

# Statistical evaluation of strain-life fatigue crack initiation predictions

M.A. Meggiolaro\*, J.T.P. Castro

*Mechanical Engineering Department, Pontifical Catholic University of Rio de Janeiro (PUC-Rio), Rua Marquês de São Vicente 225, Rio de Janeiro, RJ 22453-900, Brazil*

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## Abstract

Most of the existing methods for estimating  $\varepsilon N$  parameters are based on a relatively limited amount of experimental data. In addition, sound statistical evaluation of the popular rules of thumb used in practice to estimate fatigue properties is scarce, if available. In this work, an extensive statistical evaluation of the existing Coffin–Manson parameter estimates is presented based on monotonic tensile and uniaxial fatigue properties of 845 different metals, including 724 steels, 81 aluminum alloys, and 15 titanium alloys. The studied Coffin–Manson estimates include the methods proposed by Muralidharan and Manson, Bäumel and Seeger, Roessle and Fatemi, Mitchell, Ong, Morrow, Raske, as well as Manson's universal slope and four-point correlation methods. From the collected data, it is shown that all correlations between the fatigue ductility coefficient  $\varepsilon'_f$  and the monotonic tensile properties are very poor, and that it is statistically sounder to estimate  $\varepsilon'_f$  based on constant values for each alloy family. Based on this result, a new estimation method which uses the medians of the individual parameters of the 845 materials is proposed.

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## 1. Introduction

The so-called  $\varepsilon N$  fatigue design method correlates the number of cycles  $N$  to initiate a fatigue crack in any structure with the life of small specimens made of the same material and submitted to the same strain history that loads the critical point (generally a notch root) in service. This method does not recognize the presence of cracks; however, it models macroscopic elastic–plastic events at the notch roots and uses the local strain range (a more robust parameter to describe plastic effects) instead of the stress range to quantify them. Therefore, the  $\varepsilon N$  method must be used to model low-cycle fatigue problems, when the plastic strain range  $\Delta\varepsilon_p$  at the critical point is of the same order or larger than the elastic range  $\Delta\varepsilon_e$ , but it can be applied to predict any crack initiation life.

The classical  $\varepsilon N$  method works with real (logarithmic) stresses and strains, uses a Ramberg–Osgood description for the  $\Delta\sigma\Delta\varepsilon$  hysteresis loops, and considers the cyclic softening or hardening of the material, but not its transient behavior from the monotonic  $\sigma\varepsilon$  curve [1–5]. Hence, a single equation is used to describe all hysteresis loops:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \quad (1)$$

where  $E$  is the Young modulus,  $K'$  is the hardening coefficient and  $n'$  is the hardening exponent of the cyclically stabilized  $\Delta\sigma\Delta\varepsilon$  curve. Values for the cyclic hardening exponent  $n'$  are typically between 0.05 and 0.3, while the monotonic hardening exponent  $n$  is more disperse, varying between 0 and 0.5 in most cases.

The relationship between the strain range  $\Delta\varepsilon$  at the critical point and its fatigue initiation life  $N$  is usually

\* Corresponding author. Tel.: +55-21-3114-1638; fax: +55-21-3114-1165.

E-mail address: [meggi@mec.puc-rio.br](mailto:meggi@mec.puc-rio.br) (M.A. Meggiolaro).

given by the classical Coffin–Manson equation:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma'_f}{E}(2N)^b + \varepsilon'_f(2N)^c \quad (2)$$

where  $\sigma'_f$ ,  $\varepsilon'_f$ ,  $b$  and  $c$  are the fatigue strength and ductility coefficients and exponents measured in fully alternated tension–compression fatigue tests.

Assuming that Ramberg–Osgood’s elastic and plastic strain ranges perfectly correlate with the correspondent Coffin–Manson’s ranges, then only four of the six material parameters  $\{n', K', \sigma'_f, \varepsilon'_f, b, c\}$  would be independent. Thus, from Eqs. (1) and (2),

$$\frac{\Delta\sigma}{2} = \sigma'_f(2N)^b = K' \varepsilon'^{n'}(2N)^{c \cdot n'} \Rightarrow n' = \frac{b}{c};$$

$$K' = \frac{\sigma'_f}{\varepsilon'^{n'}} \quad (3)$$

The Ramberg–Osgood and Coffin–Manson equations describe well the cyclic response of many materials; however, they are not physical laws. Instead, Eq. (3) must be regarded as a measure of the coherence between those equations. Therefore, such “theoretical” estimates should not be used to replace experiments. Whenever possible, all six material parameters should be independently obtained from actual measurements.

However, for initial design studies, it is desirable to estimate these six  $\varepsilon N$  parameters based only on readily available monotonic tensile test data. The main estimation methods proposed in the literature are discussed next.

## 2. Estimation methods of fatigue properties

Several estimates of Coffin–Manson’s parameters have been proposed in the literature since Morrow [6], who in 1964 correlated the  $b$  and  $c$  exponents of Coffin–Manson’s equation with the cyclic hardening exponent  $n'$ , see Table 1.

Based on the experimental results on 69 metals, Manson [7] proposed in 1965 two different methods: the universal slope method, in which  $b$  and  $c$  are assumed constant for all metals (namely  $b = -0.12$  and  $c = -0.6$ ), and the four-point correlation method, defined through estimates of the elastic or the plastic strain ranges  $\Delta\varepsilon_e/2$  or  $\Delta\varepsilon_p/2$  at four different lives (namely  $N = 1/4, 10, 10^4$  and  $10^5$  cycles). Both Manson’s estimates make use of the ultimate strength  $S_U$  and the reduction in area  $RA$ .

Raske and Morrow [8] published in 1969 an estimate for the fatigue ductility coefficient  $\varepsilon'_f$  from  $\sigma'_f$ ,  $n'$ , and the cyclic yielding strength  $S'_Y$ .

Mitchell et al. [9] proposed in 1977 that the exponent  $b$  (and not only  $\sigma'_f$ ) is a function of  $S_U$ , estimated  $\varepsilon'_f$  directly from the true fracture ductility  $\varepsilon_f$ , and assumed that Manson’s slope  $c = -0.6$  is only valid for “ductile” metals, while  $c = -0.5$  should be more appropriate for “strong” alloys.

In 1988, Muralidharan and Manson [10] revisited the universal slope idea, increasing both Coffin–Manson’s exponents to  $b = -0.09$  and  $c = -0.56$ , and introducing the parameter  $S_U/E$  to estimate both coefficients,  $\sigma'_f$  and  $\varepsilon'_f$ .

Table 1  
Estimation methods for Coffin–Manson’s parameters

Estimate	$\sigma'_f$	$\varepsilon'_f$	$b$	$c$
Morrow (1964)	–	–	$\frac{-n'}{1+5n'}$	$\frac{-1}{1+5n'}$
Manson’s universal slopes (1965)	$1.9 \cdot S_U$	$0.76 \cdot \left[\ln\left(\frac{1}{1-RA}\right)\right]^{0.6}$	–0.12	–0.6
Manson’s four-point (1965)	$1.25\sigma'_f \cdot 2^b$ $\sigma'_f \cong S_U(1 + \varepsilon_f)$	$\frac{0.125}{20^c} \cdot \left[\ln\left(\frac{1}{1-RA}\right)\right]^{3/4}$	$\frac{\log(0.36 \cdot S_U / \sigma'_f)}{5.6}$	$\frac{1}{3} \log \frac{0.0066 - \sigma'_f (2 \times 10^4)^b / E}{0.239 \cdot \{\ln[1/(1-RA)]\}^{3/4}}$
Raske–Morrow (1969)	–	$0.002 \cdot (\sigma'_f / S'_Y)^{1/n'}$	–	–
Mitchell (steels, 1977)	$S_U + 345 \text{ MPa}$	$\varepsilon_f$	$\frac{1}{6} \log \frac{0.5 \cdot S_U}{S_U + 345}$	–0.6 (“ductile”) or –0.5 (“strong”)
Muralidharan–Manson (1988)	$0.623E \left(\frac{S_U}{E}\right)^{0.832}$	$0.0196(S_U/E)^{-0.53} \cdot \left[\ln\left(\frac{1}{1-RA}\right)\right]^{0.155}$	–0.09	–0.56
Bäumel–Seeger (steels, 1990)	$1.5 \cdot S_U$	$0.59$ if $S_U/E \leq 0.003$ or $0.812 - 74 \cdot S_U/E$	–0.087	–0.58
Bäumel–Seeger (Al and Ti, 1990)	$1.67 \cdot S_U$	0.35	–0.095	–0.69
Ong (1993)	$S_U \cdot (1 + \varepsilon_f)$	$\varepsilon_f$	$\frac{1}{6} \log \frac{(S_U/E)^{0.81}}{6.25 \cdot \sigma'_f / E}$	$\frac{1}{4} \log \frac{0.0074 - \sigma'_f (10^4)^b / E}{2.074 \cdot \varepsilon_f}$
Roessle–Fatemi (steels, 2000)	$4.25 \cdot HB + 225 \text{ MPa}$	$[0.32 \cdot HB^2 - 487 \cdot HB + 191,000 \text{ MPa}] / E$	–0.09	–0.56
Medians (steels, 2002)	$1.5 \cdot S_U$	0.45	–0.09	–0.59
Medians (Al alloys, 2002)	$1.9 \cdot S_U$	0.28	–0.11	–0.66

Two years later, Bäumel and Seeger [11] were the first to recognize the importance of separating the  $\varepsilon N$  estimates by alloy family, proposing different methods for low-alloy steels and for aluminum (Al) and titanium (Ti) alloys in their uniform material laws. It was found that on average, both exponents  $b$  and  $c$  are significantly lower in Al or Ti alloys than in steels, which might explain the low value of  $b$  proposed by Manson in 1965 [7], who included in his analyses several alloy families besides steels. Bäumel and Seeger were also the first to ignore any monotonic measure of the material ductility (such as the reduction in area  $RA$ ) when estimating the fatigue ductility coefficient,  $\varepsilon'_f$ .

