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jmr&t Journal of Materials Research and Technology

journal homepage: www.elsevier.com/locate/jmrt

Original Article

Modeling concrete and polymer creep using fractional calculus

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ARTICLE INFO

Article history:
Received 20 September 2020
Accepted 1 March 2021
Available online 10 March 2021

Keywords:
Concrete
Creep
Fractional calculus
Fractional Kelvin-Voigt
Scot-Blair elements

ABSTRACT

Although the strains on a given material depend on its properties and, in general, on the stresses, on the temperature and on the time under load, they may be modeled as a function of the stresses alone in most practical structural analyses at low service-to-fusion temperature ratios. Most metallic and ceramic alloys can be modeled in such a simplified way at room temperatures, but some important materials, among them polymers and concretes, can creep significantly even at room temperatures. Under relatively low stresses, they can usually be modeled as linear viscoelastic using simple rheological models based on springs and dashpots. However, in many practical cases such models cannot fit well experimental data unless many of those elements are used, a problem that can much impair their use in structural analyses. Fractional rheological elements based on fractional calculus techniques have been recently proposed as a promising modeling technique to avoid this problem, and in this work their performance is evaluated by comparing their fitting behavior with traditional modeling techniques, using representative creep data from polypropylene and from a medium strength concrete.

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1. Introduction

Both in metallic or ceramic crystalline alloys, as well as in polymers and glasses, creep is a thermo-mechanical failure mechanism that gradually accumulates inelastic strains in structural components. Creep can permanently deform and eventually break such components even under fixed loads that induce stresses well below their short-term yield strengths. At high Θ/Θ_f work-to-fusion absolute temperature ratios, creep can be the dominant failure mechanism in many practical applications.

Hence, even though strains may be modeled as a function $\varepsilon(\sigma)$ of the stresses alone in most practical structural analyses at low Θ/Θ_f ratios, in general they depend on the temperature and time under load as well, i.e. $\varepsilon = \varepsilon(\sigma, \Theta, t)$. Moreover, since creep strain rates $d\varepsilon/dt$ tend to increase exponentially with the service temperature Θ, creep strains cannot be neglected in high-Θ applications. Temperatures may be considered low in most structural analyses typically if $\Theta/\Theta_f < 0.3$ in metallic

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https://doi.org/10.1016/j.jmrt.2021.03.007
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alloys, $\Theta/\Theta_f < 0.4$ in ceramics, and $\Theta/\Theta_G < 0.5$ in polymers, where Θ_G is their glass transition temperature. However, creep strains may be relevant even at relatively low-Θ when the service stresses are remarkably high, the grains of the material are too small, and/or the dimensional tolerances are too tight [1].

Diffusive flow and dislocation movements are the main creep micromechanisms in crystalline materials. Frost and Ashby [2] claim that since plastic flow is a kinetic process, time-independent yield strengths only exist at zero Kelvin. Otherwise, the resistance to plastic flow depends on strain, strain-rate, and temperature. They classify the atomic scale kinetic processes that cause creep into five groups, namely:

(i) Plastic collapse.
(ii) Low-temperature plasticity by dislocation glide, controlled by discrete obstacles, drag of atomic particles, or Peierls' stress.
(iii) Low-temperature plasticity by twinning.
(iv) Power-law creep by dislocation glide or glide-plus-climb, caused by lattice-diffusion controlled climb (high-temperature creep), core diffusion controlled climb (low-temperature creep), power-law breakdown at the transition from glide to climb-plus-glide, Harper-Dorn creep (very slow creep rates induced by and proportional to low stresses), and creep accompanied by dynamic recrystallization.
(v) Diffusional flow, produced by lattice diffusion (Nabarro-Herring creep), grain boundary diffusion (Coble creep), and interface-reaction controlled diffusional flow.

Polymers are viscoelastic materials composed of very long chain-like macromolecules that contain many repeating units joined by strong covalent bonds, but in thermoplastics most or all bonds between their chains are weak van der Waals or H-bridge bonds. Their chains are usually not aligned, and their microstructure is analogous to an entanglement of long and sticky strings that can slip without breaking around each other, eventually with some crystalline regions that can improve a little but not modify their overall creep behavior. Polymers are true solids, but relatively low temperatures can break the weak bonds between their chains and even between parts of a single chain (the more resistant polymers soften near 250 °C, but most soften at much lower temperatures). Albeit thermosets have strong bonds between their chains, their creep behavior is similar. Structural polymers can be frequently modeled as linear viscoelastic solids for design purposes, at least when loaded by typically low service stresses.

Concrete also creeps even at room temperature, due to hydration of calcium silicate components in Portland cement, which bonds its mineral aggregates. Albeit concrete creep mechanisms are intrinsically different from crystalline metallic or ceramic alloys, or from polymeric creep mechanisms, its overall behavior is similar to the latter. It tends to be linearly dependent on the stress if the pore water content is constant.

Linear elastic materials obey Hooke's law, $\sigma(t) = E\varepsilon(t)$, where E is Young's modulus, whereas linear viscous materials obey Newton's law, $\sigma(t) = \eta \cdot d\varepsilon(t)/dt$ (both for uniaxial loads), where η is the dynamic viscosity. Linear viscoelastic materials present an intermediate behavior and have a time-dependent stress-strain ratio. They can relax when subjected to constant strains, or else creep under fixed stresses. To model such behaviors, Maxwell proposed the model described by Eq. (1)

$$\frac{d\sigma(t)}{dt} = E\frac{d\varepsilon(t)}{dt} - \frac{\sigma(t)}{\lambda} \tag{1}$$

where $\lambda = E/\eta$. This equation represents the analogous behavior of a spring and a dashpot in series, as shown in Fig. 1a. Kelvin-Voigt proposed an alternative linear viscoelastic model described by Eq. (2), which can be associated with a spring and a dashpot in parallel, as shown in Fig. 1b.

$$\sigma(t) = E\varepsilon(t) + \eta\frac{d\varepsilon(t)}{dt} \tag{2}$$

Similar simple Zener and Burger rheological models are shown in Fig. 1c and d. The general stress-strain relationship for such linear spring/dashpot rheological models is given by Eq. (3), where p_k and q_k are material constants measured by properly fitting suitable experimental data.

