50, Darmstadt, Germany (1970)

[8] Jiang, Y.; Sehitoglu, H.:
Modeling of Cyclic Ratchetting Plasticity
J Appl Mech 63 (1996), pp. 726-733

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