On the Modeling of Fatigue Damage Ahead of the Crack Tip

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ABSTRACT: Elber's assumption that ΔK_{eff} is the actual driving force for fatigue crack growth (FCG) is used by strip-yield models to quantify damage in cracked components, although it cannot explain many FCG characteristics observed in practice. To check if FCG models based on this hypothesis are indeed intrinsically better than concurrent models based on other principles, FCG rates are modeled in two ways. First using Elber's ideas, and then using the alternative view that FCG is instead due to damage accumulation ahead of the crack tip caused by the cyclic strain history that acts there. This alternative idea does not need or use the ΔK_{eff} hypothesis. To be fair, the second approach estimates FCG rates using cyclic strain ranges induced by plastic displacements calculated by the very same procedures used by strip-yield models. Moreover, to avoid the need for FCG threshold and toughness properties, both are associated with suitable strain limits. Despite based on apparently conflicting principles, both models can predict quite well FCG curves, an unexpected result. Besides confirming that data fitting cannot prove any model superiority, this result confirms that the ΔK_{eff} hypothesis is not a necessary requirement to explain the FCG behavior.

KEYWORDS: Fatigue crack growth models; strip-yield mechanics; fatigue crack closure; effective stress intensity range; damage accumulation ahead of the crack tip.

INTRODUCTION

Since Paris and Erdogan clearly demonstrated that fatigue crack growth (FCG) rates da/dN correlate well with stress intensity factor (SIF) ranges ΔK [1], residual life predictions became indispensable in structural analyses of cracked structural components. Many similar rules have been proposed since then to consider other effects that can affect FCG rates as well, such as the peak load K_{max} (or the equivalent load ratio $R = K_{min}/K_{max}$), and material limits like FCG thresholds $\Delta K_{th}(R)$ and the critical SIF K_{IC} or K_C [2]. After Elber experimentally found the crack closure phenomenon, it become an important issue for FCG modeling too [3]. Elber observed that fatigue cracks can remain partially closed along the lower portion of their load cycles even under $R > 0$, and completely open only after the applied SIF reaches the so-called crack opening load K_{op}. From this observation, he then *assumed* that FCG can only occur only after the crack tip is fully open under loads greater than K_{op} (*supposing* that only then they would be able to expose their tips to additional fatigue damage) [4]. Consequently, he postulated that ΔK_{eff} ($\Delta K_{eff} = K_{max} - K_{op}$ if $K_{op} > K_{min}$, or $\Delta K_{eff} = \Delta K$ otherwise) would be the actual FCG driving force (instead of SIF ranges ΔK or SIF combinations like $\{\Delta K, K_{max}\}$ or $\{\Delta K, R\}$).

Since the ΔK_{eff} *hypothesis* can reasonably explain many (but certainly not all) sequence effects in FCG, like crack growth delays or arrests after overloads (OL), as well as other phenomena like the R-sensitivity of FCG thresholds on non-inert environments, it has been popular among fatigue experts ever since its proposal. It has been used as the basis for many semi-empirical FCG models, in particular the so-called strip-yield models (SYM) that numerically estimate K_{op} and ΔK_{eff}, and from them FCG lives using a suitable $da/dN = f(\Delta K_{eff})$ equation properly fitted to experimental data. However, although the fatigue crack closure phenomenon is well documented and proven, its real significance for FCG is still controversial, to say the least. Indeed, the ΔK_{eff} hypothesis cannot explain many FCG peculiarities, see for instance [5] for a recent review of these topics.

Moreover, since fatigue damage is caused by cyclic elastoplastic (EP) stress/strain histories, an alternative and probably more intuitive way to model FCG is to assume fatigue cracks grow by sequentially breaking small volume elements ahead of the crack tip, after reaching all the damage they can sustain. Hence, the basic idea behind these critical damage models (CDM) is that fatigue damage is caused by the cyclic EP stress/strain fields that always accompany the crack tips, not by ΔK_{eff}. A comparative analysis between a simple CDM proposed in [6] and a classic SYM based on Newman's original formulation [7] presented in [8] needs to use a McEvily-like FCG rule to model phase I and III behaviors of typical $da/dN \times \Delta K$ curves, using FCG thresholds ΔK_{th}(R) and toughness K_C properties to limit the estimated FCG curves. This work improves that model eliminating the need for using such a reasonable albeit somehow arbitrary FCG rule. The new CDM proposed here directly estimates the entire $da/dN \times \Delta K$ behavior from simple and clear mechanical principles using only well-defined εN properties, without the need for any additional data-fitting parameter.