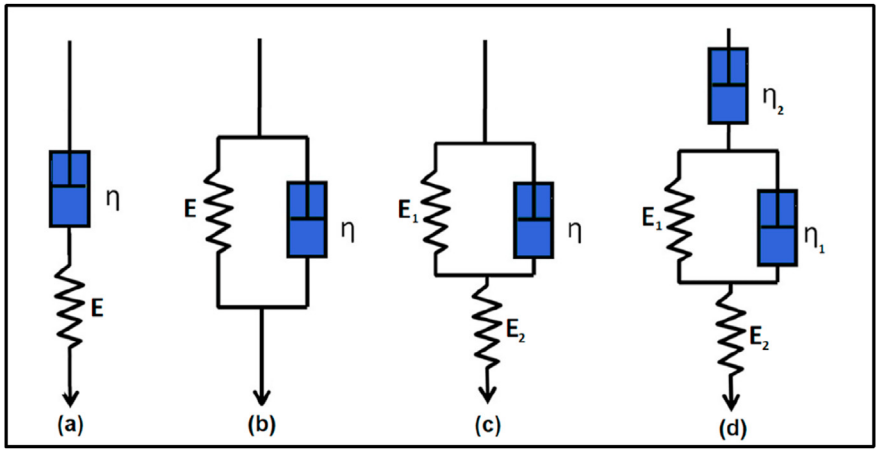

Fig. 1 – Viscoelastic models: (a) Maxwell; (b) Kelvin-Voigt; (c) Zener; (d) Burger.

$$\sum_{k=0}^{n} p_k \frac{d^k}{dt^k}\sigma(t) = \sum_{k=0}^{m} q_k \frac{d^k}{dt^k}\epsilon(t) \tag{3}$$

Recent studies have shown that rheological models based on fractional calculus can fit experimental data for viscoelastic materials [3–6] using a much smaller number of rheological elements than the number required by the linear springs and dampers used in traditional models. Recent works explore this technique to model concrete creep [7–9], to evaluate viscoelastic Euler-Bernoulli beams [10], in fatigue analyses [11], and to describe non-linear viscoelastic behavior of polymers [12,13].

Based on this fact, this paper sequentially discusses: (i) basic fractional calculus concepts, as well as the fundamental techniques needed to solve fractional differential equations; (ii) the fractional representation of viscoelastic materials; and (iii) the use of fractional elements in rheological models for such materials. Then it uses such concepts to fit both concrete and polymer creep data using very simple fractional rheological models. Finally, it compares their predictions with the predictions generated using traditional rheological models, which require more elements to fit properly the same experimental data.

2. A brief review of fractional calculus

Fractional calculus is a generalization of the classical calculus for derivatives of any order [14]. Its first mention was in a 1695 letter written by L'Hôpital and addressed to Leibniz, where he mentions the notation $D^n f(x)/Dx^n$ and questions about the possibility that the derivative order be $n = 1/2$, to which Leibniz replied, "an apparent paradox from which one day useful consequences will be drawn." In 1819, Lacroix obtained the fractional derivative from the integer derivative using the function $y = x^m$:

$$\frac{d^n y}{dx^n} = \frac{m!}{(m-n)!}x^{m-n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n} \tag{4}$$

where Γ is the Gamma function, and from this result, Lacroix used $m = 1$ and $n = 1/2$ to obtain

$$\frac{d^{1/2}y}{dx^{1/2}} = \frac{2\sqrt{x}}{\sqrt{\pi}} \tag{5}$$

Many mathematicians contributed for the development of fractional derivatives during the following centuries, and recently it has become a useful tool for engineers and scientists in many areas. Among the many definitions for fractional derivatives, the three most widely used are the Riemann-Liouville, the Caputo, and the Grünwald-Letnikov fractional derivatives, presented in the following.

2.1. The Riemann-Liouville fractional derivative

Riemann-Liouville's fractional-order integral results from the Cauchy formula for multiple integrals:

$${}_aI_t^n f(t) = \int_a^t \int_a^{\tau_{n-1}} \cdots \int_a^{\tau_1} f(\tau)d\tau d\tau_1 \cdots d\tau_{n-1} = \frac{1}{(n-1)!}\int_a^t (t-\tau)^{n-1} f(\tau)d\tau \tag{6}$$

Eq. (6) can be generalized by replacing $n \in \mathbb{Z}$ with $\alpha \in \mathbb{R}$, as shown in Eq. (7):

$${}_aI_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1} f(\tau)d\tau\,, \quad t > a \quad \alpha \in \mathbb{R} \tag{7}$$

where Γ is the gamma function.