In 1993, Ong [12] revisited Manson's four-point correlation method and proposed a few modifications to better fit the experimental data of 49 steels from the SAE J1099 technical report on fatigue properties [13]. Once again,  $b$  and  $c$  were assumed to be functions of the ultimate strength  $S_U$  and the true fracture ductility  $\varepsilon_f$ , while  $\varepsilon'_f$  was estimated in the same way as Mitchell proposed in 1977.

Recently, Roessle and Fatemi [14] proposed the hardness method, assuming the same constant slopes as Muralidharan and Manson did, while estimating both Coffin–Manson's coefficients as a function of the Brinell hardness,  $HB$ . It is no surprise that  $\sigma'_f$  can be estimated from the hardness  $HB$ , since  $S_U$  and  $HB$  present a very good correlation for steels: if  $S_U$  is given in MPa and  $HB$  in  $\text{kg}/\text{mm}^2$ ,  $S_U$  is approximately 3.4  $HB$  with a (small) coefficient of variation  $V=3.8\%$ , from a study on 1924 steels from the ViDa software database [15,16].

In the last decade, several works have been published evaluating the life prediction errors associated with each of the estimation methods discussed above [14,17–20]. In 1993, Ong [17] evaluated Manson and Mitchell's original methods based on properties of 49 steels. He concluded that Mitchell's method resulted in overly non-conservative predictions, while Manson's universal slope and four-point methods, although giving better life estimates, were only able to obtain satisfactory correlations for the fatigue strength coefficient,  $\sigma'_f$ . One year later, Brennan [18] compared all Manson's methods and concluded that Muralidharan–Manson's revised universal slopes [10] resulted in good predictions; however, his analysis was based on only six steels.

Park and Song [19] evaluated all methods proposed until 1995 using published data on 138 materials. They found that both Manson's original methods are excessively conservative for long life predictions, but slightly non-conservative for short lives. In contrast, Muralidharan–Manson's method is slightly conservative at shorter lives, but is non-conservative at long lives, being selected as the best overall estimation method

together with Bäumel–Seeger's uniform material laws. Park and Song also confirmed that Mitchell's method leads to non-conservative predictions over the entire life range.

Roessle and Fatemi [14] studied measured properties of 20 steels plus 49 steels from the SAE J1099 technical report on fatigue properties [13], arriving at basically the same conclusions as Park and Song did. In addition, no strong correlation was found between  $\sigma'_f$  and the true fracture strength. They also found that using the true fracture ductility  $\varepsilon_f$  to estimate  $\varepsilon'_f$  can result in significant error. Finally, Kim et al. [20] presented an evaluation of all available estimation methods, based on measured properties of eight steels. It was found that the best life predictions were obtained using Bäumel–Seeger, Roessle–Fatemi and Muralidharan–Manson's methods. In addition, Ong's method resulted in non-conservative predictions especially for long lives.

From the above evaluations, it is possible to conclude that the best estimation methods are all based on constant values of the exponents  $b$  and  $c$ , while in general  $\sigma'_f$  is well estimated (directly or indirectly) as a linear function of the ultimate strength  $S_U$ . It is also suggested that  $\varepsilon'_f$  does not correlate well with any monotonic measure of the material ductility, such as  $RA$  or  $\varepsilon_f$ . Comparing to the existing estimates for  $\varepsilon'_f$ , from a statistical point of view assuming it is a constant would result in better predictions. Based on these conclusions, a new  $\varepsilon N$  estimate called the medians method is proposed in this work, which assumes constant values for  $\sigma'_f/S_U$ ,  $\varepsilon'_f$ ,  $b$  and  $c$ . From a statistical study on the fatigue properties of 845 different metals, it is found that better estimates are obtained from the median values of each of these four parameters, calculated for each alloy family. A statistical evaluation of this method and all others discussed above is presented in the following sections.

### 3. Experimental program

Strain-controlled constant amplitude fatigue and monotonic tension tests at room temperature were performed on eight steels and one aluminum alloy according to the ASTM standards E606 and E8 [21,22]. The tested materials consisted of API steels 5D S-135, 5L Grade B, 5L X-60 (base and welded metals), SAE steels 1020 and 4340, USI SAR 60 (base and wet welded metals), and the aluminum alloy 7075-T6. USI SAR 60 is the commercial name of a low-C high-strength structural steel manufactured by Usiminas, with a minimum yielding strength of 460 MPa and analyzed % weight chemical composition of C 0.12, Mn 1.09, Cr 0.18, Mo 0.14, V 0.09, Al 0.04, Si 0.024, Ti 0.02, Ni 0.02, and P 0.014.

A minimum of 10  $\epsilon N$  specimens of each material were cyclically tested at strain amplitudes which ranged from 0.2% to 1.2%. All tests were made at  $R = -1$ , under strain control on servo-hydraulic testing machines at around 1 Hz. The module method [21] was used to determine the fatigue life of the steels, but the 7075-T6 aluminum specimens, due to their low fracture toughness, broke before any significant crack growth. Ramberg–Osgood curves were fitted to the cyclically stabilized hysteresis loops, and the Coffin–Manson parameters were obtained from the strain-life data. Table 2 provides a summary of the experimentally obtained material properties.

To evaluate the existing procedures for estimating fatigue lives, these nine metals were combined with the tensile and  $\epsilon N$  properties of 836 materials obtained from the literature, totaling 724 different steels, 81 aluminum, 15 titanium, nine nickel alloys, and 16 cast irons. These materials were tested under several conditions or heat treatments, at temperatures varying from 21 to 800 °C.

It should be emphasized that this sample included only the metals which reportedly had fully measured Coffin–Manson, cyclic Ramberg–Osgood, and monotonic tensile properties among the more than 13,000 different materials listed on the ViDa software database [15,16]. ViDa is a powerful PC-based academic program developed to automate all traditional local approach methods used in fatigue design, including the SN, the IIW (for welded structures) and the  $\epsilon N$  for crack initiation, and the  $da/dN$  for crack propagation. Its comprehensive materials database has been compiled from several sources in the literature and carefully filtered to avoid suspicious data. In particular, all materials considered in this study can be found in [11,13,14,18,20,23–25], and their experimental Coffin–Manson curves are shown in Figs. 1 and 2.

The 724 steels include (but are not limited to): SAE steels such as 1005, 1006, 1008, 1015, 1018, 1020, 1025, 1030, 1035, 1038, 1040, 1045, 1050, 1080, 1090, 10B21, 10B22, 10B30, 10B62, 1141, 1144, 1522, 1541, 1561, 15B27, 15B35, 4130, 4135, 4140, 4142, 4340, 5160, 52100, 8620H, 8630, 8640, 9262, 950, 950C, 950X,

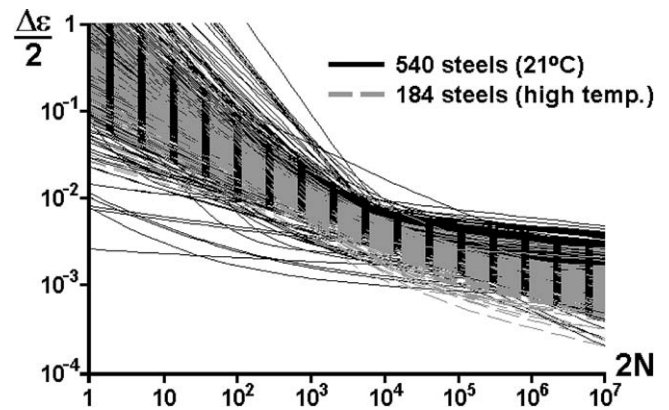


Fig. 1. Coffin–Manson curves of 724 steels under temperatures between 21 and 800 °C.

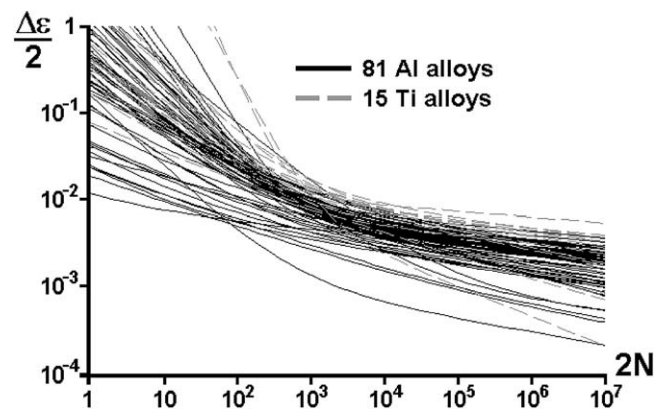


Fig. 2. Coffin–Manson curves of 81 aluminum and 15 titanium alloys.

