A Fracture Mechanics Based Model for Explaining Notch Sensitivity Effects on Fatigue Crack Initiation

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Abstract

In this work, short cracks emanating from circular holes are studied. For several combinations of notch dimensions, the smallest stress range necessary to both initiate and propagate a crack is calculated, resulting in expressions for the fatigue stress concentration factor $\mathbf{K_f}$ and therefore the notch sensitivity $\mathbf{q}$. A generalization of El Haddad-Topper-Smith's parameter, which better correlates with experimental crack propagation data from the literature, is presented.

Introduction

The distinction between "short" and "long" cracks is necessary when one attempts to use the stress intensity range fatigue crack propagation threshold $\mathbf{\Delta K_{th}}$ to calculate the safe stress range $\Delta\sigma$ that can be applied to a cracked piece. $\mathbf{\Delta K_{th}}$ certainly can be applied to long cracks, but as the crack length $\mathbf{a} \to \mathbf{0}$, the stress range that could be applied on the cracked piece would be $\Delta\sigma \to \infty$, which does not make sense, since the traditional fatigue limit of uncracked pieces $\Delta\sigma_\mathbf{0}$ is a finite value. In order to reproduce this behavior, several expressions have been proposed to model the dependency between the threshold value $\mathbf{\Delta K_{th}}$ and the crack size $\mathbf{a}$ for very small cracks [1]. Most of these expressions are based on length parameters such as El Haddad-Topper-Smith's $\mathbf{a_0}$ [2], estimated from $\mathbf{\Delta K_{th}}$ and $\Delta\sigma_\mathbf{0}$, resulting in a modified stress intensity range

$$\Delta \mathbf{K_I} = \Delta\sigma\sqrt{\pi(\mathbf{a}+\mathbf{a_0})} \tag{1}$$

where the so-called transition size for the crack, i.e., the size below which the crack must be treated as "small", is given by

$$\mathbf{a_0} = \frac{\mathbf{1}}{\pi}\left(\frac{\Delta \mathbf{K}_{th}}{\Delta\sigma_0}\right)^{\mathbf{2}} \tag{2}$$

which is able to reproduce most of the behaviour shown in the Kitagawa-Takahashi plot [3]. Yu *et al.* [4] and Atzori *et al.* [5] have also used a geometry factor α to generalize the above equation to any specimen geometry, resulting in

$$\Delta \mathbf{K}_I = \alpha \cdot \Delta\sigma \sqrt{\pi(\mathbf{a} + \mathbf{a_0})} \tag{3}$$

$$\mathbf{a_0} = \frac{\mathbf{1}}{\pi}\left(\frac{\Delta \mathbf{K}_{th}}{\alpha \cdot \Delta\sigma_0}\right)^{\mathbf{2}} \tag{4}$$

Alternatively, the stress intensity range can retain its original equation, while the threshold expression is modified by a function of the crack length $\mathbf{a}$, namely $\mathbf{\Delta K_{th}(a)}$, resulting in

$$\frac{\Delta \mathbf{K_{th}(a)}}{\Delta \mathbf{K_0}} = \sqrt{\frac{\mathbf{a}}{\mathbf{a} + \mathbf{a_0}}} \tag{5}$$

where $\mathbf{\Delta K_0}$ is the threshold stress intensity factor for a long crack.

It is well known that the notch sensitivity factor $\mathbf{q}$ can be associated with the presence of non-propagating fatigue cracks. Such cracks are present when the nominal stress range $\Delta\sigma_\mathbf{n}$ is between $\Delta\sigma_\mathbf{0}/\mathbf{K_t}$ and $\Delta\sigma_\mathbf{0}/\mathbf{K_f}$, where $\mathbf{K_t}$ is the geometric and $\mathbf{K_f}$ the fatigue stress concentration factors of the notch. Therefore, in principle it is possible to obtain expressions for $\mathbf{q}$ if the propagation behaviour of small cracks emanating from notches is known.