The formula for the fractional order derivative results from this definition of the fractional order integral. The Riemann-Liouville definition for zxc the fractional derivative of α order is obtained by performing a fractional integration of order $n -$

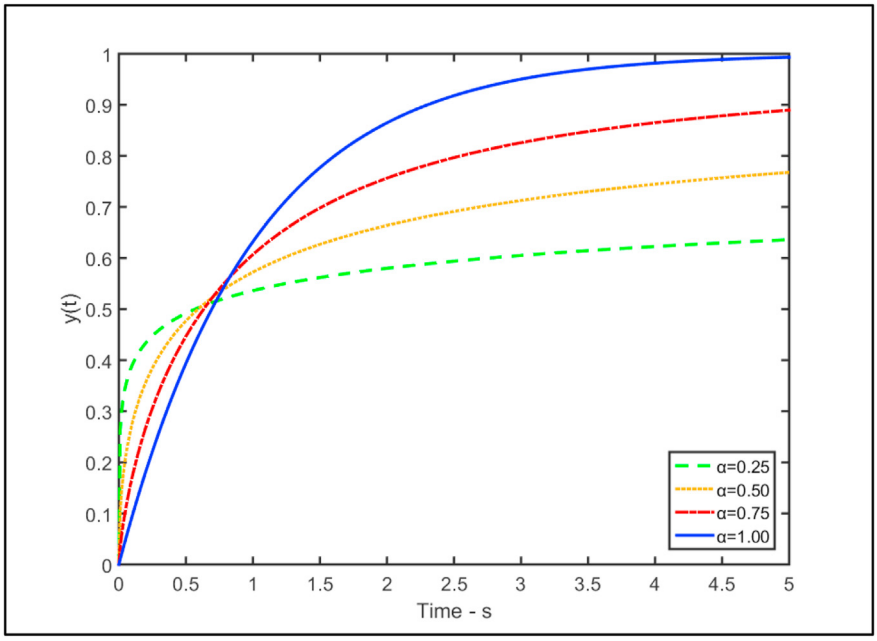

Fig. 2 – Unit step response of the system shown in Eq. (17) for some $0 < \alpha \leq 1$.

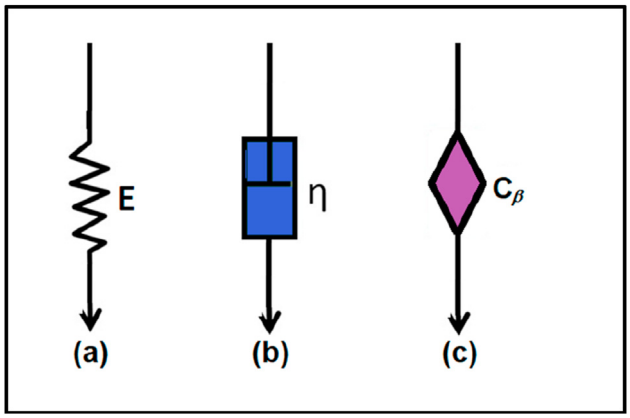

Fig. 3 – Fractional models: (a) Hooke; (b) Newton; (c) Scot-Blair (pot).

α, where n is the lowest integer larger than α, followed by a derivation of order n of the resulting function, as shown in Eq. (8).

$$
\begin{aligned}
{}_a^{RL}D_t^{\alpha} f(t) &= \frac{d^n}{dx^n}\left[{}_aI_t^{n-\alpha} f(t)\right] \\
&= \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau \\
& t > a, \quad n-1 < \alpha < n
\end{aligned} \tag{8}
$$

2.2. The Caputo fractional derivative

Riemann-Liouville's derivative has some issues: it is not null when applied to constant functions and their initial conditions are expressed in terms of derivatives of a fractional order, which do not have physical significance. To avoid such problems, Caputo defined his derivative ${}_a^{C}D_t^{\alpha}$ also from the Riemann-Liouville fractional integral, but first deriving the function and then applying n times the fractional integral of order $n-\alpha$, which again is the smallest integer larger than α, as shown in Eq. (9).

$$
\begin{aligned}
{}_a^{C}D_t^{\alpha} f(t) &= {}_aI_t^{n-\alpha}\left[\frac{d^n}{dt^n} f(t)\right] \\
&= \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^{(n)}(t)}{(t-\tau)^{\alpha+1-n}}(\tau)d\tau \\
& t > a, \quad n-1 < \alpha < n
\end{aligned} \tag{9}
$$

2.3. The Grünwald-Letnikov fractional derivative

Grünwald-Letnikov's definition is obtained by approximating the n-order derivative, assuming $f(t)$ is a well-defined function and continuous with all the derivatives up n in the interval $t \in [a, b]$, then its derivative of order n can be well approximated by:

$$
\frac{d^n f(t)}{dt^n} = \lim_{h\to 0}\frac{1}{h^n}\sum_{k=0}^{n}(-1)^k\binom{n}{k} f(t-kh) \tag{10}
$$

where:

$$
\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} \tag{11}
$$

The Grünwald-Letnikov fractional derivative is obtained by replacing $n \in \mathbb{N}$ with $\alpha \in \mathbb{R}$, as shown in Eq. (12). The notation ${}_a^{GL}D_t^{\alpha}$ means Grünwald-Letnikov fractional derivative of order α calculated using the interval $[a, t]$.