960X, 980X, Gainex; ASTM steels A36, A136, A302B, A514, A516 Gr.70, A538A, A538B, A538C, A588; stainless steels such as 304, 304L, 310, 316, 321, SUH310-B, SUH616-B, SUH660-B, SUS304-B, SUS316-B, SUS316-HP, SUS321-B, SUS347-B, SUS403-B, AM350; and also several others including 8 Mn 6, 13 Cr Mo 44, 14 Mo V 63, 15 Mo 3, 18 Ni (250) Maraging, 19 Mn 5, 2.25 Cr 1 Mo, 28 Cr Mo NiV 49, 28 Ni Cr Mo 74, 300-M, 34 Cr Ni Mo 6, 40 Cr Mo 4, 41 Cr M4B, 42 Cr Mo 4, 49 Mn VS3, 55 Cr 3, EN 8M, EN 8R, EN 16S, EN 16T,

Table 2  
Mechanical properties of the tested materials

	$E$ (GPa)	$S_U$ (MPa)	$S_Y$ (MPa)	$S'_Y$ (MPa)	$RA$ (%)	$K'$ (MPa)	$n'$	$\sigma'_f$ (MPa)	$\epsilon'_f$	$b$	$c$
API 5D S-135	200	1175	1033	800	60	1910	0.14	1620	0.49	-0.09	-0.73
API 5L Gr.B	208	423	294	277	60	1229	0.24	964	0.36	-0.145	-0.55
API 5L X-60	198	533	457	409	46	813	0.12	647	0.24	-0.049	-0.53
API 5L X-60 weld	198	576	478	475	48	890	0.098	650	0.26	-0.06	-0.77
SAE 1020	205	491	285	270	54	941	0.18	815	0.25	-0.114	-0.53
SAE 4340	200	1250	1060	700	36	1890	0.16	1180	0.092	-0.06	-0.44
SAR 60	205	620	540	500	40	1122	0.13	1010	0.45	-0.08	-0.62
SAR 60 wet weld	174	463	390	400	29	494	0.034	478	0.37	-0.037	-1.06
Al 7075-T6	71.9	576	498	494	11	787	0.07	709	0.12	-0.056	-0.75

EN 24, EN 25, H11, HI-Form 60, HT 60, HT 80, HY 80, HY 130, Incoloy 800H, Man-Ten, RQC 100, RQT 501, RQT 701, St 46, St 49, St 50, SCMV 2, SCMV 3, SCMV 4 and SPV 50.

The 81 aluminum alloys include 1050, 1100, 2014-T6, 2024-T3, 2024-T351, 2024-T4, 2219-T851, 356-T6, 5083-H12, 5083-O, 5086-F, 5182-O, 5183-O, 5454-H38, 5454-O, 5456-H311, 6005, 6061-T4, 6061-T6, 6061-T651, 6082, 6351, 7075-T6, 7075-T61, 7075-T65, 7075-T651, 7075-T73, 7075-T7351, 7175-T73, A356-T6, among others.

Some of the considered titanium alloys are: Ti 0.4Mn, Ti 5Mn, Ti 8Mn, Ti 10Mn, Ti 6Al 4V, and Ti 8Al 1Mo 1V. The cast irons include AISI A48-40B, A48-50B, A48-60B, GG 25, GG 35, GG 40, GGG 40, GGG 60, and GTS 55; and some of the nickel-base alloys are: Hastelloy X, Inconel 713C, Inconel 718, Inconel X, and Waspaloy A.

From the large size and diversity of the steel and aluminum samples, they may be considered representative of the behavior of these alloy families. Among the 724 steels, 540 were tested at room temperature, while the other 184 were tested under temperatures between 400 and 800 °C. As suggested in Fig. 1, temperature does not influence decisively on the scatter of the Coffin–Manson curves of the analyzed steels; therefore, the low- and high-temperature data are evaluated together. However, the high-cycle fatigue resistance is significantly lowered under high temperatures (Fig. 1). Part of this temperature effect can be accounted for by all discussed estimation methods, because the lower values of the ultimate strength  $S_U$  or the Brinnell hardness  $HB$  found at high temperatures always result in lower

estimates of the fatigue resistance coefficient,  $\sigma'_f$ . In the next section, the Coffin–Manson and Ramberg–Osgood parameter estimates are statistically evaluated.

#### 4. Statistical evaluation of the $\epsilon N$ parameter estimates

In this section, the Coffin–Manson and Ramberg–Osgood parameters and their estimates are individually studied based on the data of the 845 metals described above. For the statistical study, each data set is sorted in ascending order, and then each data point is associated to its mean rank. Then, each data set is fitted using 12 continuous probability distributions: beta, Birnbaum-Saunders, gamma, inverse Gauss, logistic, log-logistic, normal, log-normal, Pearson, Gumbel (extreme value), and Weibull [26,27]. The chi-square and Anderson–Darling tests [28,29] are used to evaluate the goodness-of-fit of each of the considered distributions for each set. In particular, both tests show that the log-logistic distribution [27] is the one that best fits the Coffin–Manson parameters  $b$ ,  $c$ , and  $\epsilon'_f$ , the cyclic hardening exponent  $n'$ , and the ratios  $\sigma'_f/S_U$  and  $n'/(b/c)$  of the considered steels and aluminum alloys. This does not necessarily mean that these variables follow the log-logistic distribution, it is only an indication that among the 12 considered distributions, this is the one that most likely produced the specific data sets used in this analysis. The best-fitted distributions and their mean, median, and coefficient of variation  $V$  (defined as the ratio between the standard deviation and the mean) are shown in Fig. 3.

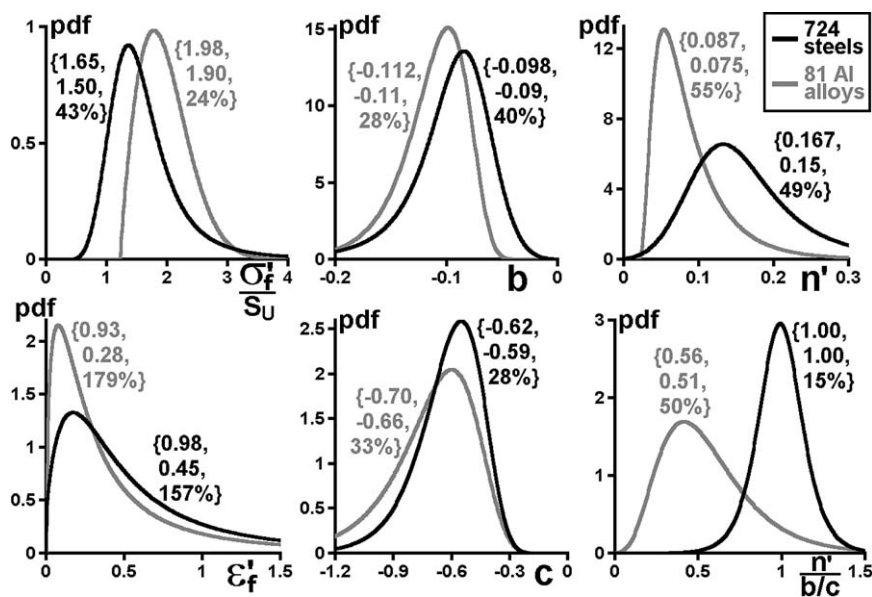


Fig. 3. Probability density functions and {mean, median, coefficient of variation} of Coffin–Manson and Ramberg–Osgood parameters of 724 steels and 81 aluminum alloys.

All 845 metals have  $\sigma'_f/S_U$  ratios between 0.5 and 10, with average 1.65 and median 1.5 for steels, suggesting that Manson's estimate  $\sigma'_f = 1.9 \cdot S_U$  is potentially non-conservative for these materials. The fatigue ductility coefficient  $\epsilon'_f$  has the greatest scatter of all studied properties (coefficient of variation  $V$  up to 179%), with values ranging from 0.001 to 400. It must be noted that  $\epsilon'_f$  values much greater than 2.3 are very likely a result of bad fitting of the Coffin–Manson curve, because such values would imply in a reduction in area  $RA$  much greater than 90% at  $2N=1$ . Also, all considered metals have cyclic hardening coefficients  $K'$  ranging between  $E/1000$  and  $E/20$ , cyclic hardening exponents  $n'$  between 0.01 and 0.6, fatigue strength exponents  $b$  between  $-0.35$  and  $-0.01$ , and fatigue ductility exponents  $c$  between  $-1.5$  and  $-0.1$ . More specifically, 93% of the steels have  $0.06 < n' < 0.35$ , 92% have  $-0.2 < b < -0.05$ , and 94% are in the range  $-0.9 < c < -0.3$ . In addition, 94% of the aluminum alloys have  $0.03 < n' < 0.2$ , 91% have  $-0.2 < b < -0.08$ , and 88% present  $-1.0 < c < -0.4$ .