DAMAGE ACCUMULATION AHEAD OF THE CRACK TIP BASED ON STRIP-YIELD MECHANICS

SYMs are based on Dugdale-Barenblatt's idea [2], modified to leave plastically deformed material around the faces of the advancing fatigue crack [7, 9]. Plastic zones pz and surface displacements are estimated by the superposition of two linear elastic solutions: a cracked plate loaded by a remote uniform nominal tensile stress σ_n, and by a uniform distributed stress s applied over crack surface segments. The numerical model developed by Newman for a M(T) specimen uses rigid-perfectly bar elements whose displacements are described by

$$V_i = \sigma_n f(x_i) - \sum_{j=1}^{n} \sigma_j \cdot g(x_i, x_j) \quad (1)$$

The influence functions $f(x_i)$ and $g(x_i, x_j)$ used in Eq. (1) are related to the plate geometry and to its width correction, and then used to calculate the bar element plastic deformation and also the contact stress required to estimate the crack opening stress. The original SYM also uses a Forman-Newman FCG rule based on the ΔK_{eff} hypothesis, which has four data-fitting parameters [9]. A home-grown SYM algorithm was developed and verified (using predictions from the literature) following these ideas, as described in [5, 8].

On the other hand, the CDM from [6] uses only physically-based hypotheses and does not need any data-fitting parameter. Its original version uses a shifted HRR strain-stress field to estimate the plastic strain ranges ahead of the crack tip, recognizing crack tip blunting to remove its singularity, and estimates the three phases of FCG curves using a McEvily-like model. This CDM was later generalized to deal with VAL conditions [10]. In the CD/SY mixed model proposed in [8], displacements calculated by SYM procedures, see Eq. (1), are used to obtain the strain field ahead of the crack tip, which replaces the shifted HRR field assumed in original CDM (but still retaining a McEvily-like rule, like the original CDM). In summary, three models are described, analyzed, and compared in [8]: (i) the original strip-yield model (SYM), (ii) the original critical damage model based on a shifted HRR field (CDM) and (iii) the mixed critical damage/strip yield model (SY-CDM).

This paper describes a significant improvement for the original SY-CDM, to avoid its need to assume a suitable, but somehow arbitrary FCG rule. This new version can simulate the three phases of typical FCG curves without the need for any other artificial tricks or arbitrary data-fitting constants. This modified strip-yield critical-damage combined model (SY-CDMmod) estimates FCG increments in a cycle-by-cycle basis considering a gradual damage accumulation process and possible crack closure effects on the cyclic strain field ahead of the crack tip. To do so, it combines Newman's strip-yield ideas [7, 9] to quantify the strain fields ahead of the crack tip with CDM routines, considering crack face contact at low loads, but not assuming ΔK_{eff} is the FCG driving force. For FCG under constant SIFs $\{\Delta K, K_{max}\}$ (fixed plastic zones) or stresses $\{\Delta\sigma, \sigma_{max}\}$ (slowly growing plastic zones), no memory effects occur during FCG. So plastic deformations at maximum (σ_{max}) and minimum (σ_{min}) applied stresses are calculated by Eqs. (2) and (3), obtained from Eq. (1), where S_F is the flow stress, a is the triaxiality constraint factor (which varies from plane stress to plane strain limit conditions), and n_{pz} is the number of bar elements inside the plastic zone. The element stress at minimum applied load, σ_j in Eq. (3), is calculated by solving the equation system Eq. (4) using a Gauss-Seidel iteration process with added restraints [7].