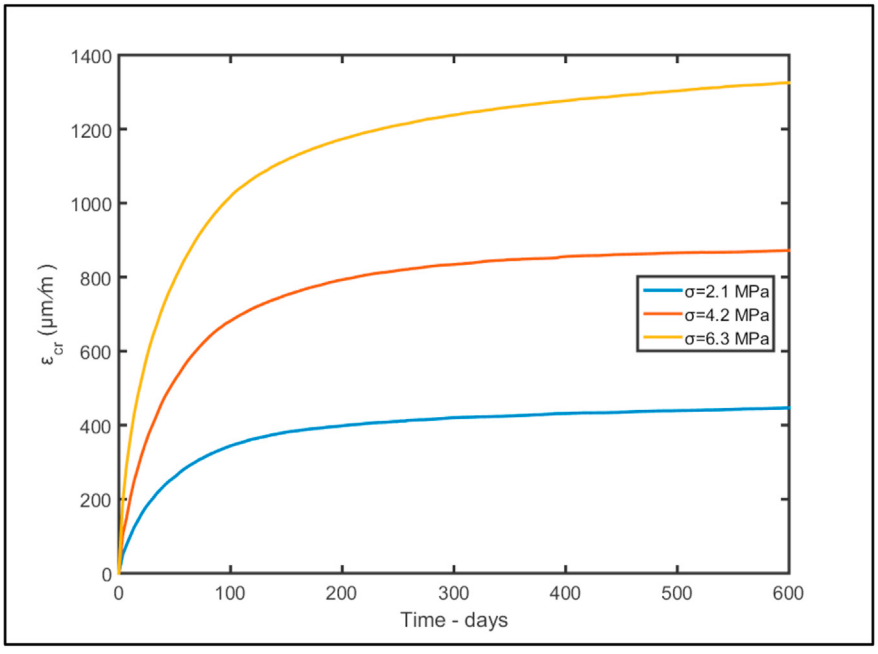

Fig. 4 – Creep strains under compressive stresses (plotted as positive for convenience) for a concrete in room temperature [15].

$$ {}_{a}^{GL}D_{t}^{\alpha}f(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{k=0}^{\left\lfloor\frac{t-a}{h}\right\rfloor}(-1)^{r}\binom{\alpha}{k}f(t-kh) $$

$$ t>a \quad \alpha\in\mathbb{R} $$

where:

$$ \binom{\alpha}{k} = \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)} \tag{13} $$

Eq. (14) shows an alternative form of the Grünwald-Letnikov fractional derivative.

$$ {}_{a}^{GL}D_{t}^{\alpha}f(t) = \lim_{N\to\infty}\frac{1}{A^{\alpha}}\sum_{k=0}^{N-1}(-1)^{r}\binom{\alpha}{k}f[t-kA] $$

$$ A = \left(\frac{t-a}{N}\right) \quad t>a \quad \alpha\in\mathbb{R} \tag{14} $$

Eq. (11) cancels out for $k>n$. Hence, the integer derivative is calculated using values close to the point t, since small values of h are used, and then the integer order derivative is a local operator. Eq. (13) does not vanish for $k>\alpha$. Thus, the fractional derivative is calculated using all the values of the function in the interval $[a,t]$, therefore, $f(t)$ must be continuous with all the derivatives up to the integer part of the fractional order (including t = 0). This provides the fractional derivative a memory character.

It can be proven that the Grünwald-Letnikov definition is equivalent to the Riemann-Liouville and to the Caputo derivatives for some functions. Therefore, it is widely used in numerical methods for solving fractional differential equations.

3. Fractional order systems

The solution of integer order differential equations is based on exponential equations. Similarly, the solution of fractional order differential equations is based on a transcendental function $E_{\alpha}(x)$ defined by Mittag-Leffler in 1903, see Eq. (15). This function is an intermediate function between the exponential for $\alpha = 0$ and the function $1/(1-x)$ for $\alpha = 1$.

$$ E_{\alpha}(x) = \sum_{k=0}^{\infty}\frac{x^{k}}{\Gamma(\alpha k+1)} \tag{15} $$

The Mittag-Leffler equation was generalized by Wilman in 1905 for two parameters, see Eq. (16).

$$ E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty}\frac{x^{k}}{\Gamma(\alpha k+\beta)} \tag{16} $$

Now it is possible to study the step response of the fractional system of interest. A fractional Kevin-Voigt model will present a fractional differential equation of the form shown in Eq. (17).

$$ a_1\frac{d^{\alpha}y(t)}{dt^{\alpha}} + a_0 y(t) = u(t) \tag{17} $$

Considering $u(t)$ a unit step function, the solution of this fractional order differential equation, considering zero initial conditions, is shown in Eq. (18). Fig. 2 shows the resulting $y(t)$ curves for some values of $0<\alpha\leq 1$, when $a_0 = a_1 = 1$.

$$ y(t) = \frac{1}{a_1}t^{\alpha}E_{\alpha,\alpha+1}\left(-\frac{a_0}{a_1}t^{\alpha}\right) \tag{18} $$

4. Fractional viscoelastic materials

The suitability of using fractional derivatives to model creep can be explained using Boltzmann's stress-strain relation [3,8], which generalizes Maxwell and Kevin-Voigt relations to consider that the current stress value depends not only on the current strain value but also on past values, to include memory effects. Eq. (19) shows Boltzmann's $J(t)$ creep function and Eq. (20) his $G(t)$ compliance function.

$$ \varepsilon(t) = \int_{0}^{t}J(t-\tau)\cdot d\sigma(\tau) = \int_{0}^{t}J(t-\tau)\cdot\dot{\sigma}(\tau)d\tau \tag{19} $$

$$ \sigma(t) = \int_{0}^{t}G(t-\tau)\cdot d\varepsilon(\tau) = \int_{0}^{t}G(t-\tau)\cdot\dot{\varepsilon}(\tau)d\tau \tag{20} $$

In addition, in the beginning of the 20th century, Nutting found that the creep and relaxation phenomena are better fitted by a power-law function, as shown in Eq. (21) for the creep function [8].

