# Using fractional calculus to model viscoelastic behavior in concrete and polymers

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Abstract: Rheological models based on linear viscoelastic concepts are commonly used to model the mechanical behavior of polymers and concrete in many practical applications. Such models use linear springs and dashpots to model elastic, creep, relaxation, and recuperation behaviors characteristic of those materials. However, their data-fitting process can become very involved when it is necessary to identify the parameters of many rheological elements to describe properly the response of real materials. This work shows that the fractional Scot-Blair element, based on fractional calculus principles, can be a better choice to fit experimental data in such cases.

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## Fundamentals and results

Both in metallic or ceramic crystalline alloys, as well as in polymers and glasses, creep is a thermo-mechanical failure mechanism that gradually accumulates inelastic strains in structural components [1]. Polypropylene (PP) is a tough and relatively cheap thermoplastic polymer used in many practical applications due not only to its versatility and low density, but also because its reputation for good fatigue performance. The creep behavior of PP is modeled using a Fractional Kevin-Voigt Transfer Function [2-3]. The resulting transfer function is shown in Eq.(1).

$$\varepsilon(s) = \varepsilon_0(s) + \varepsilon_{cr}(s) = \left(\frac{1}{E_1} + \frac{1}{E_2 + C_\beta \cdot s^\beta}\right)\sigma(s) \qquad (1)$$

Figure 1 shows the shows the simulated and experimental creep curves of (PP) at $\Theta = 20^oC$ for a period of 365 days.

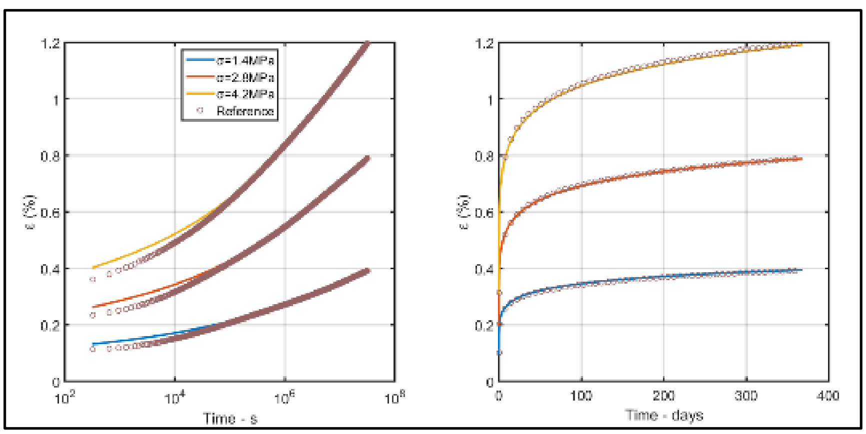


Figure 1: Simulated Polypropylene (PP) ε×t creep curves in the linear range measured at $\Theta = 20^oC$.

This very simple model can fit the experimental data with a maximum error of about 12% during the first day only, after which its performance improves significantly, making it an attractive choice to make long term creep strain predictions.

## References

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