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Short crack threshold estimates to predict notch sensitivity factors in fatigue

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Abstract

The notch sensitivity factor q can be associated with the presence of non-propagating fatigue cracks at the notch root. Such cracks are present when the nominal stress range $\Delta\sigma_n$ is between $\Delta\sigma_0/K_t$ and $\Delta\sigma_0/K_f$, where $\Delta\sigma_0$ is the fatigue limit, K_t is the geometric and K_f is the fatigue stress concentration factors of the notch. Therefore, in principle it is possible to obtain expressions for q if the propagation behavior of small cracks emanating from notches is known. Several expressions have been proposed to model the dependency between the threshold value ΔK_{th} of the stress intensity range and the crack size a for very small cracks. Most of these expressions are based on length parameters, estimated from ΔK_{th} and $\Delta\sigma_0$, resulting in a modified stress intensity range able to reproduce most of the behavior shown in the Kitagawa–Takahashi plot. Peterson or Topper-like expressions are then calibrated to q based on these crack propagation estimates. However, such q calibration is found to be extremely sensitive to the choice of $\Delta K_{th}(a)$ estimate. In this work, a generalization version of El Haddad–Topper–Smith's equation is used to evaluate the behavior of cracks emanating from circular holes and semi-elliptical notches. For several combinations of notch dimensions, the smallest stress range necessary to both initiate and propagate a crack is calculated, resulting in expressions for K_f and therefore for q. It is found that the q estimates obtained from this generalization, besides providing a sound physical basis for the notch sensitivity concept, better correlate with experimental data from the literature.

Keywords: Notch sensitivity; Short cracks; Fatigue crack growth threshold; Non-propagating cracks

1. Introduction

The purpose of this paper is to verify if and when the classical Peterson-like notch sensitivity factors, still widely used all over the world for designing mechanical components by the classical SN methodology [1–3], can be reproduced from notch stress analysis and Topper-like short crack concepts, assuming the material can be modeled as linear, elastic, isotropic and homogeneous.

The empirical notch sensitivity factor q was introduced to quantify the difference between K_t, the geometric (or linear elastic) stress concentration factor of the notch, and its actual effect in the fatigue limit, $K_f = 1 + q(K_t - 1) = \Delta\sigma_0/\Delta\sigma_f$, where K_f is the so-called fatigue (stress) concentration factor, and $\Delta\sigma_0$ and $\Delta\sigma_f$ are the fatigue limits of smooth and notched SN specimens, respectively. Small non-propagating fatigue cracks that are found at the notch roots when $\Delta\sigma_0/K_t < \Delta\sigma_n < \Delta\sigma_0/K_f$, where $\Delta\sigma_n$ is the nominal stress range applied in the notched piece [4] can, at least in some cases, explain why $K_f \leqslant K_t$. Therefore, in such cases it should in principle be possible to analytically

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predict q values based on the propagation behavior of small cracks emanating from notches.

Several expressions have been proposed to model the influence of the size a of very small fatigue cracks on their stress intensity range propagation threshold value, $\Delta K_{th}(a)$ [5]. Most of these expressions are based on length parameters such as El Haddad–Topper–Smith's a_0 [6], estimated from $\Delta\sigma_0$ and ΔK_0, the crack propagation threshold of long cracks, $\Delta K_{th}(a \to \infty)$, resulting in a modified stress intensity range for the Irwin plate

$$\Delta K_I = \Delta\sigma\sqrt{\pi(a+a_0)}, \text{ where } a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{\Delta\sigma_0}\right)^2 \tag{1}$$

These equations reproduce the Kitagawa–Takahashi plot trend [7], one of the most used tools to qualitatively understand the behavior of short cracks, as well as to design for infinite life. Lawson et al. presented a very good review of near-threshold fatigue in [8]. Yu et al. [9] and Atzori et al. [10] used a geometry factor α to generalize the above equation to other geometries, resulting in

$$\Delta K_I = \alpha\cdot\Delta\sigma\sqrt{\pi(a+a_0)}, \text{ where } a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{\alpha\cdot\Delta\sigma_0}\right)^2 \tag{2}$$

However, for a very small crack with $a << a_0$ this expression would imply that $\Delta\sigma$ tends to the fatigue limit $\Delta\sigma_0$. In the presence of notches, this would be true only if $\Delta\sigma$ is the notch root stress range, not the nominal one. But in most cases the geometry factor α used in the literature already includes the effects of the notch root stress concentration factor, defining $\Delta\sigma$ as the nominal stress. To avoid this problem, perhaps a clearer way to define the length parameter a_0 in the presence of notches is by considering $\Delta\sigma$ as the nominal stress range (away from the notch) and two factors $f(a)$ and α, where the former tends to the notch root stress concentration factor as the crack length a tends to zero, and the latter only encompasses the remaining terms, such as the free surface correction:

$$\Delta K_I = \alpha\cdot f(a)\cdot\Delta\sigma\sqrt{\pi(a+a_0)}, \text{ where } a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{\alpha\cdot\Delta\sigma_0}\right)^2 \tag{3}$$

Note that $f(a)$ does not appear in the expression of a_0, because for very small cracks ($a \to 0$) the notch root stress range $f(0)\cdot\Delta\sigma$ should be equal to and replaced by $\Delta\sigma_0$. Ciavarella and Monno [11] have recently used length parameters such as these to design not only for infinite life, but also for finite lives using an interpolation between the Basquin/Wöhler equations and the Paris law, with or without corrections for the near-threshold ΔK-controlled propagation regime. Their resulting expressions can be seen as SN curves which are a function of the initial (small) crack size.