$$ J(t) = \frac{1}{C_{\beta}\Gamma(\beta+1)}t^{\beta} \quad 0\leq\beta<1 \tag{21} $$

C_{β} is a positive real number and depends on the material properties, such as β. The compliance function is a decreasing function, given by:

$$ G(t) = \frac{C_{\beta}}{\Gamma(1-\beta)}t^{-\beta} \quad 0\leq\beta<1 \tag{22} $$

If Eq. (21) is introduced into Eq. (19), it results in:

$$ \varepsilon(t) = \frac{1}{C_{\beta}\Gamma(\beta+1)}\int_{0}^{t}(t-\tau)^{\beta}\cdot\frac{d\sigma(\tau)}{d\tau}d\tau = \frac{1}{C_{\beta}}\left[{}_{0}I_{t}^{\beta}\sigma(t)\right] \tag{23} $$

The term ${}_{0}I_{t}^{\beta}$ in Eq. (23) is the Riemann-Liouville fractional integral shown in Eq. (7). Similarly, Eq. (24) results from the introduction of Eq. (22) in Eq. (20).

$$ \sigma(t) = \frac{C_{\beta}}{\Gamma(1-\beta)}\int_{0}^{t}(t-\tau)^{-\beta}\cdot\frac{d\varepsilon(\tau)}{d\tau}d\tau = C_{\beta}\left[{}_{0}^{C}D_{t}^{\beta}\varepsilon(\tau)\right] \tag{24} $$

The term ${}_{a}^{C}D_{t}^{\alpha}$ in Eq. (24) is the Caputo fractional derivative Eq. (9) for $0<\beta<1$. Thus, the stress-strain ratio of a viscoelastic material can be modeled using the fractional derivative, as shown in Eq. (25).

$$ \sigma(t) = C_{\beta}\frac{d^{\beta}\varepsilon(t)}{dt^{\beta}} \quad 0<\beta<1 \tag{25} $$

Therefore, viscoelastic materials are intermediate materials between elastic and viscous, since $\beta = 0$ yields Hooke's equation with $C_{\beta} = E$, and $\beta = 1$ results in Newton's equation

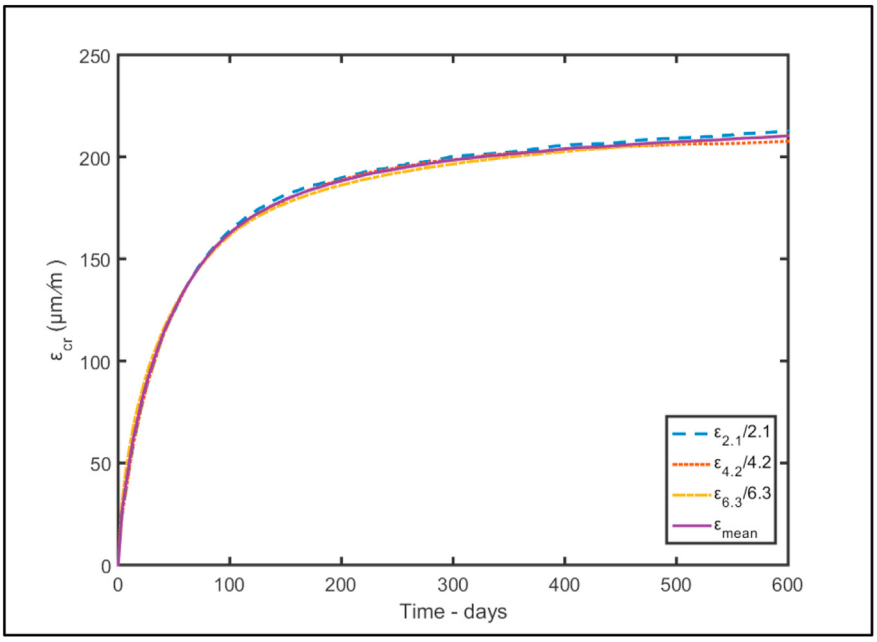

Fig. 5 – Normalized creep curves from Fig. 4 and their mean curve.

with $C_\beta = \eta$. Fig. 3 represents such materials. Fig. 3c symbol for viscoelastic materials is called a springpot or just pot. It is also called the Scot-Blair model by Mainardi et al. [5], to credit these scientists who proposed this constitutive equation for viscoelastic materials in the middle of the last century. Thus, the rheological models from Fig. 1 can be improved by replacing the spring and dashpot with the pot.

5. Modelling creep curves using fractional models

5.1. *Modelling creep curves of concrete*

Fig. 4 shows experimental creep curves of concrete, obtained from Leet [15]. Notice how the creep strains are non-negligible for this material even at room temperatures. Fig. 5 shows Fig. 4 creep curves after proper normalization (obtained by dividing them by their applied stresses), which show that this concrete creep strains are indeed linearly proportional to the applied stresses. Therefore, their mean curve can be used as a reference for obtaining its appropriate rheological model.

Castro et al. [16] tried to use the Kevin-Voigt model shown in Fig. 6a to simulate these creep curves. However, this simple model cannot fit the experimental data well. They needed to use the slightly more complex model shown in Fig. 6b to obtain a proper data fit. Then, it is interesting to investigate if the fractional Kevin-Voigt model shown Fig. 6c can do better in this case.