To verify the coherence between Coffin–Manson's and Ramberg–Osgood's elastic and plastic strain ranges, the correlations presented in Eq. (3) are evaluated for the considered steels and aluminum alloys, see Fig. 4. From this study on 724 steels, it is found that there is a reasonable (but not exact) correlation between the cyclic hardening exponent  $n'$  and the ratio  $b/c$ , with a coefficient of variation  $V=15\%$ . The cyclic hardening coefficient  $K'$  estimate based on  $n'$  and on Coffin–Manson's coefficients is also fairly good for steels, despite the somewhat significant scatter in the experimental data,  $V=15\%$  as well. However, for the considered 81 aluminum alloys, it is found that Eq. (3) tends to overestimate both  $n'$  and  $K'$ , see Figs. 3 and 4.

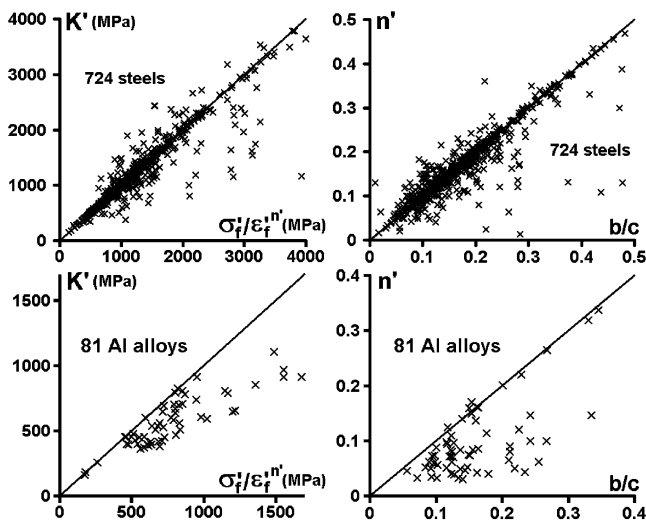


Fig. 4. Coherence between Coffin–Manson and Ramberg–Osgood parameters for steels and aluminum alloys.

This is an indication that the coherence between the stress–strain and strain–life relationships used in the traditional  $\epsilon N$  method is better verified in steels than in aluminum alloys.

The several Coffin–Manson parameter estimates are now evaluated through Figs. 5–12. As seen in Fig. 5, Manson's estimate for the fatigue strength coefficient  $\sigma'_f$  is non-conservative for most steels, while Mitchell's method results in better values. However, due to the 345 MPa offset in Mitchell's estimate,  $\sigma'_f$  is overestimated in materials with low ultimate strength  $S_U$ , such as steels under high temperatures (Fig. 5). Muralidharan–Manson's method provides a much better  $\sigma'_f$  estimate for steels; however it is overly conservative for aluminum and titanium alloys. Also, it is found that Muralidharan–Manson's  $\sigma'_f$  estimate for steels can be successfully approximated by  $1.5 \cdot S_U$ , a much simpler and equally effective expression. Interestingly, the factor 1.5 is also the median value of the  $\sigma'_f/S_U$  ratio for the 724 steels.

The correlations between the fatigue strength exponent  $b$  and  $RA$  or  $S_U$  are poor for all studied metals: Manson's four-point method underestimates  $b$  for most materials, while Mitchell's correlation has a large scatter (Fig. 6). Even though  $b$  and  $c$  correlate fairly well with the hardening exponent  $n'$ , estimating these exponents as constants results in a smaller coefficient of variation if compared to the available estimates. In addition, Morrow's  $b$  estimate is non-conservative for almost all studied aluminum and titanium alloys. A sensitivity analysis (discussed in the next section) shows that better predictions are obtained from  $b$  and  $c$  estimates based on their median values for each alloy family:  $b=-0.09$  and  $c=-0.59$  for the 724 steels, and  $b=-0.11$  and  $c=-0.66$  for the 81 aluminum alloys (Fig. 7).

As seen in Fig. 8, the fatigue ductility coefficient  $\epsilon'_f$  does not correlate well with the reduction in area  $RA$  or the true fracture ductility  $\epsilon_f$ . Mitchell and Manson's  $\epsilon'_f$  estimates are non-conservative. Also, there is a large scatter in Muralidharan–Manson and Bäumel–Seeger's  $\epsilon'_f$  estimates to justify a suitable correlation with  $S_U/E$ . One limitation of Bäumel–Seeger's method is that it is only valid if the ultimate strength  $S_U$  is much smaller than 2.2 GPa, otherwise negative values of  $\epsilon'_f$  may be obtained. Raske–Morrow's  $\epsilon'_f$  estimate also has a large scatter, because it implicitly assumes a perfect correlation between the elastic and plastic strain ranges in Ramberg–Osgood and Coffin–Manson.

Manson's method based on fixed points also results in poor estimates for the studied materials. The elastic and plastic strain ranges in Manson's four-point correlation are overestimated at  $N=1/4$ , 10 and  $10^4$  cycles for steels (Fig. 9). The only fixed point with a fair correlation is  $N=10^5$  cycles, where the elastic strain

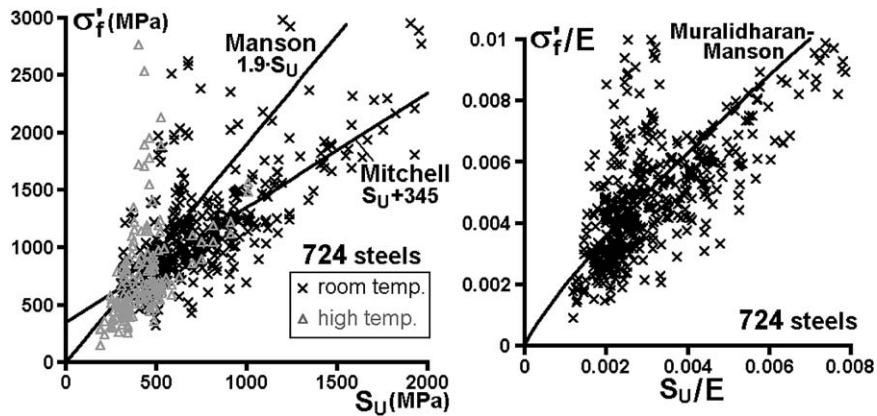


Fig. 5. Estimates of Coffin–Manson’s fatigue strength coefficient,  $\sigma'_f$ .

amplitude is slightly underestimated by  $0.45 \cdot S_U/E$ . The Coffin–Manson coefficients  $\sigma'_f$  and  $\epsilon'_f$  are overestimated from the four-point correlation method, the exponent  $b$  is underestimated, and for 93% of the steels,  $c$  results in the narrow range  $-0.7 < c < -0.5$  (Fig. 10). Ong’s proposed modification to the four-point correlation method results in better average estimates for  $\sigma'_f$ ,  $b$  and  $c$ ; however, as in Mitchell’s method, it overestimates  $\epsilon'_f$  (Fig. 11).

Roessle–Fatemi’s method results in a fair correlation between  $\sigma'_f$  and the Brinnell hardness,  $HB$ . From the good correlation  $S_U = 3.4 \cdot HB$  for steels, this  $\sigma'_f$  estimate can be rewritten as  $1.25 \cdot S_U + 225$  MPa, an intermediate function in between Manson’s and Mitchell’s. However, Roessle–Fatemi’s estimate for  $\epsilon'_f$  does not correlate well with the analyzed data. Therefore, from a statistical viewpoint, sophisticated equations surprisingly tend to increase the dispersion in  $\epsilon'_f$ . The least scatter in those cases was obtained assuming a constant value such as its median 0.45 for steels or 0.28 for aluminum alloys (Fig. 3).

Based on the above conclusions, an  $\epsilon N$  estimate called the medians method is proposed, which estimates  $\sigma'_f/S_U$ ,  $\epsilon'_f$ ,  $b$  and  $c$  as constants equal to their medians for each alloy family:

$$\frac{\Delta \epsilon}{2} = 1.5 \frac{S_U}{E} (2N)^{-0.09} + 0.45 \cdot (2N)^{-0.59} \quad \text{(from 724 steels)} \quad (4)$$

$$\frac{\Delta \epsilon}{2} = 1.9 \frac{S_U}{E} (2N)^{-0.11} + 0.28 \cdot (2N)^{-0.66} \quad \text{(from 81 aluminum alloys)} \quad (5)$$

One of the reasons why in general mean values do not produce good parameter estimates is that the mean is very much affected by the extreme values at the tails of the probability distributions (which represent only a small percentage of the considered sample). On the other hand, the median is a much more robust parameter, especially in the case of asymmetric distributions.