Alternatively, the stress intensity range ΔK_I can retain its original equation [12–17], while the threshold expression is modified by a function of the crack length a, namely $\Delta K_{th}(a)$, resulting in

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \sqrt{\frac{a}{a+a_0}} \tag{4}$$

Peterson-like [18] expressions can then be calibrated to q based on these crack propagation estimates. Topper's [6] classical approach can easily generate approximate expressions by studying the limit cases where the crack is much smaller or much larger than the notch dimensions. For instance, the stress intensity range of a semi-elliptical notch with stress concentration factor K_t and semi-axis b parallel to a crack with length a, in an infinite plate under tension, has limit values

$$\Delta K_I = 1.1215\cdot K_t\cdot\Delta\sigma\sqrt{\pi(a+a_0)}, \quad \text{for } a << b \tag{5}$$

$$\Delta K_I = 1.1215\cdot\Delta\sigma\sqrt{\pi(a+b)}, \quad \text{for } a >> b \tag{6}$$

where $\alpha = 1.1215$ is the free surface correction factor. Therefore, after some algebric manipulation,

$$\Delta\sigma_{th} = \frac{\Delta K_0}{1.1215\cdot\sqrt{\pi(a+a_0)}} \leqslant \frac{1.1215\cdot K_t\cdot\Delta\sigma_0\sqrt{\pi a_0}}{1.1215\cdot\sqrt{(a+b)/a}\cdot\sqrt{\pi(a+a_0)}} \tag{7}$$

The above expression is upper-bounded by

$$\Delta\sigma_{th} \leqslant K_t\cdot\Delta\sigma_0\cdot\sqrt{\frac{a}{a+b}\cdot\frac{a_0}{a+a_0}} \tag{8}$$

which is maximum for a critical size $a = \sqrt{b\cdot a_0}$.

Therefore, this critical size proposed by Topper is often associated with the maximum $\Delta\sigma_{th}$ for sharp notches, as well as the size of the largest non-propagating crack. However, such approximations are found to be extremely sensitive to the choice of $\Delta K_{th}(a)$ estimate.

In the following section, a generalization of El Haddad–Topper–Smith's equation is used to better fit the data on the crack size dependence of $\Delta K_{th}(a)$. This expression is then applied to cracks emanating from circular holes and semi-elliptical notches, resulting in improved estimates of the notch sensitivity q and the largest non-propagating crack size.

2. Propagation of short cracks

The El Haddad–Topper–Smith's equation can be seen as one possible asymptotic match between the short and long crack behaviors. Following Bazant's reasoning [19], a more general equation can be proposed, involving a fitting parameter n, which can be written as

$$\frac{\Delta K_{th}(a)}{\Delta K_0} = \left[1+\left(\frac{a_0}{a}\right)^{n/2}\right]^{-1/n} \tag{9}$$

Clearly, Eqs. (1)–(4) are obtained from Eq. (9) when $n = 2.0$. Also, the bi-linear estimate is obtained as n tends to infinity. The adjustable parameter n allows the ΔK_{th} estimates to better correlate with experimental crack propagation data collected from Tanaka et al. [20] and Livieri and

Tovo [21], and presented in Fig. 1. Most of the data in this figure can be bounded by two curves obtained using $n = 1.5$ and $n = 8.0$ in Eq. (9).

2.1. Short cracks from circular holes

Eq. (9) is now used to evaluate the behavior of short cracks emanating from circular holes in large plates loaded by a nominal normal stress range $\Delta\sigma$. The stress intensity range of a single crack with length a emanating from a circular hole with radius ρ is expressed, within 1%, by [22]

$$\Delta K_I = 1.1215 \cdot f(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} \tag{10}$$

where the factor $f(a/\rho)$, related to the hole stress concentration, is

$$f\left(\frac{a}{\rho}\right) \equiv f(x) = \left(1 + \frac{0.2}{(1+x)} + \frac{0.3}{(1+x)^6}\right) \cdot \left(2 - 2.354\frac{x}{1+x} + 1.2056\left(\frac{x}{1+x}\right)^2 - 0.2211\left(\frac{x}{1+x}\right)^3\right), \quad x \equiv \frac{a}{\rho} \tag{11}$$

Note that when the crack size a tends to zero, Eq. (10) becomes

$$\lim_{a\to 0} \Delta K_I = 1.1215 \cdot 3 \cdot \Delta\sigma\sqrt{\pi a} \tag{12}$$

as expected, since the above equation combines the solution for an edge crack in a semi-infinite plate with the stress concentration factor of a circular hole, K_t equal to 3 (i.e., $f(0) = 3$). Note also that the other limit, when a tends to infinity, results in

$$\lim_{a\to\infty} \Delta K_I = \Delta\sigma\sqrt{\pi a/2} \tag{13}$$

which is the solution for a crack with length a in an infinite plate, where one of its edges is far enough from the circular hole not to suffer its influence in the stress field (in fact, the equivalent crack length would be $a + 2\rho$, however as a tends to infinity the ρ value disappears from the equation). Therefore, for a circular hole $f(x = 0) = 3$ and $f(x \to \infty) = 1/1.1215\sqrt{2} \cong 0.63$, and from Eqs. (9)–(11), it follows that the crack will propagate when

$$\Delta K_I = \alpha \cdot f(a/\rho) \cdot \Delta\sigma\sqrt{\pi a} > \Delta K_{th} = \Delta K_0 \cdot [1 + (a_0/a)^{n/2}]^{-1/n} \tag{14}$$

where $\alpha = 1.1215$ is the free surface correction. Knowing that $\Delta K_{th} \equiv \Delta K_0$ for a long crack, the crack length parameter a_0 from the above equation is

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_0}{1.1215 \cdot \Delta\sigma_0}\right)^2 \tag{15}$$

Note in the above equation that, as discussed before, the factor $f(a/\rho)$ does not appear in the definition of a_0. Therefore, the crack propagation criterion based on the dimensionless functions $f(a/\rho)$ and $g(a/\rho,\ \Delta\sigma_0/\Delta\sigma,\ \Delta K0/\Delta\sigma_0\sqrt{\rho},n)$ is:

$$f\left(\frac{a}{\rho}\right) > \frac{\left(\frac{\Delta K_0}{\Delta\sigma_0\sqrt{\rho}}\right) \cdot \left(\frac{\Delta\sigma_0}{\Delta\sigma}\right)}{\left[\left(\alpha\sqrt{\frac{\pi a}{\rho}}\right)^n + \left(\frac{\Delta K_0}{\Delta\sigma_0\sqrt{\rho}}\right)^n\right]^{1/n}} \equiv g\left(\frac{a}{\rho}, \frac{\Delta\sigma_0}{\Delta\sigma}, \frac{\Delta K_0}{\Delta\sigma_0\sqrt{\rho}}, n\right) \tag{16}$$

If $x \equiv a/\rho$ and $k \equiv \Delta K_0/\Delta\sigma_0\sqrt{\rho}$, then the crack grows whenever $f(x) > g(x,\ \Delta\sigma_0/\Delta\sigma,\ k,\ n)$.