The Transfer Function of the Kevin-Voigt model is given by

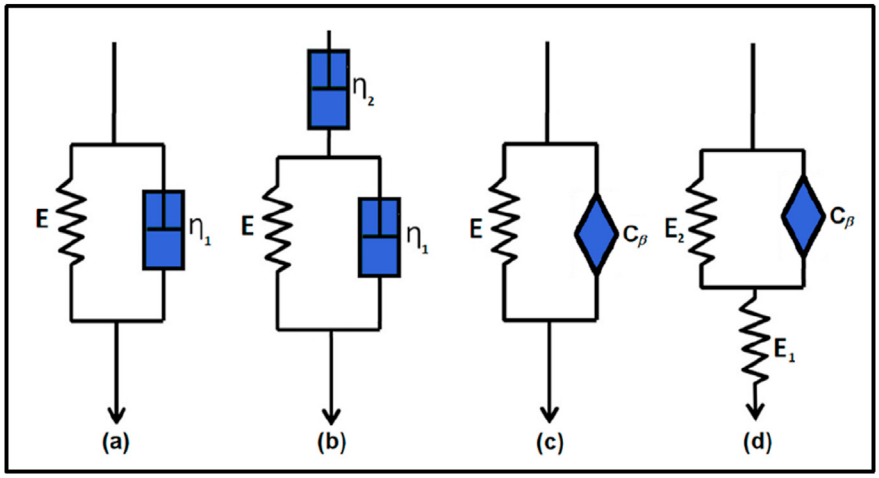

Fig. 6 – Rheological models used for the modeling of creep.

Table 1 – Parameters of the concrete calculated using the Matlab function fminsearch.

	Parameters					Error (%)
	E (GPa)	η_1 (GPa.day)	η_2 (TPa.day)	C_β(GPa.day$^\beta$)	β	
2 elements IO	4.95	282.06	–	–	–	3.55
3 elements IO	5.61	239.87	16.75	–	–	1.75
2 elements FO	4.54	–	–	100.33	0.77	0.31

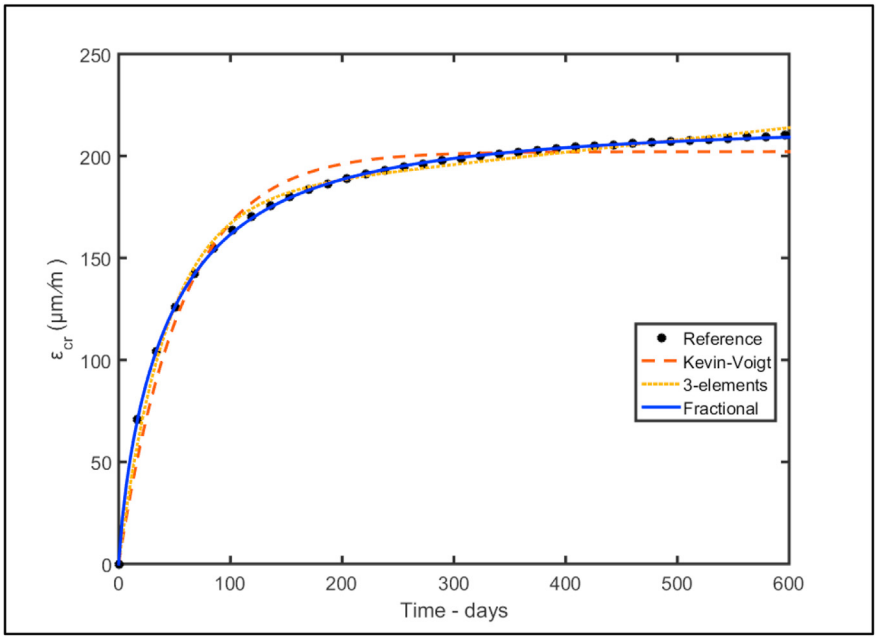

Fig. 7 – Results of the simulations compared to the reference curve.

$$\varepsilon_{cr}(s) = \frac{1}{\eta_1 \cdot s + E}\sigma(s) \tag{26}$$

The Kevin-Voigt in series with dashpot has the Transfer Function given by

$$\varepsilon_{cr}(s) = \frac{(\eta_1 + \eta_2)\cdot s + E}{\eta_2 \cdot s \cdot (\eta_1 \cdot s + E)}\sigma(s) \tag{27}$$

Since Laplace Transform rules remain valid for fractional

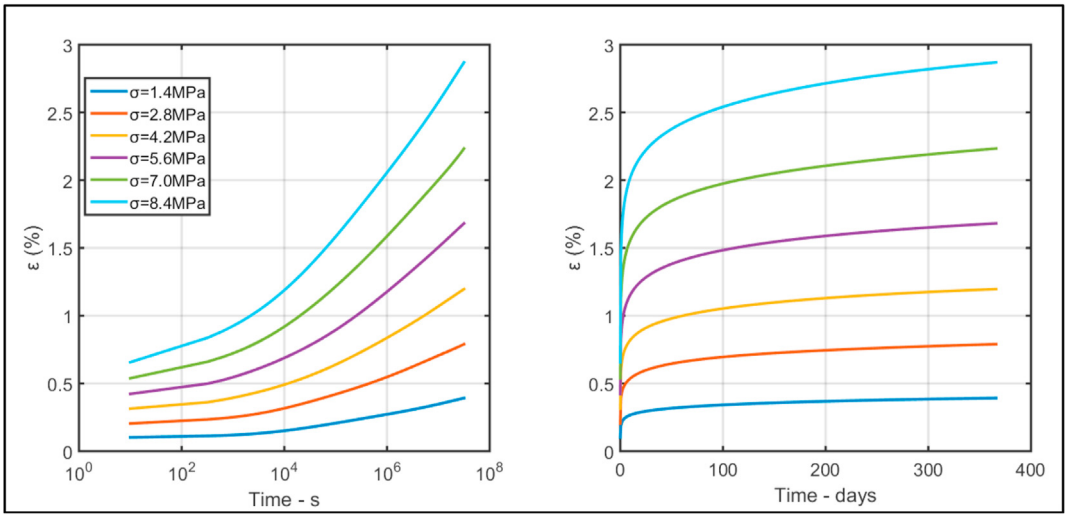

Fig. 8 – Polypropylene (PP) $\varepsilon \times t$ creep curves measured at $\Theta = 20\ ^\circ C$ [18].