Several expressions have been proposed to model this crack size dependence [6-8]. Peterson-like expressions are then calibrated to $\mathbf{q}$ based on these crack propagation estimates. However, such $\mathbf{q}$ calibration is found to be extremely sensitive to the choice of $\mathbf{\Delta K_{th}(a)}$ estimate.

In the following section, a generalization of El Haddad-Topper-Smith's equation is proposed to better model the crack size dependence of $\mathbf{\Delta K_{th}}$. This expression is then applied to a single crack emanating from a circular hole, resulting in improved estimates of $\mathbf{q}$.

Analytical Development

First a new expression for the threshold stress intensity factor of short cracks is proposed, based on El Haddad-Topper-Smith's equation:

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \left[1 + \left(\frac{a_0}{a}\right)^{n/2}\right]^{-1/n} \qquad (6)$$

In the above equation, **n** is typically found to be between **1.5** and **8.0**. Clearly, Eqs. (1), (3) and (5) are obtained from Eq. (6) when **n = 2.0**. Also, the classical bi-linear estimate is obtained as **n** tends to infinity. The main advantage of the adjustable parameter **n** is to allow the ΔK_{th} estimates to better correlate with experimental crack propagation data collected from Tanaka et al. [9] and Livieri and Tovo [10], see Fig. 1.

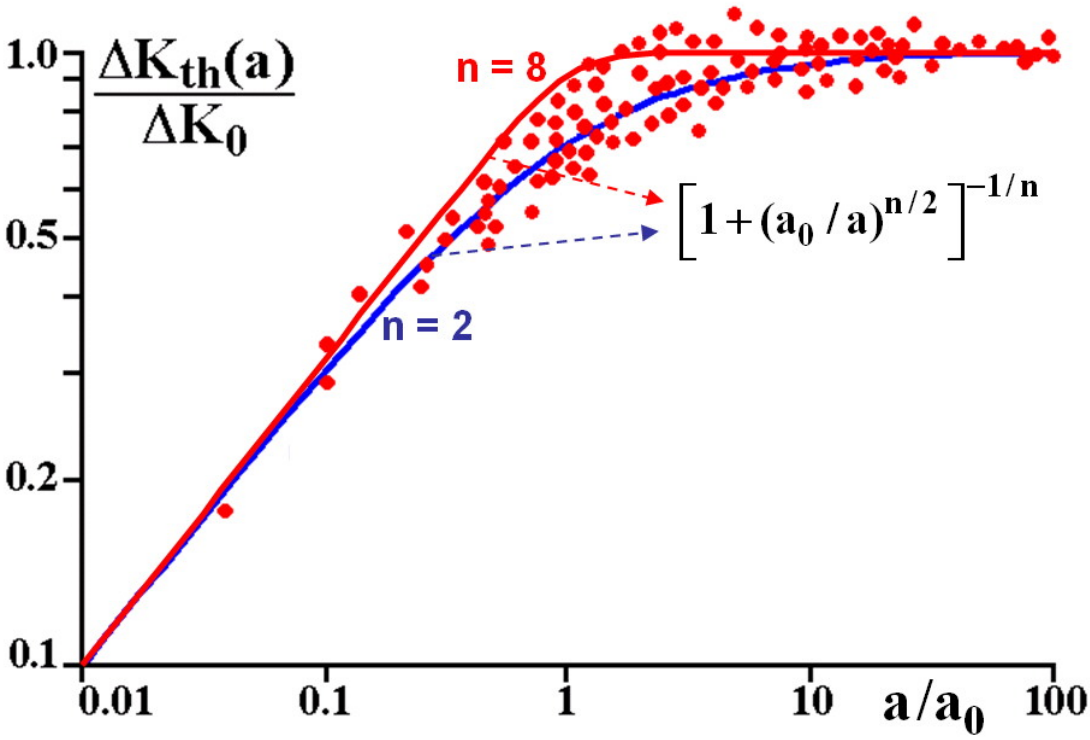

Figure 1: Ratio between short and long crack propagation thresholds as a function of $\mathbf{a/a_0}$.