Another interesting property is that the median estimate for steels is almost insensitive to the operating temperature. The only parameter with a significant temperature dependence is the fatigue ductility coefficient  $\epsilon'_f$ : the median value for 540 steels at room temperature is  $\epsilon'_f = 0.51$ , while 184 steels at temperatures between 400 and 800 °C have  $\epsilon'_f = 0.35$ . Using these values, separate median estimates can then be proposed for high- and low-temperature steels. The fatigue strength coefficient  $\sigma'_f$  has also a significant temperature dependence; however, the median of the  $\sigma'_f/S_U$  ratio remains unchanged.

As shown in Table 3, other median estimates for  $\{\sigma'_f, \epsilon'_f, b, c\}$  are obtained for three alloy families:  $\{1.9 \cdot S_U, 0.50, -0.10, -0.69\}$  from a study on 15 titanium alloys;  $\{1.2 \cdot S_U, 0.04, -0.08, -0.52\}$  calculated from 16 cast irons; and  $\{1.4 \cdot S_U, 0.15, -0.08, -0.59\}$  from nine nickel alloys. However, these three estimates should be used with caution, because they were based on a very limited sample.

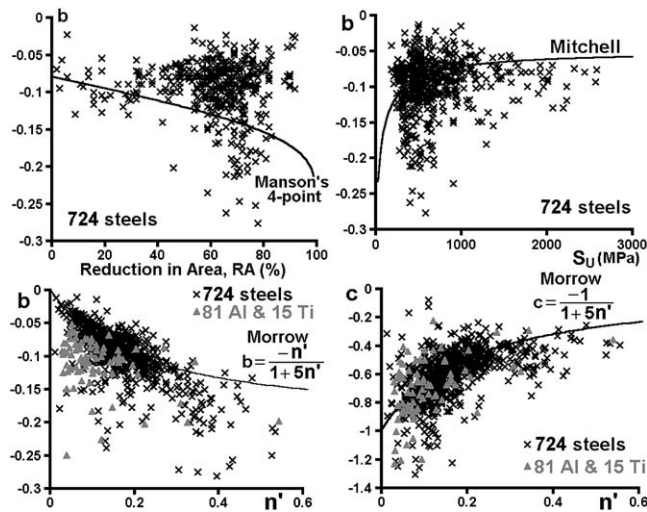


Fig. 6. Estimates of Coffin–Manson’s exponents,  $b$  and  $c$ .

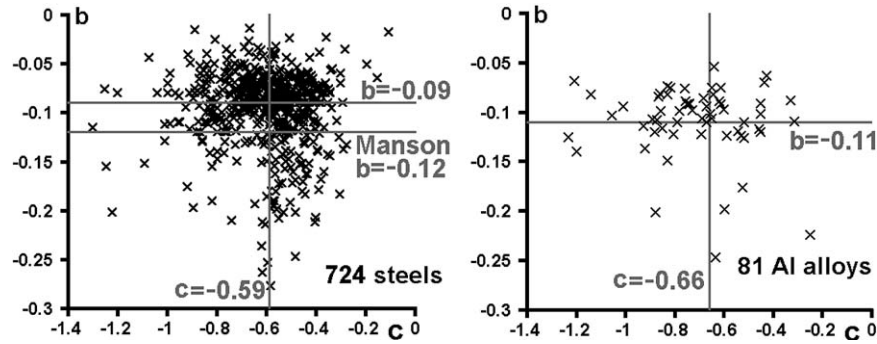


Fig. 7. Coffin–Manson’s exponents  $b$  and  $c$  for 724 steels and 81 aluminum alloys.

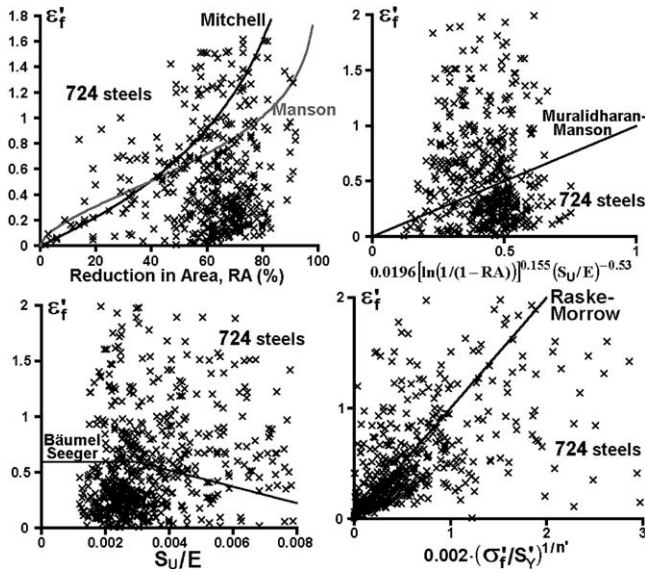


Fig. 8. Estimates of Coffin–Manson’s fatigue ductility coefficient,  $\epsilon_f'$ .

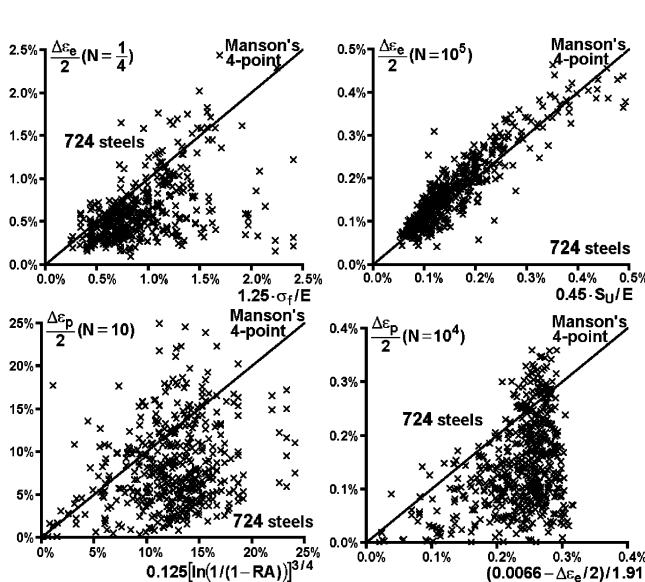


Fig. 9. Evaluation of the strain range estimates used by Manson’s four-point correlation method.

Other useful estimates based on median values are  $E = 205$  GPa (median value of 3157 steels at room temperature from the ViDa database [15,16], with a coefficient of variation  $V = 3.1\%$ ),  $E = 71$  GPa (from 551 Al alloys,  $V = 4.0\%$ ),  $E = 108$  GPa (139 Ti alloys,  $V = 7.4\%$ ),  $E = 140$  GPa (22 cast irons,  $V = 24\%$ ), and  $E = 211$  GPa (376 Ni alloys,  $V = 3.4\%$ ).

The cyclic strain hardening exponent can also be estimated in the same way:  $n' = 0.15$  (823 steels,  $V = 49\%$ ),  $n' = 0.09$  (237 Al alloys,  $V = 41\%$ ),  $n' = 0.10$  (43 Ti alloys,  $V = 64\%$ ),  $n' = 0.145$  (16 cast irons,  $V = 37\%$ ), and  $n' = 0.14$  (eight nickel alloys,  $V = 26\%$ ). However, the very high scatter in these  $n'$  estimates should be noted.

### 5. Statistical evaluation of the $\epsilon N$ fatigue life estimates

In the previous section, all fatigue estimates were evaluated by treating the  $\epsilon N$  parameters as independent

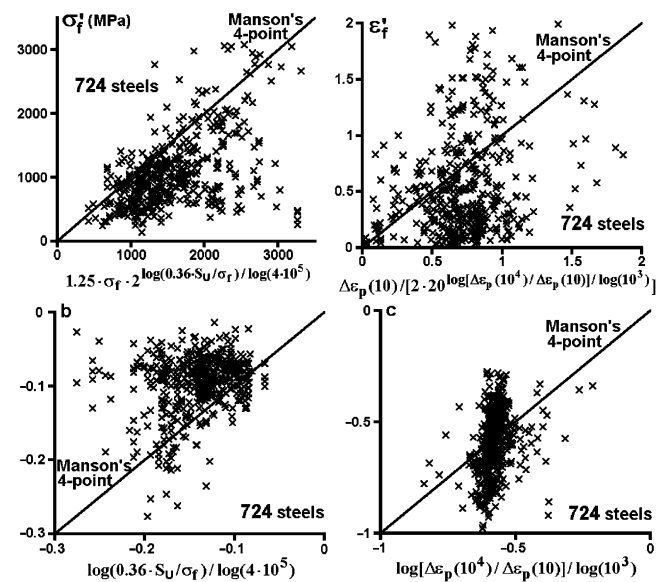


Fig. 10. Resulting Coffin–Manson parameters from Manson’s four-point correlation method.