Fig. 2 plots f and g, assuming a material/notch combination with $k = 1.5$ and $n = 6$, as a function of the normalized crack length x. For a high applied $\Delta\sigma$, the ratio $\Delta\sigma_0/\Delta\sigma$ becomes small, and the function g is always below f, meaning that a crack of any length will propagate. The lower curve in Fig. 2 shows the function g obtained from a ratio $\Delta\sigma_0/\Delta\sigma = 1.4$, never crossing f. On the other hand, for a $\Delta\sigma$ small enough such that $\Delta\sigma_0/\Delta\sigma \geqslant K_t = 3$, g is always

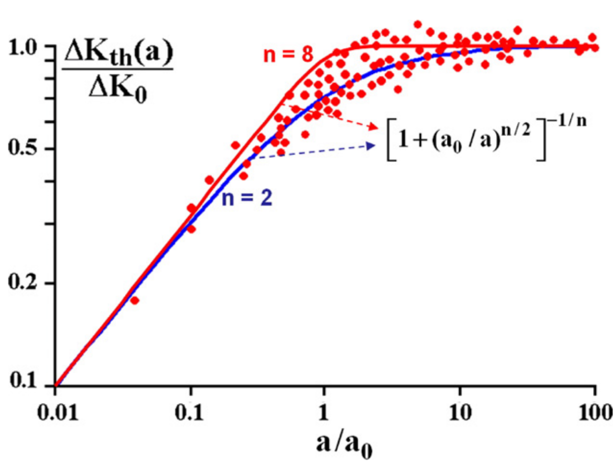

Fig. 1. Ratio between short and long crack propagation thresholds as a function of a/a_0.

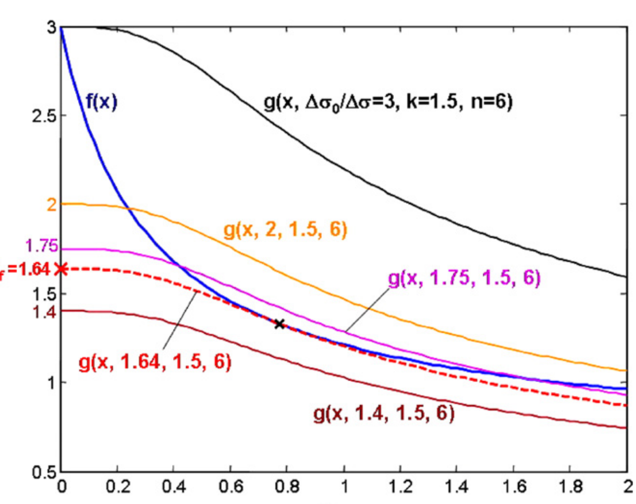

Fig. 2. Calculation of the fatigue stress concentration factor K_f from the plots of the functions $f(x)$ and $g(x,\ \Delta\sigma_0/\Delta\sigma,\ k,\ n)$, where $x \equiv a/\rho$ and $k \equiv \Delta K_0/\Delta\sigma_0\sqrt{\rho}$.

above f and no crack will initiate nor propagate, as shown by the top curve in the figure.

But three other cases must be noted. The first one, illustrated by the g curve with $\Delta\sigma_0/\Delta\sigma = 2$ in Fig. 2, has only one intersection point with f. This means that such stress levels cause a crack to initiate at the notch, however it will only propagate until a size $a = x \cdot \rho$ obtained from the x value at the intersection point. Therefore, non-propagating cracks will appear at the notch root.

The second case, illustrated by the g curve with $\Delta\sigma_0/\Delta\sigma = 1.75$ in Fig. 2, has two intersection points with f. Therefore, non-propagating cracks will also appear, with maximum sizes obtained from the first intersection point (on the left). Interestingly, cracks longer than the value defined by the second intersection will re-start propagating until fracture. However, crack growth between the two intersections should be caused by a different mechanism, e.g., corrosion or creep.

Finally, in the third case, both f and g functions are tangent, thus meet at a single point (such as the curve with $\Delta\sigma_0/\Delta\sigma = 1.64$ in Fig. 2). This $\Delta\sigma_0/\Delta\sigma$ value is therefore associated with the smallest stress range $\Delta\sigma$ that can cause crack initiation *and* propagation without arrest. So, by definition, this specific $\Delta\sigma_0/\Delta\sigma$ is equal to the fatigue stress concentration factor K_f. To obtain K_f, it is then sufficient to guarantee that both functions f and g are tangent at a single point with $x = x_{max}$. This x_{max} value is associated with the largest non-propagating flaw that can arise from fatigue alone. So, given n and k from the material and notch, x_{max} and K_f can be solved from the system of equations:

$$\begin{cases} f(x_{max}) = g(x_{max}, K_f, k, n) \\ \frac{\partial}{\partial x} f(x_{max}) = \frac{\partial}{\partial x} g(x_{max}, K_f, k, n) \end{cases} \tag{17}$$

This system can be solved numerically for each combination of k and n values, and the notch sensitivity factor q is then obtained from

$$q(k,n) \equiv \frac{K_f(k,n) - 1}{K_t - 1} \tag{18}$$

The above approach has two main advantages. First, it considers the material-dependent data fit parameter n, which has a significant influence on the calculations. And second, it is an exact procedure, not an approximation such as the ones based on the limit case inequalities. In the next section, the same approach will be applied to semi-elliptical notches.