Equation (6) is now used to evaluate the behavior of short cracks emanating from circular holes. The stress intensity range of a single crack with length *a* emanating from a circular hole with radius ρ is expressed, within 1%, by [11]

$$\Delta K_I = 1.1215 \cdot \Delta\sigma\sqrt{\pi a} \cdot f\left(a/\rho\right) \qquad (7)$$

$$f\left(\frac{a}{\rho}\right) \equiv f(x) = \left(1 + \frac{0.2}{(1+x)} + \frac{0.3}{(1+x)^6}\right) \cdot \left(2 - 2.354\frac{x}{1+x} + 1.2056\left(\frac{x}{1+x}\right)^2 - 0.2211\left(\frac{x}{1+x}\right)^3\right) \qquad (8)$$

where $\mathbf{x} \equiv \mathbf{a}/\rho$ is the normalized crack length. Note that, when the crack size $\mathbf{a}$ tends to zero, Eq. (7) becomes

$$\lim_{\mathbf{a}\to\mathbf{0}} \Delta\mathbf{K}_I = \mathbf{1.1215}\cdot\Delta\sigma\sqrt{\pi\mathbf{a}}\cdot\mathbf{3} \tag{9}$$

as expected, since the above equation combines the solution for an edge crack in a semi-infinite plate with the stress concentration factor of a circular hole, $\mathbf{K_t = 3}$. Note also that the other limit, when $\mathbf{a}$ tends to infinity, results in

$$\lim_{\mathbf{a}\to\infty} \Delta\mathbf{K}_I = \Delta\sigma\sqrt{\pi\,\mathbf{a}/\mathbf{2}} \tag{10}$$

which is the solution for a crack with length $\mathbf{a}$ in an infinite plate, where one of its edges is far enough from the circular hole not to suffer its influence in the stress field (in fact, the equivalent crack length would be $\mathbf{a} + \rho$, however as $\mathbf{a}$ tends to infinity the ρ value disappears from the equation). Therefore, it follows that for a circular hole $\mathbf{f(x=0) = 3}$ and $\mathbf{f(x\to\infty) = 1/1.1215\sqrt{2} \cong 0.63}$.

From Eqs. (4-6), it follows that the crack will propagate when

$$\Delta\mathbf{K_I} = \mathbf{1.1215}\cdot\Delta\sigma\sqrt{\pi\mathbf{a}}\cdot\mathbf{f}\left(\frac{\mathbf{a}}{\rho}\right) > \Delta\mathbf{K_{th}} = \Delta\mathbf{K_0}\cdot\left[\mathbf{1}+\left(\frac{\mathbf{a_0}}{\mathbf{a}}\right)^{\mathbf{n/2}}\right]^{-\mathbf{1/n}} \tag{11}$$

Using $\alpha = \mathbf{1.1215}$ and $\Delta\mathbf{K_{th}} \equiv \Delta\mathbf{K_0}$ for a long crack, then the crack length parameter from the above equation is

$$\mathbf{a_0} = \frac{\mathbf{1}}{\pi}\left(\frac{\Delta\mathbf{K}_0}{\mathbf{1.1215}\cdot\Delta\sigma_0}\right)^{\mathbf{2}} \tag{12}$$

Using (11) and (12) a crack propagation criterion based on two adimensional functions $\mathbf{f}$ and $\mathbf{g}$, which can be regarded as the loading and the resisting functions for this specific cracked geometry, can be proposed