$$L_{max}(i) = \sigma_{max} \cdot f(x_i) - \sum_{j=1}^{npz} \alpha \cdot S_F \cdot g(x_i, x_j) \quad (2)$$

$$L_{min}(i) = \sigma_{min} \cdot f(x_i) - \sum_{j=1}^{n} \sigma_j \cdot g(x_i, x_j) \tag{3}$$

$$(\sigma_i)_I = \left[Sf_i - L_{max,i} - \sum_{j=1}^{i-1} (\sigma_j)_I g_{ij} - \sum_{j=i+1}^{n} (\sigma_j)_{I-1} g_{ij} \right] / g_{ii} \tag{4}$$

Figure 1 shows the plastic deformation ahead of the crack tip estimated by the SY-CDMmod at maximum and minimum loads, using Eqs. (2-3) in two conditions: (i) considering the elements at the crack surface and (ii) assuming no plastic deformation around the crack surfaces, thus no crack closure. Like in the SYMs, the broken elements are kept along the crack surfaces and are used to consider possible crack closure effects in the cyclic strain field ahead of the crack tip. This figure also shows how the contact of the crack surfaces affects their values. Hence, the FCG rates calculated by the proposed SY-CDMmod are affected (but not controlled) by crack closure (which reduces the reverse plastic zone size, and the plastic strain ranges inside it as well).

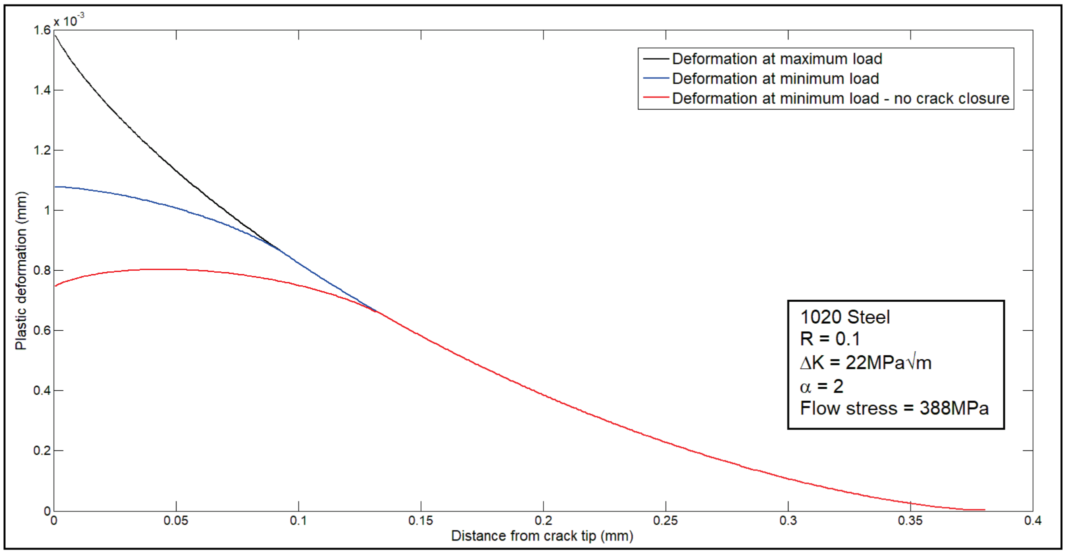

Figure 1: Plastic displacement ahead of the crack tip from SY-CDMmod.

The SY-CDMmod divides the plastic zone *pz* into small rigid-plastic bar elements, assumed analogous to tiny *εN* specimens. Due to the simplified material behavior assumed by the SYM mechanics (rigid-perfectly-plastic, neglecting elastic and strain-hardening effects), damage occurs only into the reverse plastic zone pz_r. Fatigue damage must be calculated, accumulated, and stored in a cycle-by-cycle basis. To reduce numerical errors, all bar elements have the same initial width. SYMs estimate plastic deformation and stresses at the center of each bar element, a characteristic kept in the SY-CDMmod. Crack increments can be located between two adjacent bar elements, due to the critical damage value (usually assumed as 1). Hence, an interpolation routine is needed to locate them and to correctly store the damage information in each one of its 400 bar elements inside the monotonic plastic zone (since $n_{pz} = 400$ is enough, as discussed in [5]).