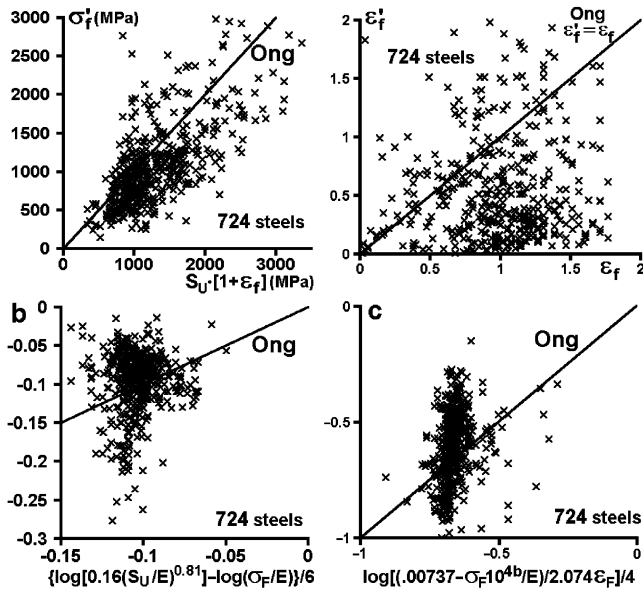


Fig. 11. Resulting Coffin–Manson parameters from Ong’s modified four-point correlation method.

random variables. However, for fatigue life estimation purposes, Coffin–Manson’s coefficients and exponents are not independent. For instance, it is possible to obtain fair life predictions using a method that overestimates the fatigue strength coefficient  $\sigma'_f$  while underestimating the corresponding exponent  $b$ , since both errors may cancel each other. Therefore, to validate  $\epsilon N$  estimates, a statistical study must be performed comparing the predicted lives (and not only the individual Coffin–Manson parameters) with the experimentally measured ones.

The best-fitted probability density functions (pdf) of the  $\epsilon N$  specimen lives under several strain amplitudes  $\Delta\epsilon/2$  are shown in Figs. 13 and 14, calculated from measured Coffin–Manson data on 724 steels and 81 aluminum alloys. The scatter in the  $\epsilon N$  specimen lives for the different materials is minimum between 1000 and 3000 cycles, which is perhaps a good reason to continue esti-

imating Wöhler’s curve using  $N = 10^3$  cycles as a fixed point in the SN methodology. Also, the average strain amplitude at  $10^3$  cycles in both steels and aluminum alloys is approximately  $\Delta\epsilon(10^3)/2 = 0.8\%$ . Even though the scatter is minimum around 0.8%,  $\epsilon N$  specimen lives varying from less than 50 cycles (for a few wet welds) up to  $2 \times 10^4$  cycles (for a hot-worked H11 tool steel) can be obtained at this strain amplitude. The high scatter observed at lives greater than  $10^5$  cycles is expected, due to the large variation in the fatigue resistance of several steels and aluminum alloys.

One limitation of the presented analysis is that the data used in the evaluation are not direct experimental data (which are very difficult to obtain in the literature for such a large sample of test materials), but calculated values from the experimentally obtained fatigue properties. However, it can be assumed that the Coffin–Manson parameters generate reasonable data points at least in the range where most experiments were performed, typically  $0.3\% < \Delta\epsilon/2 < 2\%$  for metals. Outside this range, the calculated values from the Coffin–Manson parameters might include significant extrapolation errors, degrading the accuracy of this analysis. But in any case, it would not be simple to obtain reliable experimental data outside this range. First, due to the high cost of the  $\epsilon N$  test machines, very few specimens are tested under very low strain amplitudes (e.g. a servo-hydraulic testing machine at 40 Hz would take over 144 days to reach  $5 \times 10^8$  cycles). Also, most commercial clip-gages do not have adequate resolution to control tests with strain amplitudes smaller than 0.1%, generating significant measurement errors. And second, very high strain amplitude tests are difficult to perform in practice, since  $\epsilon N$  specimens may buckle under such conditions. Therefore, the presented analysis cannot be extended to very short or very long life predictions, but it can be successfully applied to a significant range of strain amplitudes.

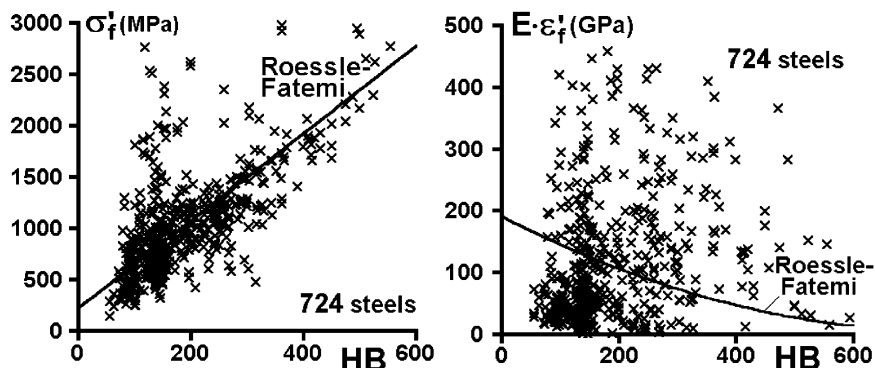


Fig. 12. Evaluation of Roessle–Fatemi’s estimates for Coffin–Manson coefficients  $\sigma'_f$  and  $\epsilon'_f$  based on the Brinnell hardness, HB.

Table 3  
Median and coefficient of variation of the stress amplitudes at  $10^3$  cycles and Coffin–Manson parameters for the studied materials

Alloy family	$\sigma_a$ ( $10^3$ cycles)		$\sigma'_f$		$\epsilon'_f$		$b$		$c$		$E$ (GPa)	
	Median	$V$ (%)	Median	$V$ (%)	Median	$V$ (%)	Median	$V$ (%)	Median	$V$ (%)	Median	$V$ (%)
724 steels	$0.76 \cdot S_U$	18	$1.5 \cdot S_U$	43	0.45	157	-0.09	40	-0.59	28	205	3.1
81 Al alloys	$0.82 \cdot S_U$	10	$1.9 \cdot S_U$	24	0.28	179	-0.11	28	-0.66	33	71	4.0
15 Ti alloys	$0.89 \cdot S_U$	9	$1.9 \cdot S_U$	36	0.5	123	-0.10	37	-0.69	24	108	7.4
9 Ni alloys	$0.76 \cdot S_U$	31	$1.4 \cdot S_U$	30	0.15	171	-0.08	28	-0.59	22	211	3.4
16 cast irons	$0.65 \cdot S_U$	28	$1.2 \cdot S_U$	28	0.04	127	-0.08	29	-0.52	30	140	24

The performance of each fatigue estimate is now evaluated through the life prediction ratio (LPR), defined as the ratio between the life (in cycles) predicted by any of the presented methods,  $N_{\text{predicted}}$ , and the observed experimental life,  $N_{\text{observed}}$ . Therefore, LPR values between 0 and 1.0 are a result of conservative estimates, while values greater than 1.0 are non-conservative. It must be noted that all mean values and standard deviations of the LPR will be calculated based on the logarithmic representation of  $N_{\text{predicted}}/N_{\text{observed}}$ , in order to give equal weight to, e.g. ratios 3 and 1/3, since both imply on a factor of 3 in the life estimation error.

The pdf that best-fitted the  $\epsilon N$  specimen LPR of the 724 steels are shown in Fig. 15, obtained under the strain amplitude  $\Delta\epsilon/2 = 1.0\%$ . Under such strain amplitude, Manson’s universal slope method results in average non-conservative prediction errors of 97% (since its mean LPR is 1.97), Bäuml–Seeger’s in 38%, and the medians method in 3%, with similar standard deviations. Except for Mitchell’s method, which presents a high scatter in the LPR, it is found that all estimates shown in Table 1 result in roughly the same standard deviations when represented in the logarithmic scale at each strain range level. However, these standard deviations do vary with the strain amplitude

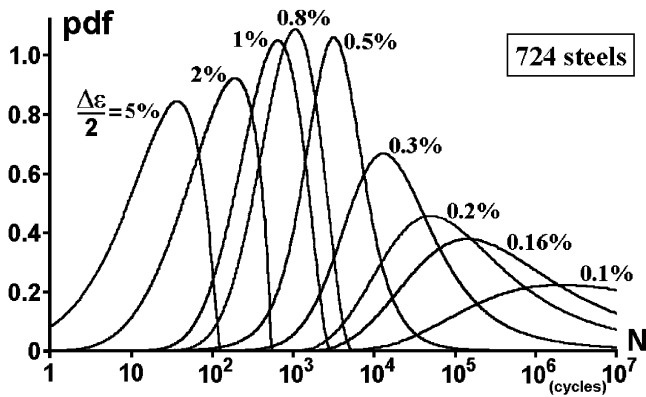


Fig. 13. Probability density functions of the  $\epsilon N$  test specimen lives under various strain amplitudes  $\Delta\epsilon/2$ , calculated from experimental Coffin–Manson curves of 724 steels.