2.2. Short cracks from semi-elliptical notches

The behavior of short cracks emanating from semi-elliptical notches can be evaluated in the same way. The stress intensity range of a single crack with length a emanating from a semi-elliptical notch with semi-axes b and c (where b is in the same direction as a) can be written as

$$\Delta K_I = \alpha \cdot f\left(\frac{a}{b}, \frac{c}{b}\right) \cdot \Delta\sigma\sqrt{\pi a} \tag{19}$$

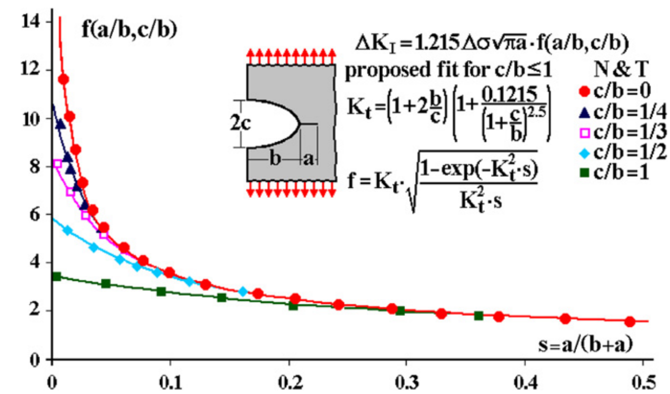

Fig. 3. Finite element calculations and proposed fit for the geometry factor of semi-elliptical notches with $c \leqslant b$.

where $\alpha = 1.1215$ is the free surface correction, and $f(a/b, c/b)$ is a geometry factor associated with the notch stress concentration. The geometry factor can be expressed as a function of the dimensionless parameter $s = a/(b + a)$ and the notch root stress concentration factor K_t

$$K_t = \left(1 + 2\frac{b}{c}\right) \cdot \left[1 + \frac{0.1215}{(1 + c/b)^{2.5}}\right] \tag{20}$$

Values for f have been calculated by Nishitani and Tada [19], with results available only in graph form, however equations are necessary to perform the analyses. To obtain expressions for f, finite element (FE) calculations were performed using the Quebra2D program [23] considering several cracked semi-elliptical notch configurations. The numerical results, which agreed well with [22], were fitted within 3% using empirical equations, resulting in

$$f\left(\frac{a}{b}, \frac{c}{b}\right) \equiv f(K_t, s) = K_t \cdot \sqrt{\frac{1 - \exp(-K_t^2 \cdot s)}{K_t^2 \cdot s}} \quad \text{for } c \leqslant b \tag{21}$$

$$f\left(\frac{a}{b}, \frac{c}{b}\right) \equiv f(K_t, s) = K_t \cdot \sqrt{\frac{1 - \exp(-K_t^2 \cdot s)}{K_t^2 \cdot s}} \cdot [1 - \exp(-K_t^2)]^{-s/2} \quad \text{for } c \geqslant b \tag{22}$$

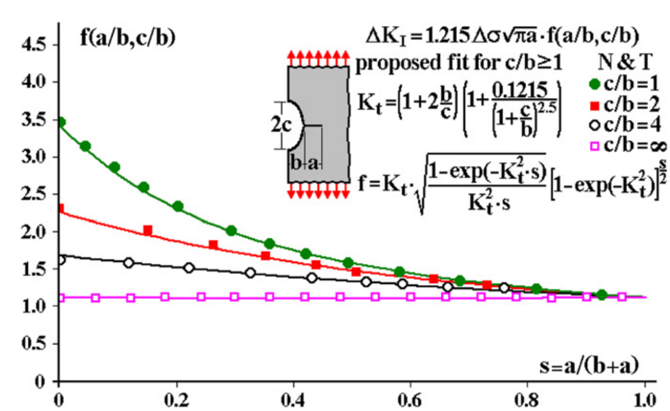

Fig. 4. Finite element calculations and proposed fit for the geometry factor of semi-elliptical notches with $c \geqslant b$.

Figs. 3 and 4 compare the proposed equations (solid lines) with the FE calculations.

Using Eqs. (17)–(22), the same procedure to evaluate the notch sensitivity in circular holes is then applied to the semi-elliptical notch geometry. The results are presented next.

3. Results

For several combinations of k and n, the smallest stress range necessary to both initiate and propagate a crack is calculated from Eq. (17), resulting in expressions for K_f and therefore q. The following sections present the results for circular holes and semi-elliptical notches.

3.1. Results for circular holes

Fig. 5 shows the calculated notch sensitivity factors for circular holes as a function of the dimensionless parameter $1/k \equiv \Delta\sigma_0\sqrt{\rho}/\Delta K_0$. Note from the figure that q is approximately linear with $1/k$ for $q > 0$. This results in the proposed estimate:

$$q(k,n) \cong \frac{q_1(n)}{k} - q_0(n) = q_1(n)\frac{\Delta\sigma_0\sqrt{\rho}}{\Delta K_0} - q_0(n) \tag{23}$$

where $q_0(n)$ and $q_1(n)$ are functions of n, and $q_1(n)$ is typically between 0.85 and 1.15. Note that if the estimate above results in q larger than 1, then $q = 1$. This will happen at holes with a very large radius ρ_{upper} such that

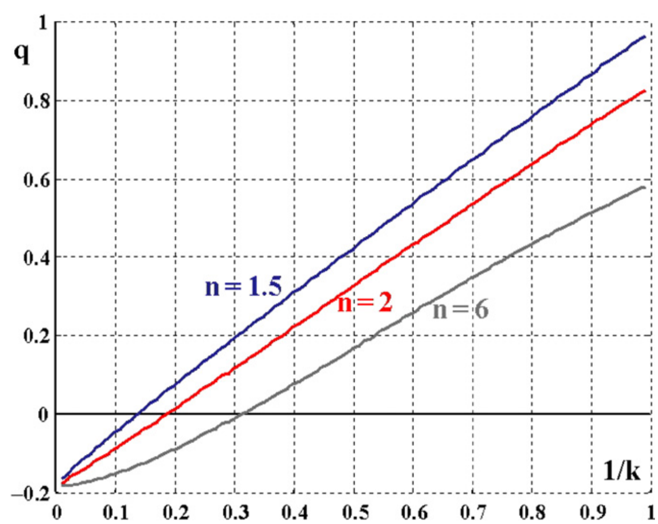

Fig. 5. Notch sensitivity factors q as a function of the dimensionless parameters k and n.