Original SYM procedures calculate peak and residual plastic *displacements* at each bar element, hence they must be adapted to generate the strain field needed by the SY-CDMmod, using a formulation proposed by Rice to estimate cyclic strains for tensile cracks based on crack opening displacements [11]. Rice assumes an idealized elastic-perfectly plastic material and proportional plastic flow, so plastic strain tensor components that remain proportional in all bar elements inside the *pz*. Rice´s formulation is properly modified to consider the calculated plastic strain acting at the various bar elements ahead of the crack tip. The positions of the elements starting from the crack tip, $x_{ct}(i)$, are located at the center of each bar element. For details, see [5].

The SY-CDMmod eliminates the need the original CDM and SY-CDM [8] have to suppose $da/dN \times \Delta K$ curves described by a suitable (but nevertheless arbitrary) McEvily-like FCG rule. It does so using two new reasonable hypotheses. The first assumes that if a fatigue limit exists, then there is a limit strain range $\Delta\varepsilon_{y,th}$ below which the crack does not grow, associated to the SIF threshold range ΔK_{th}. So, any applied load range smaller than the threshold induces a strain range that does not cause damage to the crack. The second assumes the crack becomes unstable at a maximum plastic strain related to the material toughness. The effective plastic strain range $\Delta\varepsilon_{y,eff}$ that acts at the center of each element ahead of the crack tip can be correlated with the number of cycles $N(i)$ that would be required to break that element if that range was kept constant. These $N(i)$ can be calculated from the plastic part of Coffin-Manson's rule using Eq. (5), or from the SWT rule using Eq. (6):

$$N(i) = (1/2)\left(\Delta\varepsilon_{y,eff}(i)/2\varepsilon_c\right)^{1/c} \tag{5}$$

$$N(i) = (1/2)\left(\sigma_{max}(i)\cdot\Delta\varepsilon_{y,eff}(i)/2\sigma_c\varepsilon_c\right)^{1/(b+c)} \tag{6}$$

Notice that the effective strain range in Eqs. (5-6) is not associated to ΔK_{eff}, but to the maximum and minimum strains at each bar element. This SY-CDMmod formulation only considers plastic strain ranges, because strains calculated from SYM-estimated deformations assume a rigid-perfectly-plastic material ahead of the crack tip. The fatigue damage at each bar element is accumulated at every load cycle by Palmgren-Miner's rule (or by any other suitable rule). Crack increments are assumed equal to the distance where the accumulated damage reaches *1.0* (or any other suitable value). The stress σ_{max} from Eq. (6) is calculated considering tri-axial restrictions near the crack tip ($\sigma_{max} = \alpha\cdot S_F$). Although not considered in this work, this SY-CDMmod model is able to deal with VAL changing the width of the first and of the last elements inside the plastic zone, see [5] for details.

EXPERIMENTAL RESULTS AND DISCUSSION

These four models (SYM, CDM, SY-CDM, and SY-CDMmod) are compared with experimental $da/dN\times\Delta K$ data measured at $R = 0.1$ and $R = 0.7$ for two materials, a 7075-T6 Al alloy and a 1020 low carbon steel, following standard ASTM E647 procedures as described elsewhere [8]. Due to space limitations, only the 7075 Al data is presented here. Since the SYM formulation was developed for a center cracked plate and the data was measured in C(T) specimens, the *K*-analogy is used to define the applied stress as explained in [9]. Since the tests were made under plane strain conditions, a constraint factor $\alpha = 3$ is adopted for modeling purposes (the SY-CDM and the SY-CDMmod use strains calculated from SYM procedures, so it is necessary to define this parameter.) For each material and load condition, the simulation stopped only after reaching a FCG rate fluctuation lower than 0.1% with a minimal crack increment of 5mm.

Figures 2 and 3 show the measured $da/dN\times\Delta K$ data points and the curves predicted by six models. First, by the original CDM based on Creager and Paris (C&P). Second, by the SYM assuming a plastic constraint $\alpha = 3$ as mentioned above. Third, by two SY-CDMs (SY-CDM C&M and SY-CDM SWT) proposed in [8]. Finally, by two SY-CDMmod proposed in [5] (SY-CDMmod C&M and SY-CDMmod SWT). As explained before, all SYM-CDM curves are predicted from the εN damage induced by the cyclic strain fields estimated by strip-yield procedures using either Coffin-Manson or SWT εN rules, using only the plastic part of those εN models. This simplification is needed for a fair comparison, since the numerical procedures used in the SYMs discretize the *pz* ahead of the crack tip assuming rigid-perfectly-plastic VE elements. Recall that Coffin-Manson does not recognize mean or maximum stress effects, whereas SWT does. Recall as well that the SYM-CDMmod does not need to use a previously chosen da/dN rule, due to the two limiting strains introduced in this new model.