$$\mathbf{f}\left(\frac{\mathbf{a}}{\rho}\right) > \frac{\left(\dfrac{\Delta\mathbf{K_0}}{\Delta\sigma_\mathbf{0}\sqrt{\rho}}\right)\cdot\left(\dfrac{\Delta\sigma_\mathbf{0}}{\Delta\sigma}\right)}{\left[\left(\mathbf{1.1215}\sqrt{\dfrac{\pi\mathbf{a}}{\rho}}\right)^{\mathbf{n}}+\left(\dfrac{\Delta\mathbf{K_0}}{\Delta\sigma_\mathbf{0}\sqrt{\rho}}\right)^{\mathbf{n}}\right]^{\mathbf{1/n}}} \equiv \mathbf{g}\left(\frac{\mathbf{a}}{\rho},\frac{\Delta\sigma_\mathbf{0}}{\Delta\sigma},\frac{\Delta\mathbf{K_0}}{\Delta\sigma_\mathbf{0}\sqrt{\rho}},\mathbf{n}\right) \tag{13}$$

In this way, using the normalized crack length $\mathbf{x} \equiv \mathbf{a}/\rho$ and defining $\mathbf{k} \equiv \Delta\mathbf{K_0}/\Delta\sigma_0\sqrt{\rho}$, it is possible to predict that the crack propagates whenever

$$\mathbf{f}(\mathbf{x}) > \mathbf{g}\left(\mathbf{x}, \frac{\Delta\sigma_0}{\Delta\sigma}, \mathbf{k}, \mathbf{n}\right) \tag{14}$$

Figure 2 plots **f** and **g**, assuming a material/notch combination with $\mathbf{k} = \mathbf{1.5}$ and $\mathbf{n} = \mathbf{6}$, as a function of the normalized crack length **x**. For a high applied stress range $\Delta\sigma$, the ratio $\Delta\sigma_0/\Delta\sigma$ becomes small, and the function **g** is always below **f**, meaning that a crack of any length will propagate. The lower curve in Fig. 2 shows the function **g** obtained for a ratio $\Delta\sigma_0/\Delta\sigma = \mathbf{1.4}$ which, as expected, never crosses **f**, since the maximum stress range at the hole root $\mathbf{K_t}\Delta\sigma > \Delta\sigma_0$ is bigger than the fatigue limit of the material in this case. On the other hand, for a $\Delta\sigma$ small enough such that $\Delta\sigma_0/\Delta\sigma \geq \mathbf{K_t} = \mathbf{3}$, then **g** is always above **f** and no crack will initiate nor propagate, as shown by the top curve in the figure.