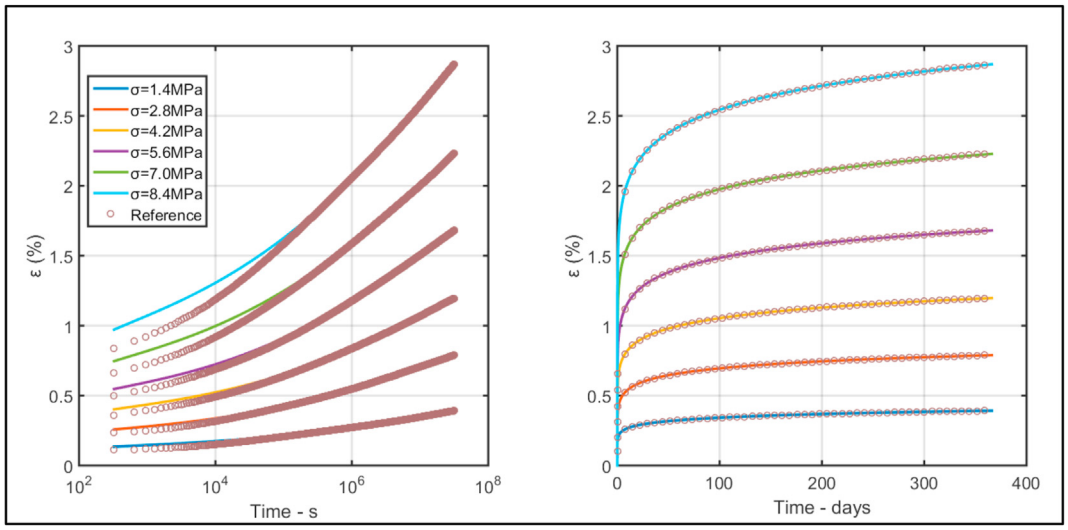

Fig. 9 – Simulated Polypropylene (PP) ε × t creep curves measured at Θ = 20 °C.

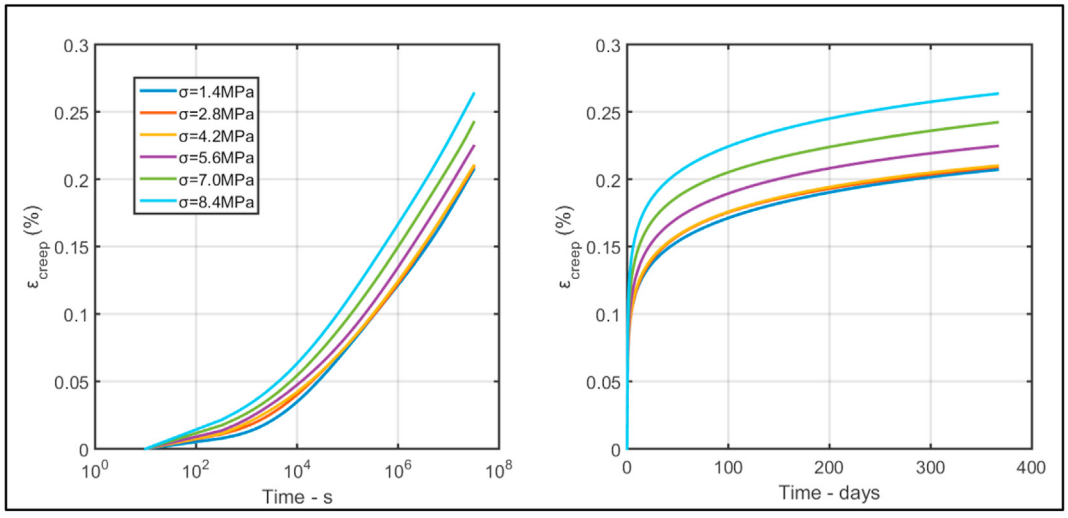

Fig. 10 – Normalized Polypropylene (PP) ε × t creep curves measured at Θ = 20 °C.

differential equations, the Fractional Kevin-Voigt model has the Transfer Function shown in Eq. (28).

$$\varepsilon_{cr}(s) = \frac{1}{E + C_\beta \cdot s^\beta}\sigma(s) \tag{28}$$

The reference creep curve is the unit step response and the identification of the parameters is obtained minimizing the function shown in Eq. (29) using the Matlab *fminsearch* function.

$$J_\varepsilon = \sum_{i=1}^{N} [\varepsilon_{actual}(i) - \varepsilon_{simulated}(i)]^2 \tag{29}$$

The fractional Transfer Function can be simulated using the FOMCOM [17] toolbox for Matlab. The results obtained for the proposed models are shown in Table 1. These results show

Table 2 – PP parameters calculated using the Matlab function fminsearch.

Load (MPa)	Parameters				Error (%)
	E_1 (GPa)	E_2 (GPa)	C_β (GPa.day$^\beta$)	β	
1.4	1.37	0.18	1.22	0.22	0.27
2.8	1.37	0.25	1.12	0.26	0.15
4.2	1.34	0.23	1.16	0.24	0.17
5.6	1.32	0.24	1.05	0.25	0.16
7.0	1.30	0.20	0.90	0.22	0.23
8.4	1.28	0.17	0.79	0.20	0.21

Table 3 – Parameters of the PP in the linear range.