Figure 2 shows that the FCG curves estimated by the SY-CDM based on Coffin-Manson (C&M) and by the original C&P CDM from [6] are essentially equal. Both are quite reasonable for $R = 0.1$, albeit not as good for $R = 0.7$ (Fig. 3). The original SYM curve (estimated assuming $\alpha = 3$) describes better the data points measured at $R = 0.7$ (Fig. 3), but generates non-conservative predictions for lower ΔK at $R = 0.1$ (Fig. 2). As expected, critical damage FCG rate estimates based on SWT are higher than estimates based on Coffin-Manson for both *R*-ratios. The model proposed here that uses a Coffin-Manson damage calculation (SY-CDMmod C&M) yield the best estimates for $R = 0.1$ (Fig. 2) and has a reasonable performance (similar to the original SYM) for the higher $R = 0.7$ (Fig. 3). The SYM-CDMmod has a better performance at the higher ΔK ranges, where the original models systematically estimated FCG rates higher than the measured data. It is important to emphasize that the SY-CDMmod does not need to assume a pre-defined FCG curve. It does not need to use any adjustable data-fitting constant either. It only uses εN properties and suitable strain limits associated to the FCG threshold and the toughness of the material. The original CDM and the SY-CDM need to assume a pre-chosen McEvilytype $da/dN \times \Delta K$ curve, whose single adjustable parameter can however be calculated by εN procedures. The former also needs to assume a displaced HRR field to describe the strain field ahead of the crack tip and to eliminate the (unreal) crack tip singularity, as explained in [8]. The original SYM, on the other hand, assumes a ΔK_{eff}-based Forman-Newman FCG curve with four adjustable data-fitting parameters [9].

The quite reasonable performance of the CDMs certainly is not a coincidence, since their FCG predictions are based on εN properties and use no adjustable data-fitting parameters. In fact, when compared to SYM estimates based on ΔK_{eff} concepts and on a FCG rule that needs 4 adjustable parameters, not to mention the constraint factor α that in practice is frequently used as a 5th data-fitting parameter, the CDM performance could be even qualified as quite impressive for such a simple model. Even though a reasonable data-fitting cannot be considered as proof of the SY-CDMmod validity, it at least indicates that the CDM hypotheses are reasonable.