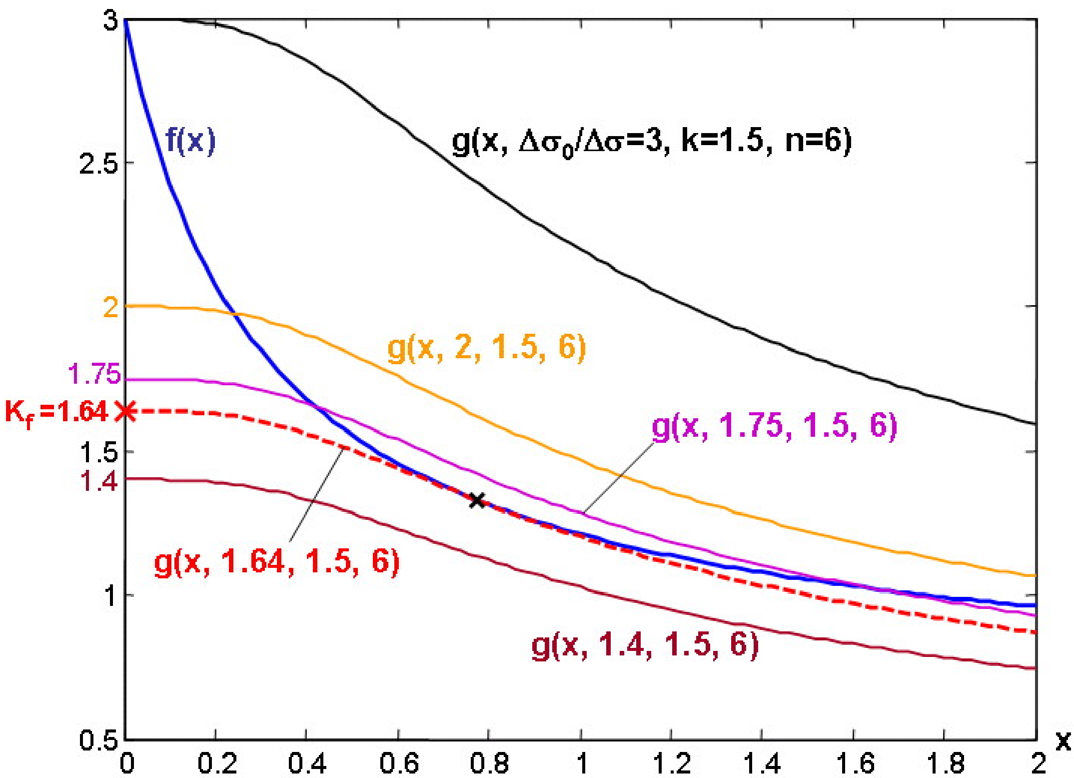

Figure 2. Calculation of the fatigue stress concentration factor $\mathbf{K_f}$ from the functions **f** and **g**.

But three other cases can be noted in this figure, as follows. In the first case, illustrated by the **g** curve with $\Delta\sigma_0/\Delta\sigma = \mathbf{2}$ in the figure above, **g** has only one intersection point with **f**. This means that stress levels of that order can cause a small fatigue crack to initiate at the notch root, but this crack will stop propagating by fatigue when it reaches a size $\mathbf{a = x \cdot \rho}$, obtained from the $\mathbf{x}$ value at that one intersection point. Therefore, at such stress levels non-propagating fatigue cracks will appear at the notch root (if fatigue is the sole damage mechanism able to increase the crack size). In other words, the piece can live forever with a non-damaging crack at the notch root.

In the second case, illustrated by the $\mathbf{g}$ curve with $\Delta\sigma_0/\Delta\sigma = \mathbf{1.75}$ in the figure above, $\mathbf{g}$ has two intersection points with $\mathbf{f}$. Therefore, non-propagating fatigue cracks will also appear in this case, with maximum sizes obtained from the first intersection point (on the left). Interestingly, cracks longer than the value defined by the second intersection will re-start propagating by fatigue until fracture. However, the crack growth between the two intersection points would need to be caused by a different damage mechanism, e.g. corrosion or creep.

Finally, the third case that can be seen in Figure 2 is for the g curve with $\Delta\sigma_0/\Delta\sigma = \mathbf{1.64}$. In this case, the $\mathbf{f}$ and $\mathbf{g}$ functions are tangent and meet in a single point. This $\Delta\sigma_0/\Delta\sigma$ value is therefore associated with the smaller stress range $\Delta\sigma$ that can cause crack initiation *and* propagation without arrest. So, by definition, this specific $\Delta\sigma_0/\Delta\sigma$ is equal to the fatigue stress concentration factor $\mathbf{K_f}$. Thus, to obtain $\mathbf{K_f}$ it is then sufficient to guarantee that the functions $\mathbf{f}$ and $\mathbf{g}$ are tangent at a single point, where $\mathbf{x = x_{max}}$. This $\mathbf{x_{max}}$ value is associated with the largest non-propagating flaw that can arise from fatigue alone. So, given $\mathbf{n}$ and $\mathbf{k}$ from the material and notch, $\mathbf{x_{max}}$ and $\mathbf{K_f}$ can be found from the system of equations:

$$\begin{cases} \mathbf{f}\left(\mathbf{x_{max}}\right) = \mathbf{g}\left(\mathbf{x_{max}}, \mathbf{K_f}, \mathbf{k}, \mathbf{n}\right) \\ \dfrac{\partial}{\partial \mathbf{x}}\mathbf{f}\left(\mathbf{x_{max}}\right) = \dfrac{\partial}{\partial \mathbf{x}}\mathbf{g}\left(\mathbf{x_{max}}, \mathbf{K_f}, \mathbf{k}, \mathbf{n}\right) \end{cases} \tag{15}$$