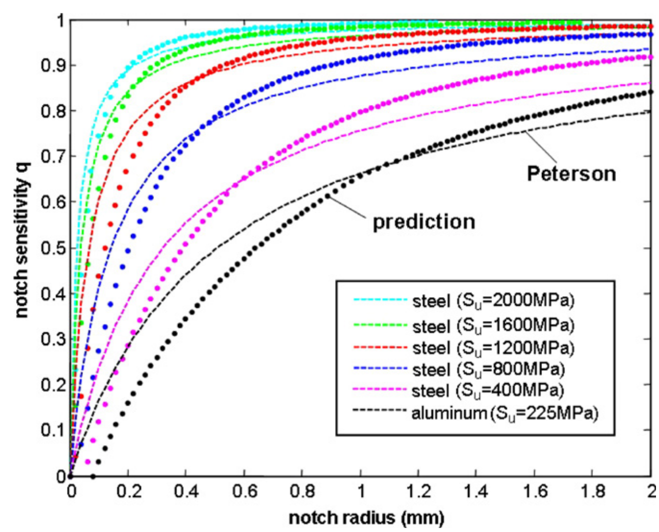

Fig. 6. Notch sensitivity factors for circular holes as a function of their radii.

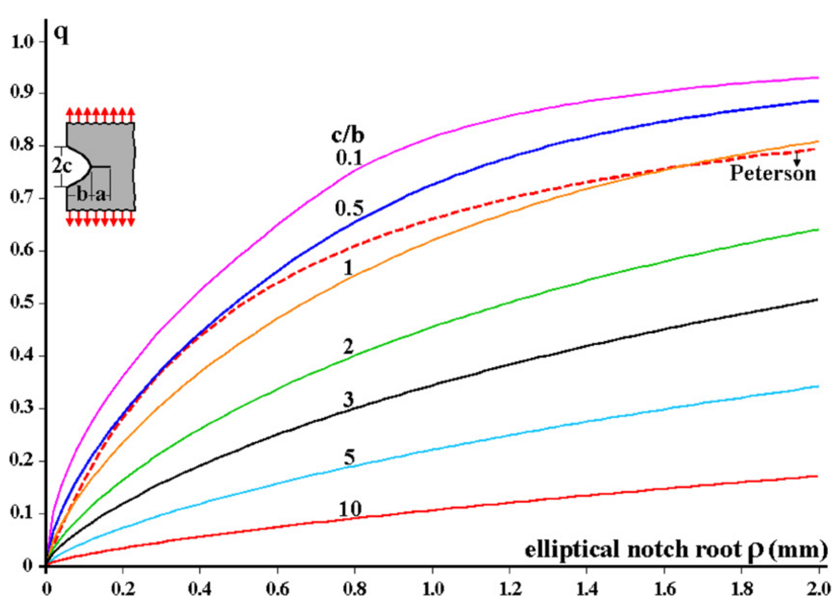

Fig. 7. Notch sensitivity q as a function of the semi-elliptical notch root radius ρ for aluminium alloys with $a_0 = 0.26$ mm ($S_u \cong 225$ MPa).

$$\frac{\Delta\sigma_0\sqrt{\rho_{\text{upper}}}}{\Delta K_0} > \frac{1+q_0(n)}{q_1(n)} \Rightarrow \rho_{\text{upper}} > \left(\frac{1+q_0(n)}{q_1(n)}\cdot\frac{\Delta K_0}{\Delta\sigma_0}\right)^2 \tag{24}$$

Therefore, it is impossible to generate a non-propagating crack under constant amplitude loading in notches with a very large radius, regardless of the stress level. The stress gradient is so small in this case that any crack that initiates will cut through a long region still influenced by the stress concentration, preventing any possibility of crack arrest. Eq. (17) will not have a solution for $x_{\max} > 0$, because $\partial g/\partial x$ in this case will be more negative than $\partial f/\partial x$ at $x = 0$.

On the other hand, it is possible to obtain a value of q smaller than zero, down to $q = -0.2$ for a circular hole, see Fig. 5. This can indeed happen for holes with a very small radius ρ_{lower} such that

$$\frac{\Delta\sigma_0\sqrt{\rho_{\text{lower}}}}{\Delta K_0} < \frac{q_0(n)}{q_1(n)} \Rightarrow \rho_{\text{lower}} < \left(\frac{q_0(n)}{q_1(n)}\cdot\frac{\Delta K_0}{\Delta\sigma_0}\right)^2 \tag{25}$$

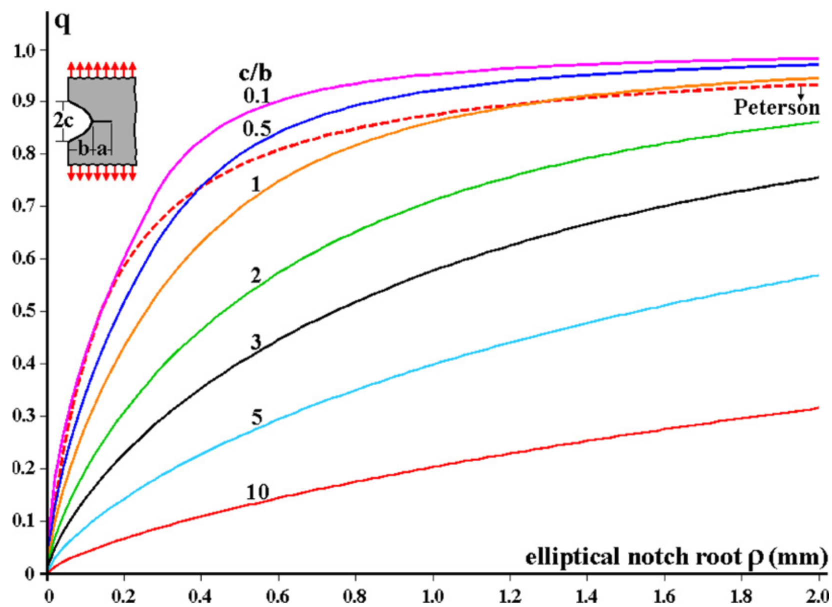

Fig. 8. Notch sensitivity q as a function of ρ for steels with $a_0 = 0.10$ mm ($S_u \cong 800$ MPa).