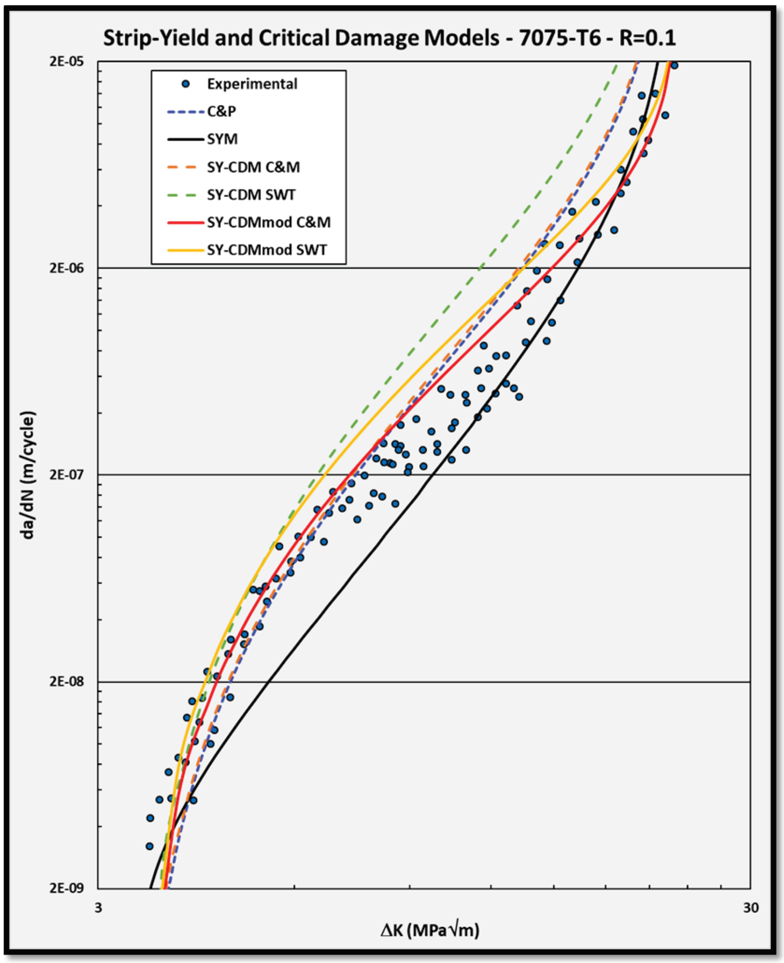

Figure 16: Strip-yield and critical damage models for the Al 7075-T6 at $R = 0.1$.

CONCLUSION

FCG models based on critical damage and on strip-yield/ΔK_{eff} contradictory ideas, and combined SYM/CDM models that join the SYM mechanics with CDM procedures, are used to estimate FCG curves, which are compared against 7075-T6 Al alloy FCG data, measured following standard ASTM E647 procedures. The εN properties were measured by standard ASTM E606 procedures, using coupons machined from the same material lot, to avoid any inconsistency in the data. The quite reasonable performance of the predictions indicates that, although apparently contradictory, such models are not incompatible. It also indicates that the good fitting of some properly obtained data set is not enough to prove which one is the best.

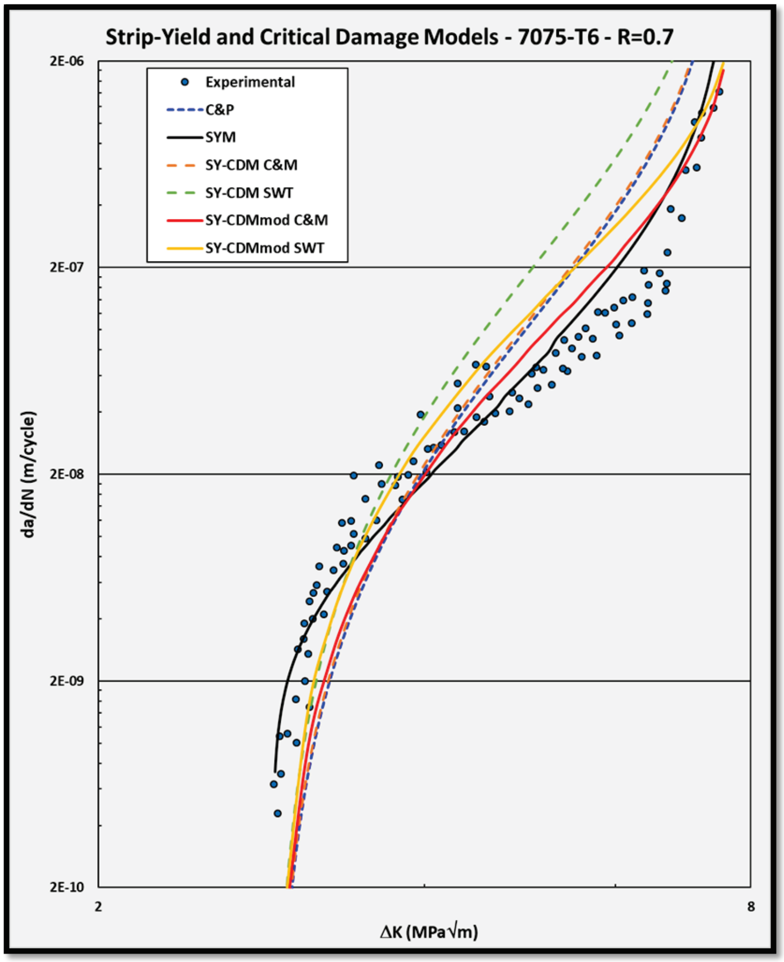

Figure 3: Strip-yield and critical damage models for the Al 7075-T6 at $R = 0.7$.

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