This system can be solved numerically for each combination of $\mathbf{k} \equiv \Delta\mathbf{K_0}/\Delta\sigma_0\sqrt{\rho}$ and $\mathbf{n}$ values, that define the material and the hole size influence on the fatigue behavior of the plate. Thus the notch sensitivity factor $\mathbf{q}$ can be obtained from

$$\mathbf{q(k,n)} \equiv \frac{\mathbf{K_f(k,n) - 1}}{\mathbf{K_t - 1}} \tag{16}$$

Results

For several combinations of $\mathbf{k}$ and $\mathbf{n}$, the smallest stress range necessary to both initiate and propagate a crack is calculated from Equation (15), resulting in expressions for $\mathbf{K_f}$ and therefore $\mathbf{q}$, see Figure 3. Note in this figure that $\mathbf{q}$ is approximately linear with $\mathbf{1/k}$ for $\mathbf{q} > \mathbf{0}$. This results in the proposed estimate:

$$\mathbf{q(k,n) \cong \frac{q_1(n)}{k} - q_0(n) = q_1(n)\frac{\Delta\sigma_0\sqrt{\rho}}{\Delta K_0} - q_0(n)} \tag{17}$$

where $\mathbf{q_0(n)}$ and $\mathbf{q_1(n)}$ are functions of $\mathbf{n}$, and $\mathbf{q_1(n)}$ is typically between $\mathbf{0.85}$ and $\mathbf{1.15}$. Note also that if the estimate above results in $\mathbf{q}$ larger than 1, then $\mathbf{q = 1}$. This will happen at holes with a very large radius $\rho_{\mathbf{upper}}$ such that

$$\mathbf{\frac{\Delta\sigma_0\sqrt{\rho_{upper}}}{\Delta K_0} > \frac{1+q_0(n)}{q_1(n)} \quad \Rightarrow \quad \rho_{upper} > \left(\frac{1+q_0(n)}{q_1(n)} \cdot \frac{\Delta K_0}{\Delta\sigma_0}\right)^2} \tag{18}$$

Therefore, it is impossible to generate a non-propagating crack under constant amplitude loading in notches with a very large radius, regardless of the stress level. The stress gradient is so small in this case that any crack that initiates will cut through a long region still influenced by the stress concentration, preventing any possibility of crack arrest. Equation (15) will not have a solution for $\mathbf{x_{max}} > \mathbf{0}$, because $\partial\mathbf{g}/\partial\mathbf{x}$ in this case will be more negative than $\partial\mathbf{f}/\partial\mathbf{x}$ at $\mathbf{x = 0}$.

On the other hand, it is possible to obtain a value of $\mathbf{q}$ smaller than zero, down to $\mathbf{q = -0.2}$ for a circular hole, see Fig. 3. This can indeed happen for holes with a very small radius $\rho_{\mathbf{lower}}$ such that

$$\mathbf{\frac{\Delta\sigma_0\sqrt{\rho_{lower}}}{\Delta K_0} < \frac{q_0(n)}{q_1(n)} \quad \Rightarrow \quad \rho_{lower} < \left(\frac{q_0(n)}{q_1(n)} \cdot \frac{\Delta K_0}{\Delta\sigma_0}\right)^2} \tag{18}$$

The physical meaning of a negative $\mathbf{q}$ is that it is easier to initiate and propagate a fatigue crack at a notchless border of the plate than at a very small hole inside the plate. The $\mathbf{\Delta K_I}$ of a crack at the small hole will soon tend to Eq. (10) due to the large stress gradient, while the stress intensity solution for an edge crack will be larger, since it includes the $\mathbf{1.1215}$ surface factor. In addition, for most materials, the size of this critical radius $\rho_{\mathbf{lower}}$ is just a few micrometers. This leads to the conclusion that internal defects with equivalent radius smaller than such $\rho_{\mathbf{lower}}$ of a few micrometers are harmless, since its $\mathbf{K_f}$ will be smaller than 1, and the main propagating crack will initiate at the surface.