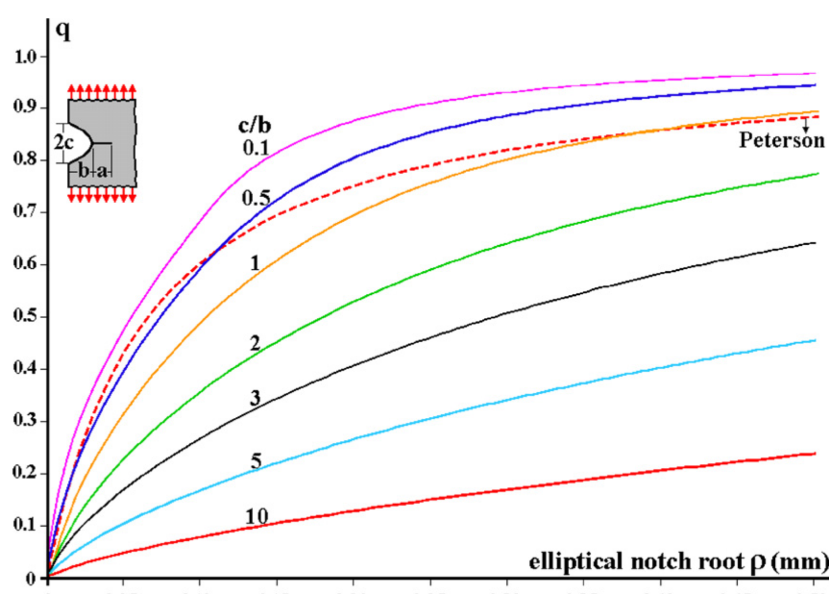

Fig. 9. Notch sensitivity q as a function of ρ for steels with $a_0 = 0.040$ mm ($S_u \cong 1200$ MPa).

The physical meaning of a negative q is that it is easier to initiate and propagate a fatigue crack at a notchless border of the plate than at a very small hole inside the plate. The ΔK_I of a crack at the small hole will soon tend to Eq. (13) due to the large stress gradient, without the 1.1215 free surface factor, while the stress intensity solution for an edge crack will be larger since it includes the 1.1215 factor. In addition, for most materials, the size of this critical radius ρ_{lower} is just a few micrometers. This leads to the conclusion that internal defects with equivalent radius smaller than such ρ_{lower} of a few micrometers are harmless, since its K_f will be smaller than 1, and the main propagating crack will initiate at the surface. Murakami's results support this claim [24].

Note that Peterson's [18] and similar estimates assume that the notch sensitivity q is only a function of notch radius ρ and the ultimate strength of the material S_u. Eq. (23), however, suggests that q depends not only on ρ, $\Delta\sigma_0$ and ΔK_0, as observed by Topper [6], but also on the short crack threshold data-fitting parameter n. Even though there are reasonable estimates relating $\Delta\sigma_0$ and S_u, there is no clear relationship between ΔK_0 and S_u. This means,

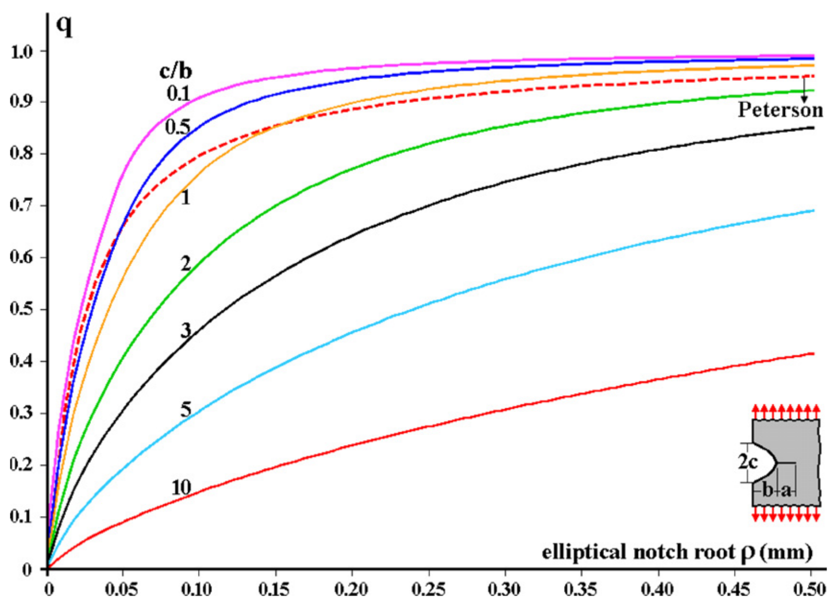

Fig. 10. Notch sensitivity q as a function of ρ for steels with $a_0 = 0.016$ mm ($S_u \cong 2000$ MPa).