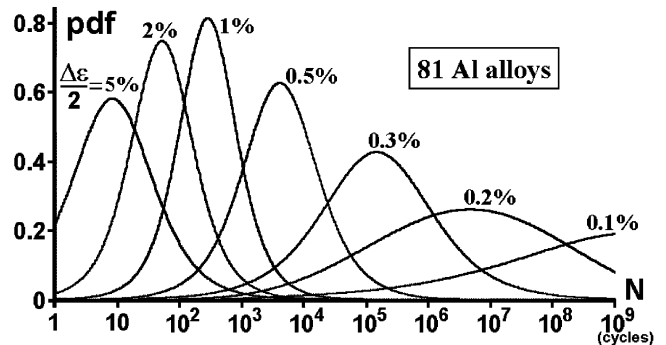


Fig. 14. Probability density functions of the  $\epsilon N$  test specimen lives under various strain amplitudes  $\Delta\epsilon/2$ , calculated from experimental Coffin–Manson curves of 81 aluminum alloys.

level, presenting a minimum near  $\Delta\epsilon/2 = 1.0\%$ . The poor performance of Mitchell’s method in this study is mainly a result of its non-conservative  $\epsilon'_f$  estimate, since the great majority of steels and aluminum alloys have  $\epsilon'_f$  much smaller than the true fracture ductility,  $\epsilon_f$ .

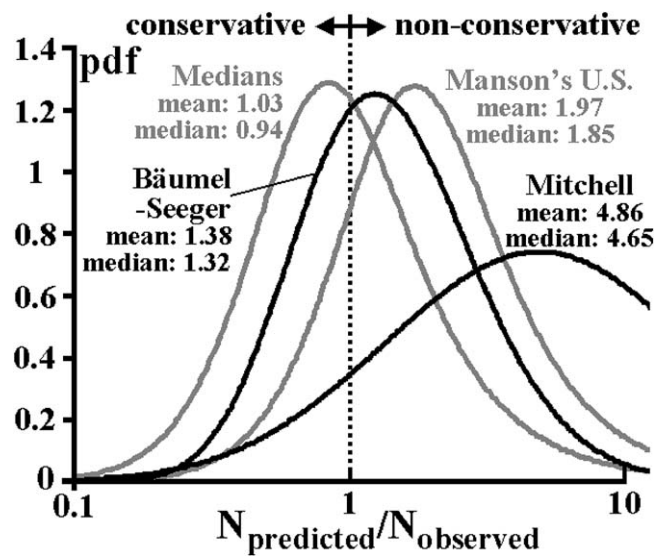


Fig. 15. Statistics of the life prediction ratio obtained by a few estimation methods for 724 steels, obtained under the strain amplitude  $\Delta\epsilon/2 = 1.0\%$ .

As follows, each estimation method is further evaluated through the average values of the LPR probability density functions obtained under several strain amplitudes  $\Delta\epsilon/2$ , see Fig. 16. Mitchell’s method is not represented in this figure, because its average LPR is greater than 4.0 in the entire life range.

Manson’s universal slope and four-point correlation methods are non-conservative for short lives, with average life prediction errors of over 100%. Also, these two methods are highly conservative for long lives, underestimating the elastic strain amplitude  $\Delta\epsilon_e/2$  at  $10^5$  cycles using  $0.44 \cdot S_U/E$  or  $0.45 \cdot S_U/E$ . A better correlation for the 724 steels is obtained from the median estimate:

$$\frac{\Delta\epsilon_e}{2}(10^5 \text{ cycles}) = 1.5 \frac{S_U \cdot (2 \times 10^5)^{-0.09}}{E} = 0.5 \frac{S_U}{E} \quad (6)$$

Muralidharan–Manson and Roessle–Fatemi’s methods result in reasonable average LPR for steels, even though significantly non-conservative predictions may be obtained at strain amplitudes  $\Delta\epsilon/2$  below 1.0% (Fig. 16). Bäuml–Seeger and Ong’s methods also result in fair predictions; however, they are slightly non-conservative at high  $\Delta\epsilon/2$  levels because of the poor estimates for  $\epsilon'_f$ , which do not correlate with  $S_U/E$  or  $\epsilon_f$  for the 724 steels. The lowest average prediction errors are obtained from the median estimate for steels, with LPR very close to 1.0 in all strain amplitudes between 0.4% and 5%, and conservative errors below this interval. However, this could be expected as the parameters from the median method were calibrated using the same material data set used in the comparisons.

For aluminum and titanium alloys, it is found that better predictions are obtained from the medians method, followed by Bäuml–Seeger’s uniform material law, very likely because both are based on constant

estimates for  $\sigma'_f/S_U$ ,  $\epsilon'_f$ ,  $b$  and  $c$ . Also, Bäuml–Seeger’s estimate  $c = -0.69$  may be appropriate for titanium but a little low for aluminum alloys. Therefore, it is always a good idea to consider separate estimates for each alloy family, separating the aluminum from the titanium alloys such as in the medians method.

Finally, a sensitivity analysis is performed varying each of the individual Coffin–Manson parameters. More specifically, all combinations of ratios  $\sigma'_f/S_U$  between 0 and 4,  $\epsilon'_f$  values between 0 and 2,  $-0.4 < b < 0$ , and  $-1.5 < c < 0$  are evaluated in steps of 0.01 (0.001 for the  $b$  exponent) for each alloy family. For each parameter combination, the average life prediction ratio and its standard deviation (in the logarithmic scale) are evaluated for the studied metals at a few selected strain amplitude levels. For the studied steels and aluminum alloys, it is found that the individual medians of the Coffin–Manson parameters are the optimal values that result in the best average predictions with the smallest standard deviations. In addition, it is found that the introduction of an offset into the  $\sigma'_f$  estimate (such as 345 MPa in Mitchell or 225 MPa in Roessle–Fatemi’s methods) does not improve the life predictions if compared to the ones obtained from constant  $\sigma'_f/S_U$  ratios. In the next section, the studied metals and the medians method are used to evaluate traditional estimates in the SN methodology.

### 6. Statistical evaluation of the SN fatigue life estimates

One of the most popular estimates used in the SN stress–life methodology is based on the stress amplitudes associated with fatigue lives of  $10^3$  and  $10^6$  cycles,  $\sigma_a(10^3) = 0.9 \cdot S_U$  and  $\sigma_a(10^6) = 0.5 \cdot k_a \cdot k_b \cdot \dots \cdot k_e \cdot S_U$  (for steels with  $S_U \leq 1400$  MPa), where the constants  $k_a, k_b, k_c, k_d$  and  $k_e$  are the endurance limit modifying factors [30,31]. These SN estimates can be verified from Coffin–Manson’s strain amplitudes  $\Delta\epsilon/2$  calculated at  $N = 10^3$  or  $10^6$  cycles. The cyclic Ramberg–Osgood curve of each material is then applied to the  $\Delta\epsilon/2$  values to obtain the associated elastic–plastic stress amplitude,  $\sigma_a$ .

Surface finish does not significantly affect low-cycle fatigue lives. In addition, the ASTM standard does not require the  $\epsilon N$  specimens to be polished—it only requires, as a final operation, a machining or other “finishing” procedure that would introduce minimal surface metal distortion consistency in the surface finish [21]. Thus, it seems safe to assume that the  $\epsilon N$  specimens might have been machined, as opposed to the SN ones, which are always polished, because these are primarily used in high cycle fatigue studies, where surface finish plays a major role. Therefore, a surface

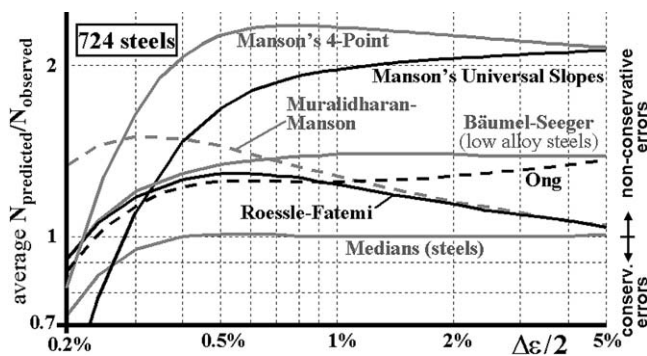


Fig. 16. Average life prediction ratios obtained by several estimation methods for 724 steels, under strain amplitude levels between 0.2% and 5%.

finish factor  $k_a$  was considered in this analysis for the  $\varepsilon N$  specimens, estimated for steels by  $4.51 \cdot (S_U)^{-0.265}$ . Therefore, the 0.5 factor at  $10^6$  cycles can be verified through the ratio  $\sigma_a(10^6)/(k_a \cdot S_U)$ , while the 0.9 factor at  $10^3$  cycles is checked using  $\sigma_a(10^3)/S_U$ .

Both mean and median values of the ratio  $\sigma_a(10^6)/(k_a \cdot S_U)$  are 0.48 for the 654 steels in this study with  $S_U \leq 1400$  MPa, a value very close to the traditional 0.5 factor. The assumption of a machined surface finish in the  $\varepsilon N$  specimens indeed resulted, for the studied materials, in a better agreement with the traditional  $0.5 \cdot S_U$  estimate. However, as would be expected for high cycle estimates, there is significant scatter at  $10^6$  cycles, with a coefficient of variation  $V = 27\%$ . If all 724 steels are considered, then both mean and median values are slightly increased to 0.49, an indication that the 700 MPa estimate for steels with  $S_U > 1400$  MPa is quite conservative, see Fig. 17.