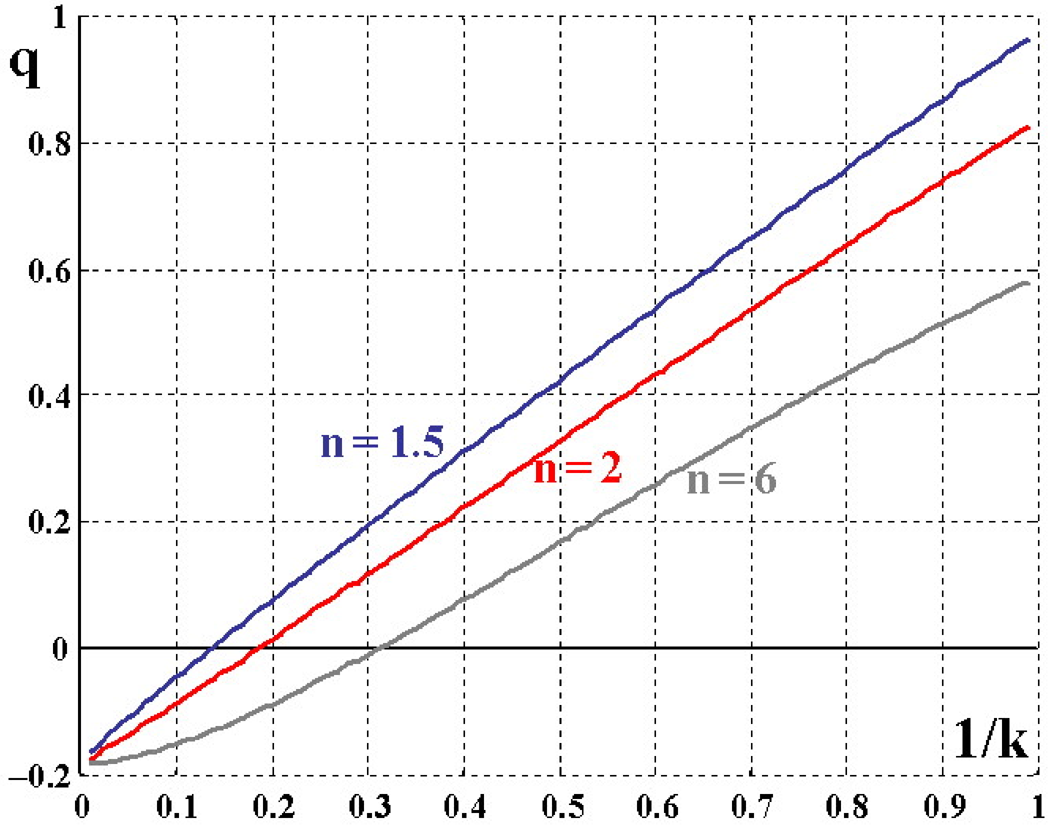

Figure 3. Notch sensitivity factors **q** as a function of the adimensional parameters **k** and **n**.

Note that several estimates, such as Peterson's, assume that the notch sensitivity is only a function of the hole radius ρ and the material ultimate strength $\mathbf{S_u}$. Equation (17), however, suggests that **q** depends basically on ρ, $\Delta\sigma_0$ and $\Delta\mathbf{K_0}$, and also on **n**. Even though there are reasonable estimates relating $\Delta\sigma_0$ and $\mathbf{S_u}$, there is no clear relationship between $\Delta\mathbf{K_0}$ and $\mathbf{S_u}$. This means, e.g., that two steels with same $\mathbf{S_u}$ but very different $\Delta\mathbf{K_0}$ would have different behaviors, a fact that Peterson's equation would not be able to reproduce. Therefore, notch sensitivity experiments should always include a measure of the $\Delta\mathbf{K_0}$ of the material.