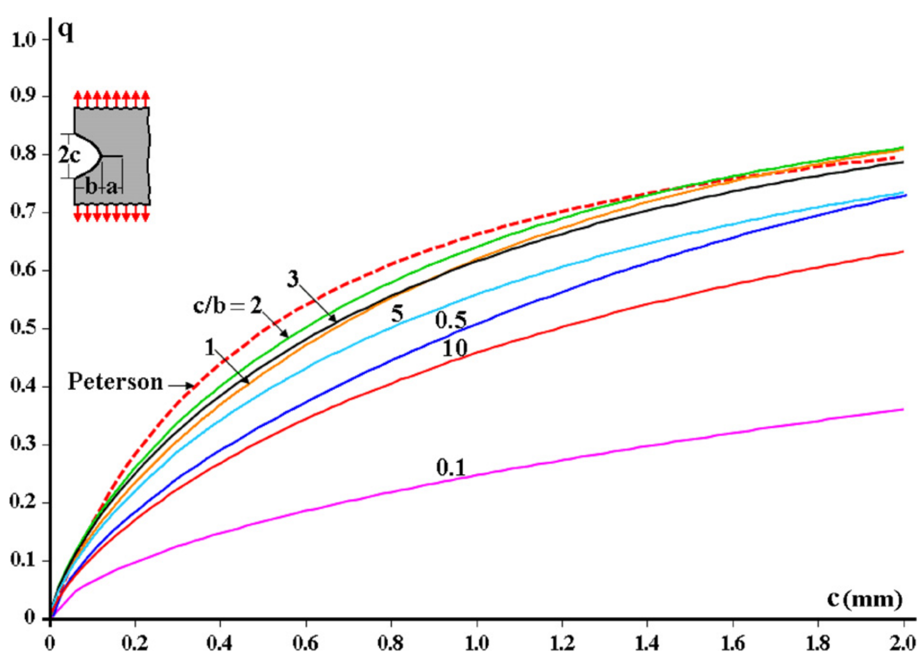

Fig. 11. Notch sensitivity q as a function of the semi-axis c for aluminium alloys with $a_0 = 0.26$ mm ($S_u \cong 225$ MPa).

e.g., that two steels with same S_u but very different ΔK_0 would have different behaviors that a Peterson's-like equation would not be able to reproduce. Thus, notch sensitivity experiments should always include a measure of the ΔK_0 of the material.

Finally, the ViDa software database [23] is used to collected data on 450 steels and aluminum alloys with fully measured S_u, material fatigue limit S_L for $R = -1$, and ΔK_0 (for $R = 0$). Their average values of S_L and ΔK_0 are evaluated for steels with S_u near the ranges 400, 800, 1200, 1600 and 2000 MPa, and for aluminum alloys near 225 MPa. In order to estimate $\Delta\sigma_0$, the stress range associated with the fatigue limit at $R = 0$, the Goodman equation is used considering alternate and mean stress components $\sigma_a = \sigma_m = \Delta\sigma_0/2$. The resulting Eq. (18) is then plotted as a function of the notch radius ρ, using the above averages and assuming $n = 6$, see Fig. 6. Note that Peterson's equations, which were originally fitted to notch sensitivity experiments, can be reasonably predicted and reproduced using the proposed analytical approach.

3.2. Results for semi-elliptical notches

Notch sensitivity factors are also evaluated for semi-elliptical notches. As expected, the results depend on ρ, $\Delta\sigma_0$ and ΔK_0, in addition to n. Moreover, a significant dependency is observed with respect to the aspect ratio

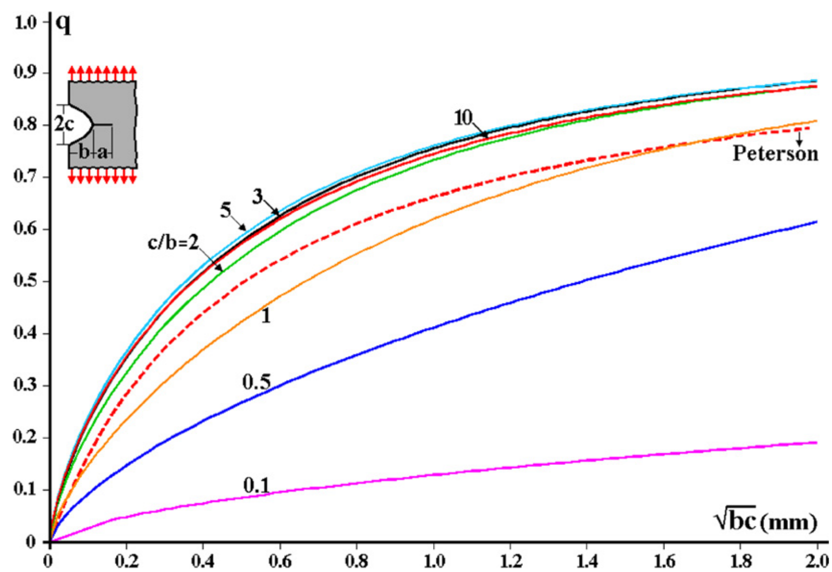

Fig. 12. Notch sensitivity q as a function of the square root of the product between the semi-axes for aluminium alloys with $a_0 = 0.26$ mm ($S_u \cong 225$ MPa).

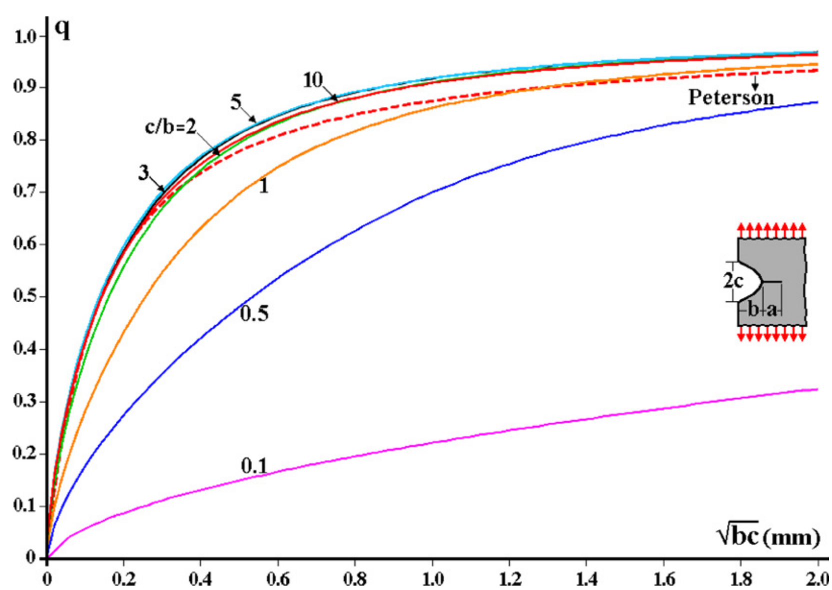

Fig. 13. Notch sensitivity q as a function of the square root of the product between the semi-axes for steels with $a_0 = 0.10$ mm ($S_u \cong 800$ MPa).

c/b. Therefore, the entire notch geometry, not only its radius, is an important factor when evaluating its sensitivity.