On the other hand, the  $0.9 \cdot S_U$  estimate at  $10^3$  cycles would be highly non-conservative if applied to  $\varepsilon N$  specimens, as observed in Fig. 17 from the mean 0.76 and median 0.75 of the ratio  $\sigma_a(10^3)/S_U$  (with  $V = 18\%$ ). This difference is because  $0.9 \cdot S_U$  is a *purely elastic* stress associated to the bending moment that would result in a fatigue life of  $10^3$  cycles for rotating bending SN specimens. However, in 85% of 7492 metals sampled from the ViDa database [15,16], the yielding strength is below  $0.9 \cdot S_U$ ; therefore, such stress level cannot be considered as purely elastic. The maximum elastic–plastic stress  $\sigma_{max}$  that is actually applied to the SN specimen is then estimated by equating the externally applied moment  $\pi d^3(0.9 \cdot S_U)/32$  with the resisting moment of the specimen cross-section (where  $d$  is

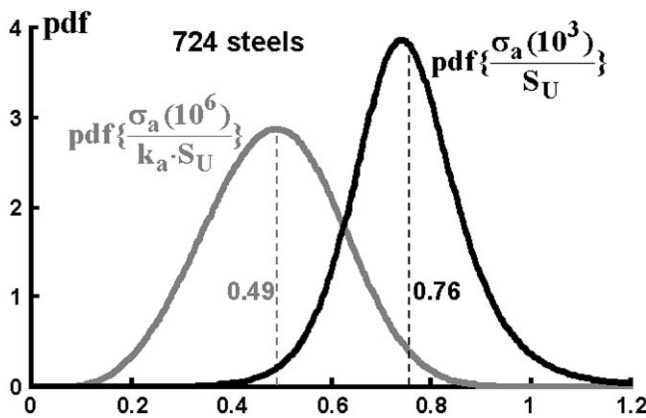


Fig. 17. Probability density functions of the SN estimate coefficients at  $10^3$  and  $10^6$  cycles.

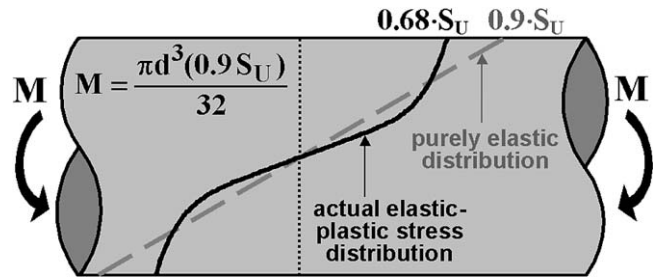


Fig. 18. Schematic of the actual elastic–plastic stress distribution along the cross-section of an SN rotating bending specimen.

its diameter):

$$\frac{0.9 S_U \pi}{16} = \int_0^{\sigma_{max}} \frac{\left[ \frac{\sigma}{E} + \frac{1}{n'} \left( \frac{\sigma}{K'} \right)^{1/n'} \right] \frac{\sigma}{E} + \left( \frac{\sigma}{K'} \right)^{1/n'}}{\left[ \frac{\sigma_{max}}{E} + \left( \frac{\sigma_{max}}{K'} \right)^{1/n'} \right]^2} \cdot \sqrt{1 - \left[ \frac{\frac{\sigma}{E} + \left( \frac{\sigma}{K'} \right)^{1/n'}}{\frac{\sigma_{max}}{E} + \left( \frac{\sigma_{max}}{K'} \right)^{1/n'}} \right]^2} d\sigma \quad (7)$$

Eq. (7) is then solved numerically using the Ramberg–Osgood properties of each of the 724 steels, resulting in an *average*  $\sigma_{max}$  value of  $0.68 \cdot S_U$ , see Fig. 18. Such elastic–plastic analysis confirms the inadequacy of the purely elastic approach; however, it is a little conservative if compared to the (better)  $0.76 \cdot S_U$  estimate obtained from Coffin–Manson data, due to the following two reasons. First, the purely elastic  $0.9 \cdot S_U$  estimate was originally conservative [30]; therefore, a higher externally applied moment of  $\pi d^3 S_U/32$  would be more appropriate at  $10^3$  cycles. And second, plasticity effects cause a slight misalignment between the stress and the strain neutral axes in the rotating bending specimen, as described in [7], which is not modeled in Eq. (7).

Finally, estimates of the  $\sigma_a(10^3)/S_U$  ratio can also be obtained for other alloy families using the medians method. For idealized materials in which Eq. (3) is valid (i.e. there is a perfect correlation between Coffin–Manson and Ramberg–Osgood’s elastic and plastic strain ranges), the elastic–plastic stress amplitude  $\sigma_a$  at  $N = 10^3$  cycles can be calculated multiplying the Young modulus  $E$  by the elastic strain amplitude  $\Delta \varepsilon_e/2$ , resulting in:

$$\sigma_a(N = 10^3) = \frac{\Delta \sigma}{2} \cong E \frac{\Delta \varepsilon_e}{2} = \sigma'_f (2 \times 10^3)^b \quad (8)$$

From the median estimate for steels, Eq. (8) predicts that  $\sigma_a(10^3) = 0.757 \cdot S_U \cong 0.76 \cdot S_U$ , as expected, which agrees with Juvinall’s estimate for uniaxial tension–compression tests [30]. As shown in Table 3, the medians method also predicts  $\sigma_a(10^3) \cong 0.82 \cdot S_U$  for

aluminum alloys,  $0.89 \cdot S_U$  for titanium alloys,  $0.65 \cdot S_U$  for cast irons, and  $0.76 \cdot S_U$  for nickel alloys, allowing for improved estimates in the SN methodology.

## 7. Conclusions

A statistical evaluation of the existing Coffin–Manson parameter estimates was presented in this work, based on monotonic tensile and uniaxial fatigue properties of 845 different metals from ViDa's database. Based on this analysis, the following conclusions can be drawn:

- On average, steels present significantly higher  $b$  and  $c$  exponents than aluminum and titanium alloys. Therefore, different estimates for the Coffin–Manson parameters should be considered for each alloy family.
- Correlations between Coffin–Manson's exponents and the monotonic tensile test properties are poor. Even though the cyclic hardening exponent  $n'$  is well estimated by the ratio  $b/c$  for steels, estimates for  $b$  and  $c$  based on  $n'$  are detrimental to all studied methods.
- The fatigue strength coefficient  $\sigma'_f$  presents a fair correlation with the ultimate strength  $S_U$  (and consequently with the Brinnell hardness,  $HB$ ). The relatively large scatter in this correlation does not justify the use of non-linear estimates (such as Muralidharan–Manson's) or linear estimates with offsets (such as Mitchell or Roessle–Fatemi's, which overestimate  $\sigma'_f$  for low values of  $S_U$  or  $HB$  due to such offsets). Constant estimates for the ratio  $\sigma'_f/S_U$  were found to better agree with the studied data. The correlation between  $\sigma'_f$  and  $\sigma_f$  is not as good and should not be used.
- The fatigue ductility coefficient  $\epsilon'_f$  does not correlate well with monotonic tensile test properties. Most  $\epsilon'_f$  correlations proposed in the literature are based on a limited number of materials and cannot be statistically justified. In particular,  $\epsilon'_f$  should not be estimated from the true fracture ductility,  $\epsilon_f$ .
- Keep it simple: better life predictions are obtained simply from constant estimates of the parameters  $b$ ,  $c$ ,  $\sigma'_f/S_U$  and  $\epsilon'_f$ , such as in the proposed medians method, which combines good average life predictions with one of the lowest standard deviations.
- Other estimates that resulted in good predictions are Roessle–Fatemi, Bäuml–Seeger, and Muralidharan–Manson's methods for steels. However, the estimates of the fatigue ductility coefficient  $\epsilon'_f$  in these three methods are not very good. The main reason for the good performance of these methods is the combination of constant values for the  $b$  and  $c$  exponents and reasonable estimates for the fatigue strength coefficient. In fact, it is found that replacing the  $\epsilon'_f$  estimates by a constant value in these three methods

slightly improves the life predictions and reduces the associated scatter. Ong's method also results in reasonable predictions, despite its poor  $\sigma'_f$  and  $\epsilon'_f$  estimates. It must also be noted that Muralidharan–Manson's method should not be applied to aluminum or titanium alloys, which present significantly lower  $b$  and  $c$  exponents.

- Manson's universal slope and four-point correlation methods are very conservative for steels at long lives, as pointed out by Park and Song. Also, both methods result on average in significantly non-conservative life predictions at short lives.
- The classical SN estimates for steels at  $10^3$  and  $10^6$  cycles were evaluated, resulting in average stress amplitudes of  $0.76 \cdot S_U$  and  $0.49 \cdot S_U$ , respectively. Other estimates at  $10^3$  cycles were also proposed for cast irons, aluminum, titanium, and nickel alloys, based on the respective medians method parameters. For future work, improved median estimates could be obtained for both uniaxial and torsional fatigue properties using larger samples of material data.
- Finally, it must be pointed out that all the presented estimates should never be used in design, because for some materials, even the best methods may result in life prediction errors of an order of magnitude. The use of such estimates, even the proposed medians method, is only admissible during the first stages of design; otherwise all fatigue properties should be experimentally obtained.

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