Finally, data on 450 steels and aluminum alloys with (sic) fully measured $\mathbf{S_u}$, $\Delta\sigma_0$ and $\Delta\mathbf{K_0}$ are collected from the ViDa software database [12]. The average values of $\Delta\sigma_0$ and $\Delta\mathbf{K_0}$ are evaluated for steels with $\mathbf{S_u}$ near the ranges **400**, **800**, **1200**, **1600** and **2000MPa**, and aluminum alloys near

225MPa. Equation (17) is then plotted as a function of the notch radius ρ, using the above averages and assuming **n = 6**, see Fig. 4. Note that Peterson's equations, which were originally fitted to notch sensitivity experiments, can be quite reasonably predicted and reproduced using the proposed analytical approach.

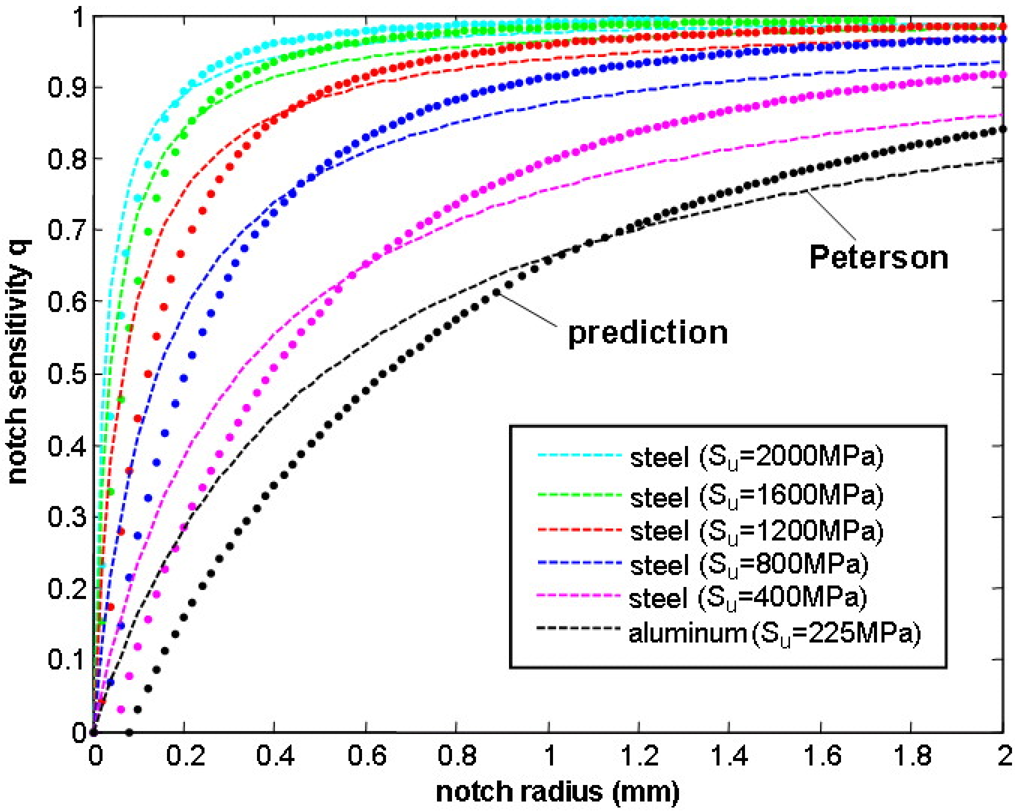

Figure 4: Predicted and experimentally fitted notch sensitivity factors as a function of notch radius for several materials.

Conclusions

A generalization of El Haddad-Topper-Smith's parameter was presented to model the crack size dependence of the threshold stress intensity range for short cracks. The proposed expressions were used to calculate the behavior of non-propagating cracks. New estimates for the notch sensitivity factor **q** were obtained and compared with Peterson's results. It was found that the **q** estimates obtained from this generalization correlate well with crack initiation data.

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