Figs. 7–10 show the calculated notch sensitivity q as a function of the semi-elliptical notch root radius ρ for several aspect ratios c/b. These figures show representative values for aluminum alloys with S_u near 225 MPa, selected from ViDa [23], as well as for steels with S_u near 800, 1200 and 2000 MPa. The associated crack length parameters according to Eq. (15) are, respectively, $a_0 = 0.26$ mm, 0.10 mm, 0.040 mm and 0.016 mm, averaged from the selected sample. It can be seen that q has a strong dependence on the notch geometry through the c/b ratio.

Fig. 11 shows that q is also dependent on c/b when plotted against c. However, for c/b ratios between 1 and 3, the notch sensitivity is found to be dependent mainly on the semi-axis c.

For semi-elliptical notches with larger aspect ratios (c/b between 2 and 10), another interesting dependence is found, with the square root of the product between the semi-axes, see Figs. 12–15. This dependence is in agreement with Murakami's factor [24], which states that the notch sensitivity associated with internal defects depends on the square root of the area. For c/b ratios outside this range, however, there's a significant influence of c/b in the resulting q values.

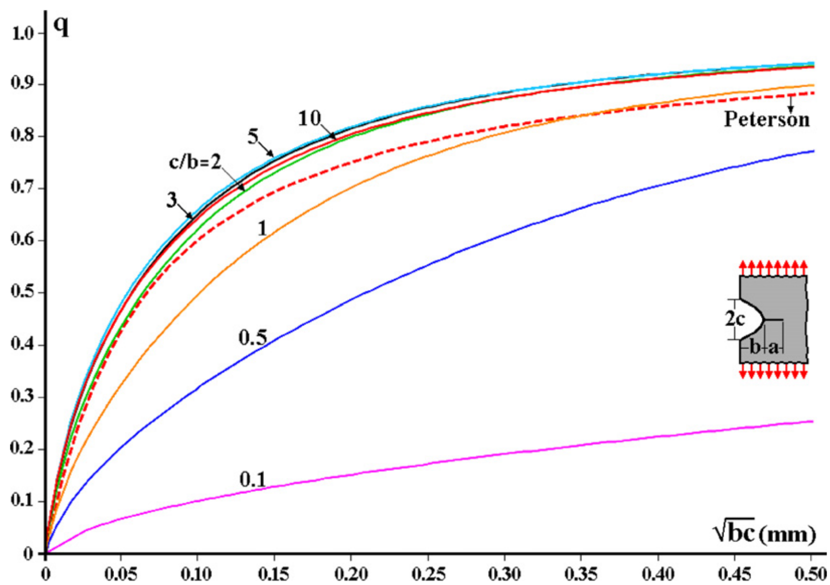

Fig. 14. Notch sensitivity q as a function of the square root of the product between the semi-axes for steels with $a_0 = 0.040$ mm ($S_u \cong 1200$ MPa).

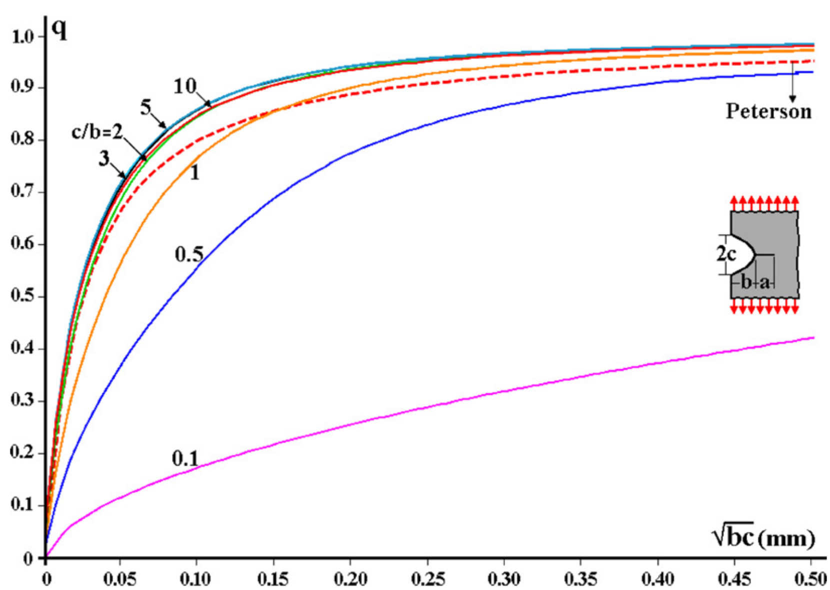

Fig. 15. Notch sensitivity q as a function of the square root of the product between the semi-axes for steels with $a_0 = 0.016$ mm ($S_u \cong 2000$ MPa).

4. Conclusions

A generalization of El Haddad–Topper–Smith's parameter was presented to model the crack size dependence of the threshold stress intensity range for short cracks. The proposed expressions were used to calculate the behavior of non-propagating cracks. New estimates for the notch sensitivity factor q were obtained for cracked holes and semi-elliptical notches, and compared with results from the literature. It was found that the notch sensitivity has a strong dependence on the notch aspect ratio. For semi-elliptical notches with aspect ratio between 2 and 10, a good agreement was found between q and the square root of the product between the semi-axes.

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