Load (MPa)	Parameters				Error (%)
	E_1 (GPa)	E_2 (GPa)	C_β (GPa.day$^\beta$)	β	
1.4	1.36	0.22	1.19	0.24	0.94
2.8					0.29
4.2					0.63

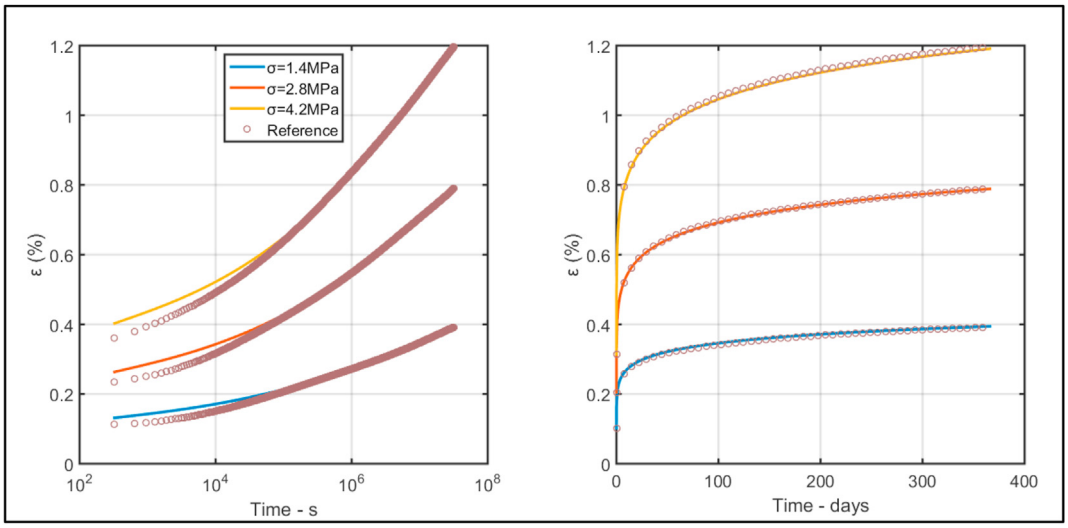

Fig. 11 – Simulated Polypropylene (PP) $\varepsilon \times t$ creep curves in the linear range measured at $\Theta = 20\ ^{\circ}C$.

that the fractional Kevin-Voigt presents the better results. The error has been calculated using Eq. (30). Fig. 7 shows the results of the simulation of the proposed models, compared to the reference curve.

$$E_{\varepsilon} = \sqrt{\frac{\sum_{i=1}^{N}\left[\varepsilon_{actual}(i) - \varepsilon_{simulated}(i)\right]^2}{\sum_{i=1}^{N}\left[\varepsilon_{actual}(i)\right]^2}} \times 100 \tag{30}$$

5.2. *Modeling creep curves of polypropylene (PP)*

Polypropylene (PP) is a tough and relatively cheap thermoplastic polymer used in many applications because of its typical basic properties and a reputation of good fatigue performance. Fig. 8 shows the creep curves of (PP) at $\Theta = 20°C$ for a period of 365 days obtained by Crawford [18]. Notice how much PP can creep under relatively low stresses even at room temperature.

The deformation of the PP is modeled using the Kevin-Voigt Transfer Function shown in Fig. 6d, which has a spring in series to account for the initial deformation and given by Eq. (31).

$$\varepsilon(s) = \varepsilon_0(s) + \varepsilon_{cr}(s) = \left(\frac{1}{E_1} + \frac{1}{E_2 + C_{\beta} \cdot s^{\beta}}\right)\sigma(s) \tag{31}$$

The PP creep parameters are obtained similarly. The results are shown in Table 2.

Fig. 9 shows the simulated and measured curves, with an error of about 12% during the first of the 365 days of the data.

The creep behavior of the PP can be considered linear viscoelastic under low stress. Fig. 10 shows the normalized curve where the curves for $\sigma \leq 4.2$MPa are equivalent. Table 3 shows the parameters for a linear model calculated using the mean curve among the curves of $\sigma =$ 1.4, 2.8 and 4.2MPa.

The simulated curves are shown in Fig. 11, compared to the actual curves. An error of about 12% can be verified during the first of the 365 days of the data.

6. Conclusions

Structural materials in general exhibit a creep behavior that is best modeled using a power function. The classical rheological models used extensively by structural engineers lead to exponential functions, which adjust the real behavior to a limited extent. Models based on fractional calculus techniques are potentially much more suitable, as they naturally lead to power functions. Despite this, they are still not used in structural analysis. One of the reasons may be associated with the difficulty of physically interpreting the fractional derivatives of the quantities involved in the process. Another reason may be the lack of knowledge on this subject, which normally still is not addressed in undergraduate or even in graduate calculus courses. Nevertheless, the various published studies and computer codes already available for solving fractional differential equations show that Fractional calculus is a mature enough subject to be used more often in structural engineering analyses. This work has shown that fractional rheological models with just one single element can indeed describe the viscoelastic behavior of two very different materials, a polymer (polypropylene) and a low strength concrete. In fact, such a simple fractional model was able to describe their creep data even better than more complex traditional models that need several springs and dashpots to properly fit it.

Declaration of Competing Interest

The authors declare that they have no significant competing financial, professional, or personal interests that might have

influenced the performance or presentation of the work described in this manuscript.

Acknowledgement

"This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001"

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